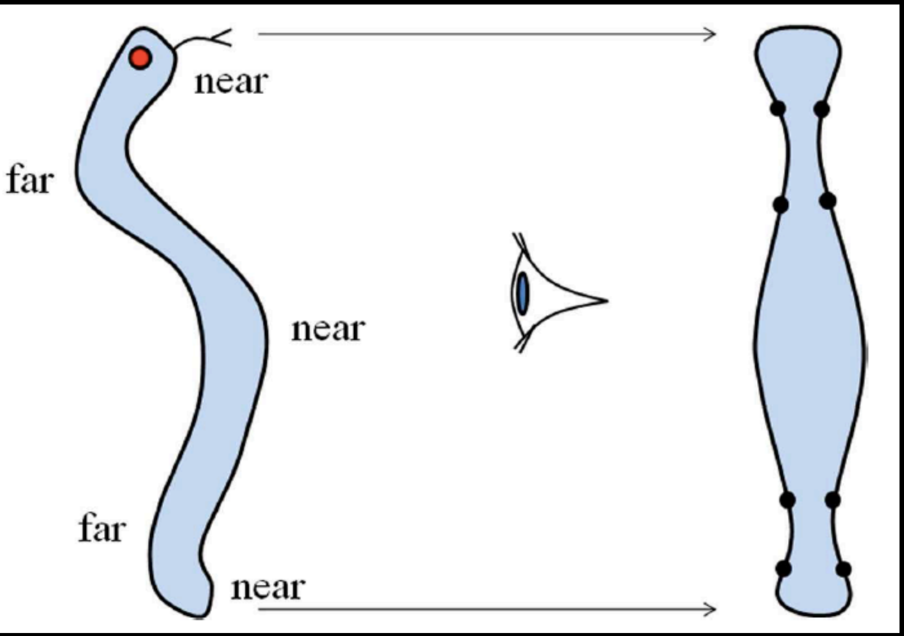
On Image Contours of Projective Shapes

Jean Ponce
Département d'Informatique
Ecole Normale Supérieure

Martial Hebert
Robotics Institute
Carnegie-Mellon University

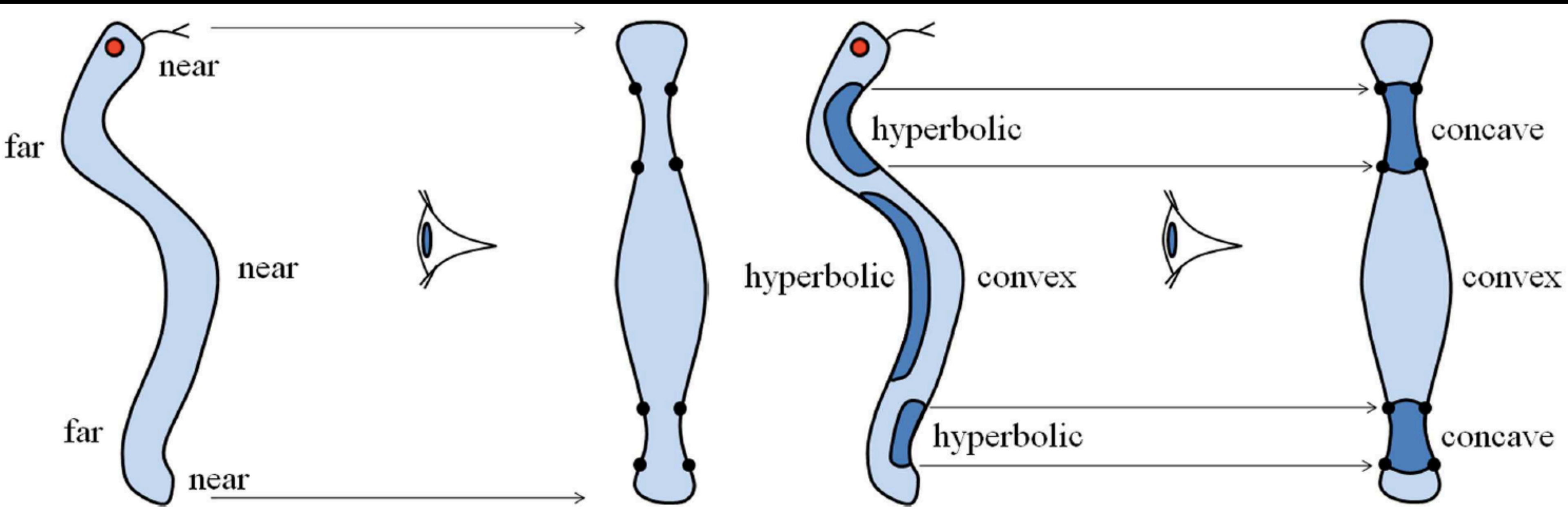
What does the occluding contour tell us about shape?

Nothing (Marr'77) ?



What does the occluding contour tell us about shape?

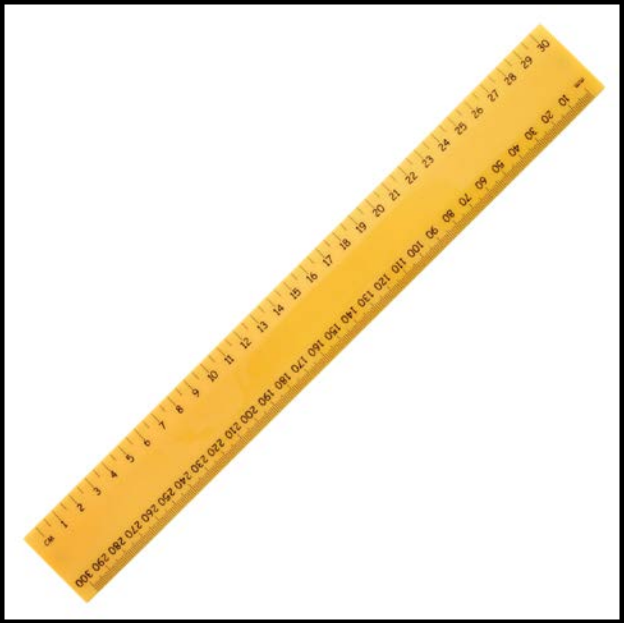
Nothing (Marr'77) ?

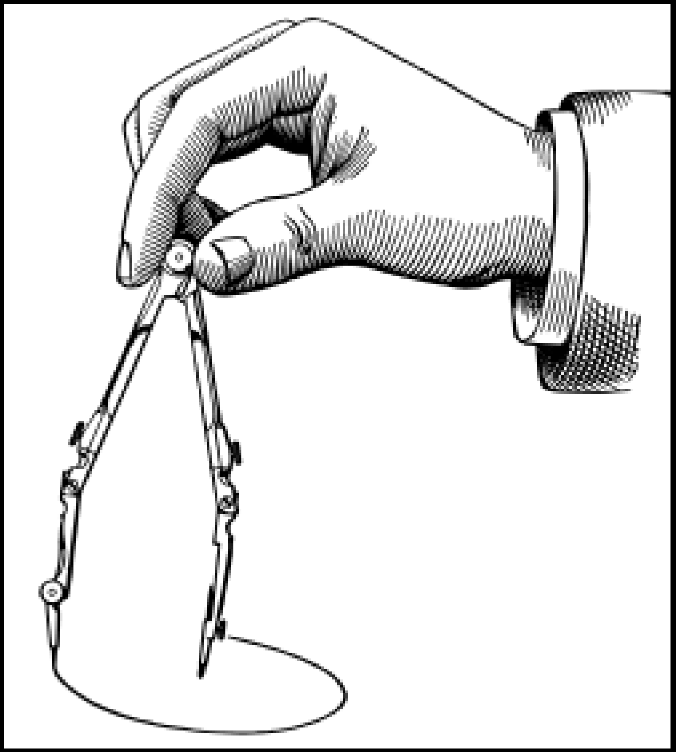


Or rather quite a bit
(Koenderink'84) ?

(Arbogast & Mohr'91;Cipolla & Blake'92;Vaillant & Faugeras'92;Boyer & Berger'96;Lazebnik'02)

Are quantitative measurements necessary? No!

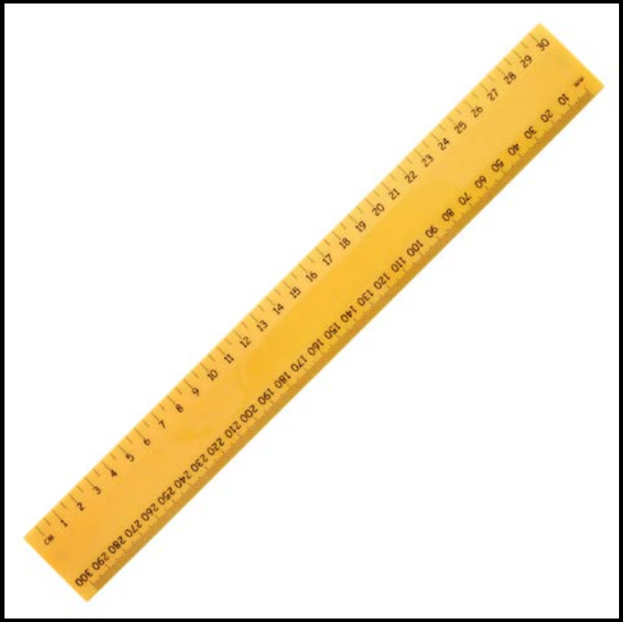


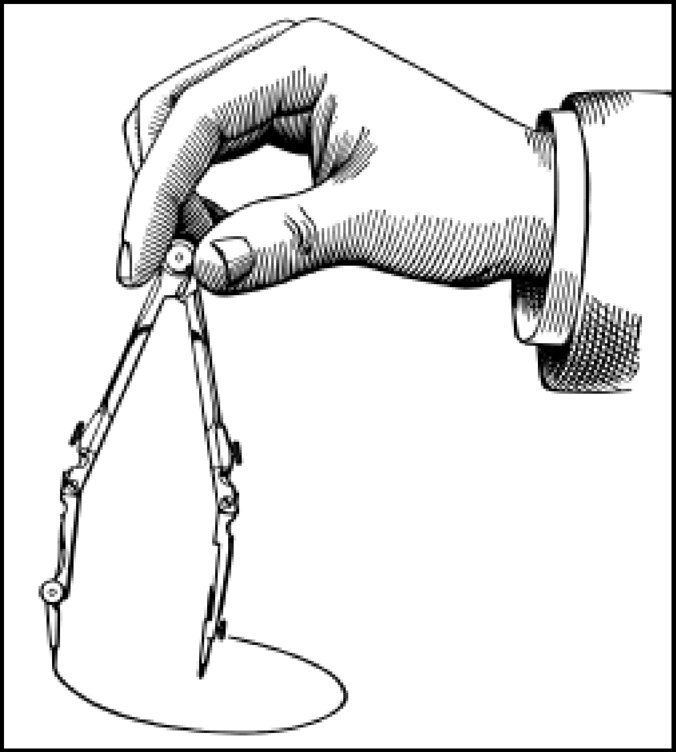


- Move from Euclidean to projective geometry.
- Thus also give up orientation and convexity.

[See (Lazebnik'02; Lazebnik et al.'05) for the oriented projective case.]

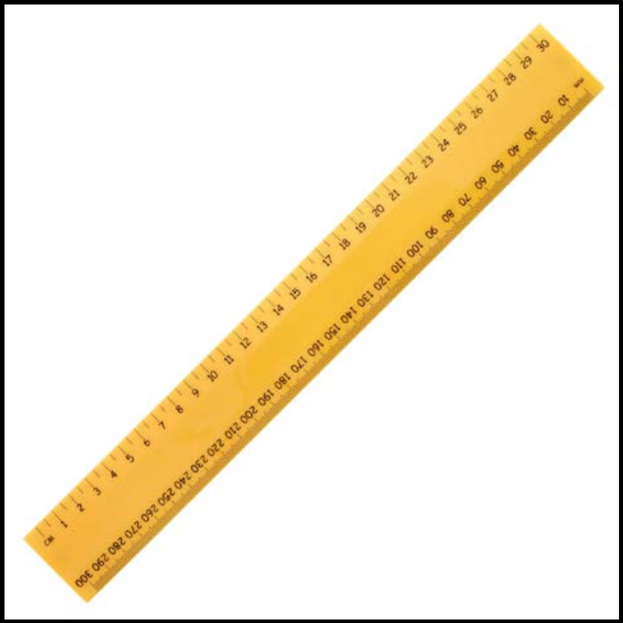
Projective geometry

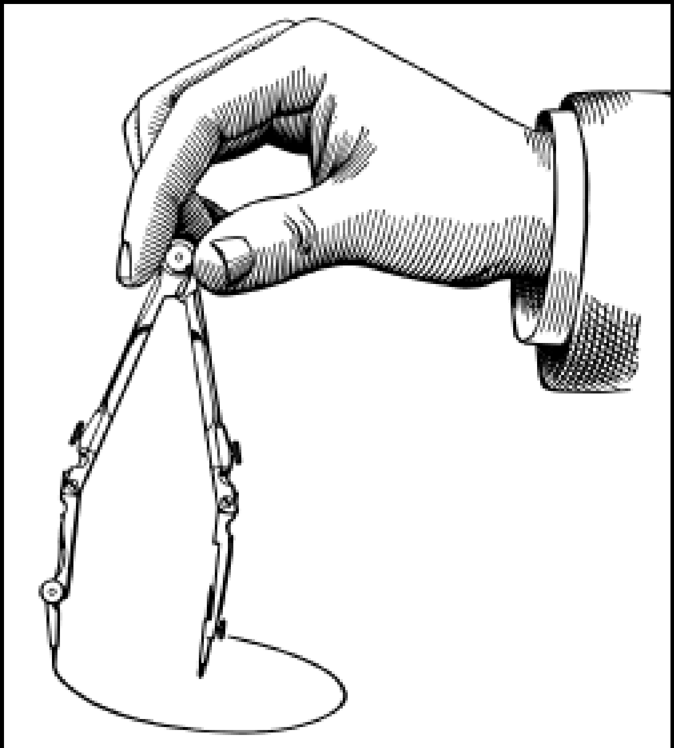




- Not just an analytical device for linearizing SFM.
- A natural framework for studying the relationship between solid shapes and their projections.

Why should we care?

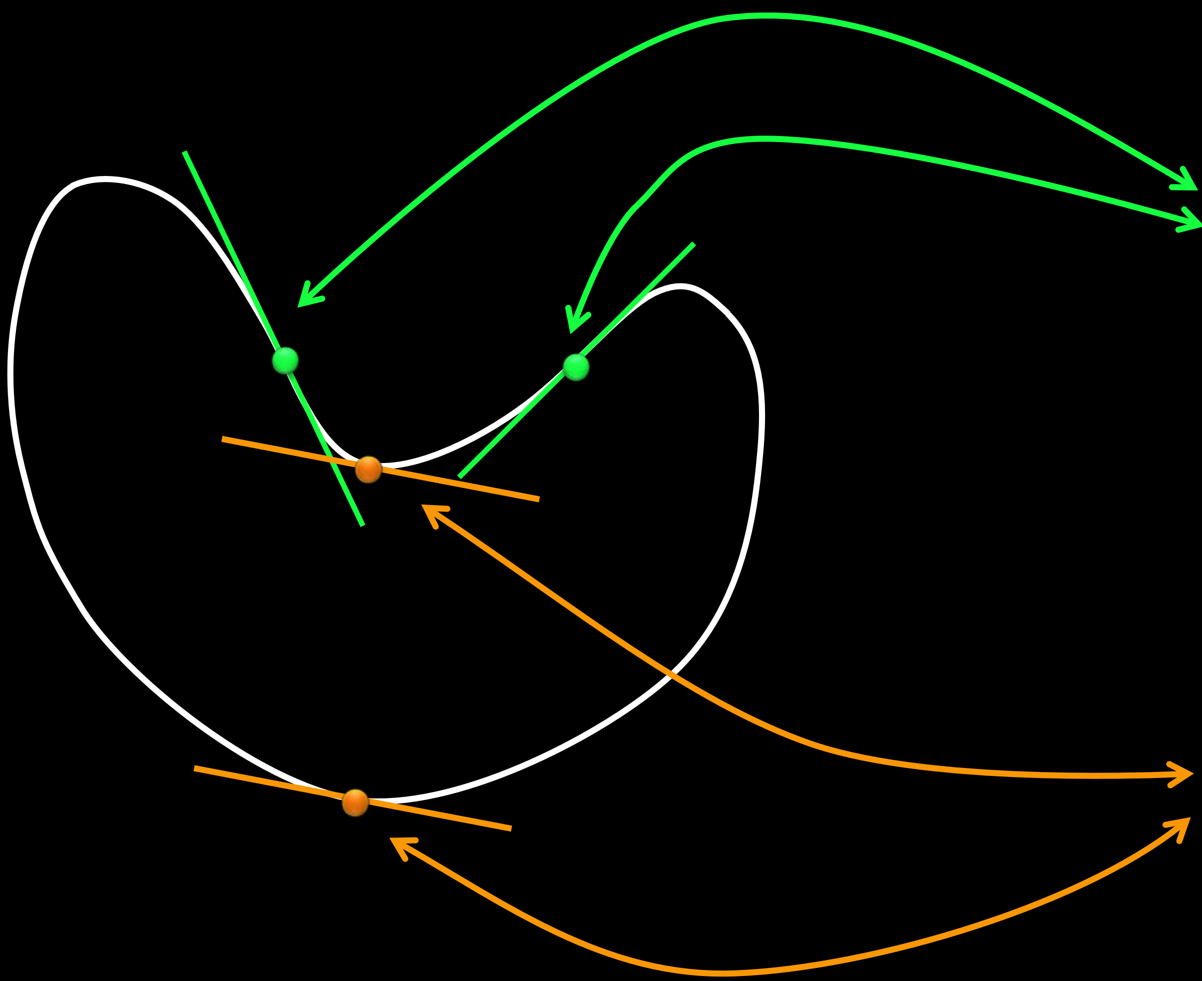




- To understand the optical principles of computer vision.
- To understand those of human visual perception.

[The influence of Koenderink and Van Doorn should be obvious.]

The local projective shape of smooth curves

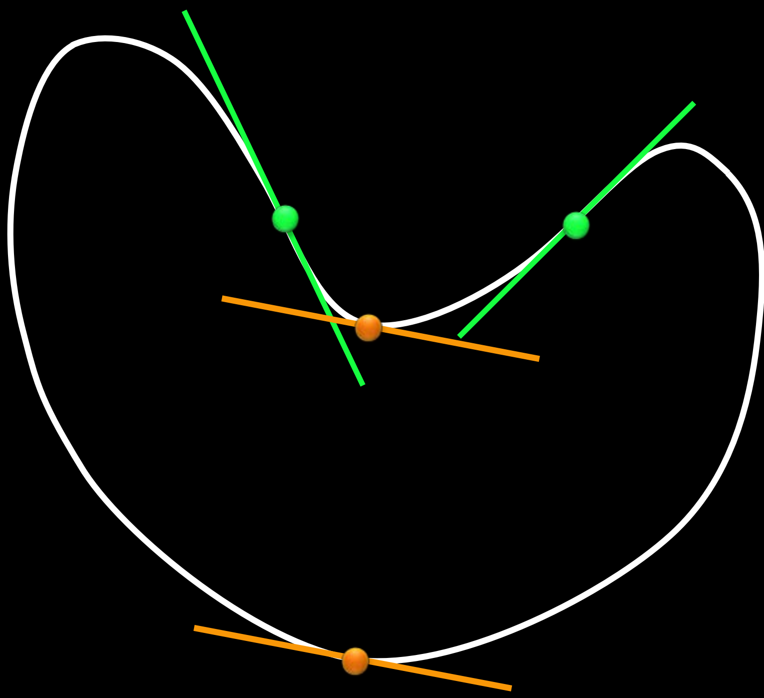


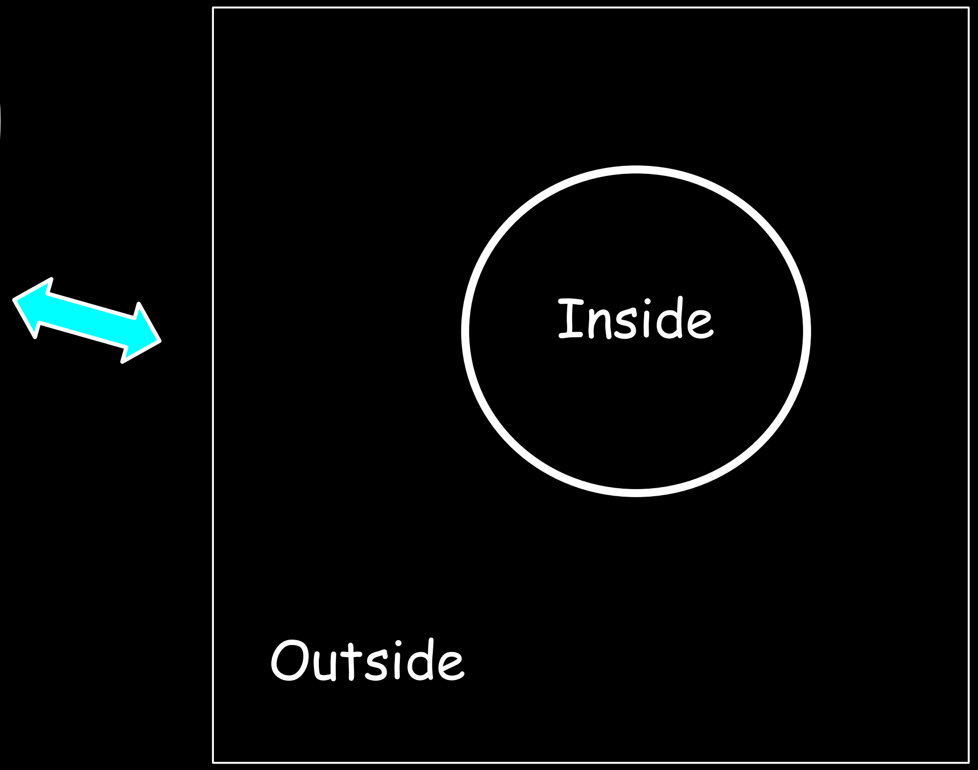
Singular points

- Inflections
- Cusps of the 1st kind
- Cusps of the 2nd kind

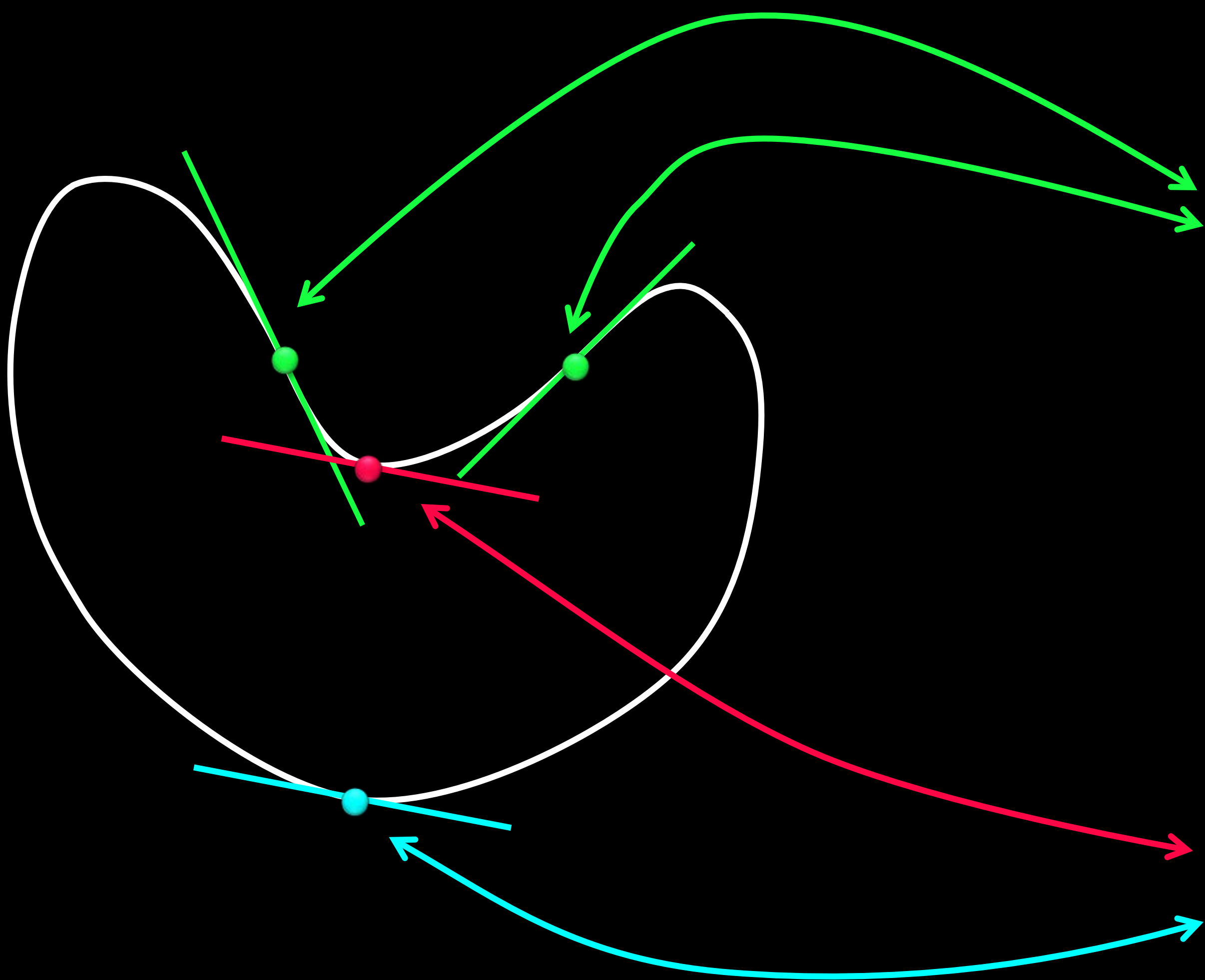
Regular points

Smooth curves bounding solid regions have a well defined inside and outside





Smooth curves bounding solid regions have a well defined inside and outside



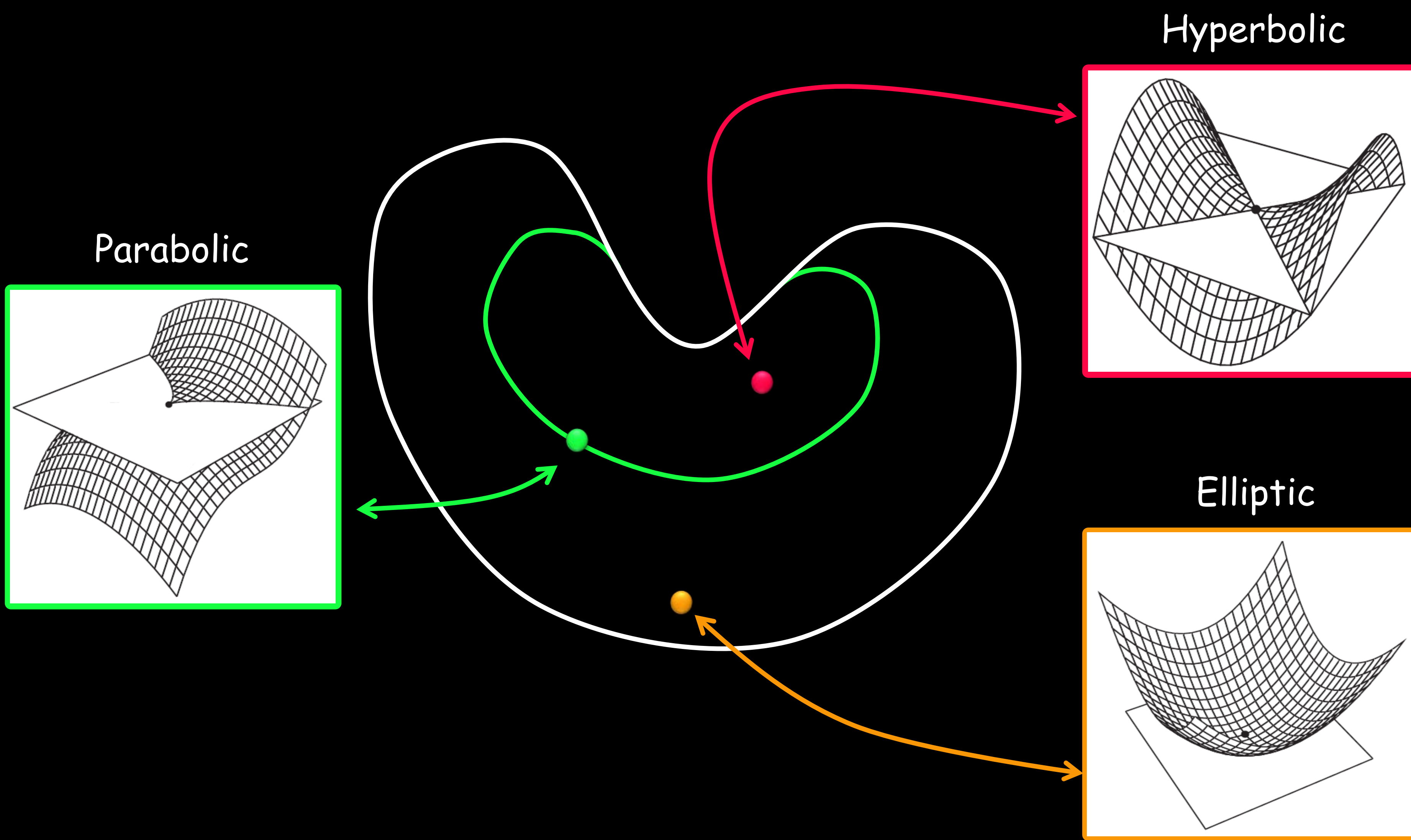
Singular points

- Inflections
- Cusps of the 1^{st} kind
- Cusps of the 2^{nd} kind

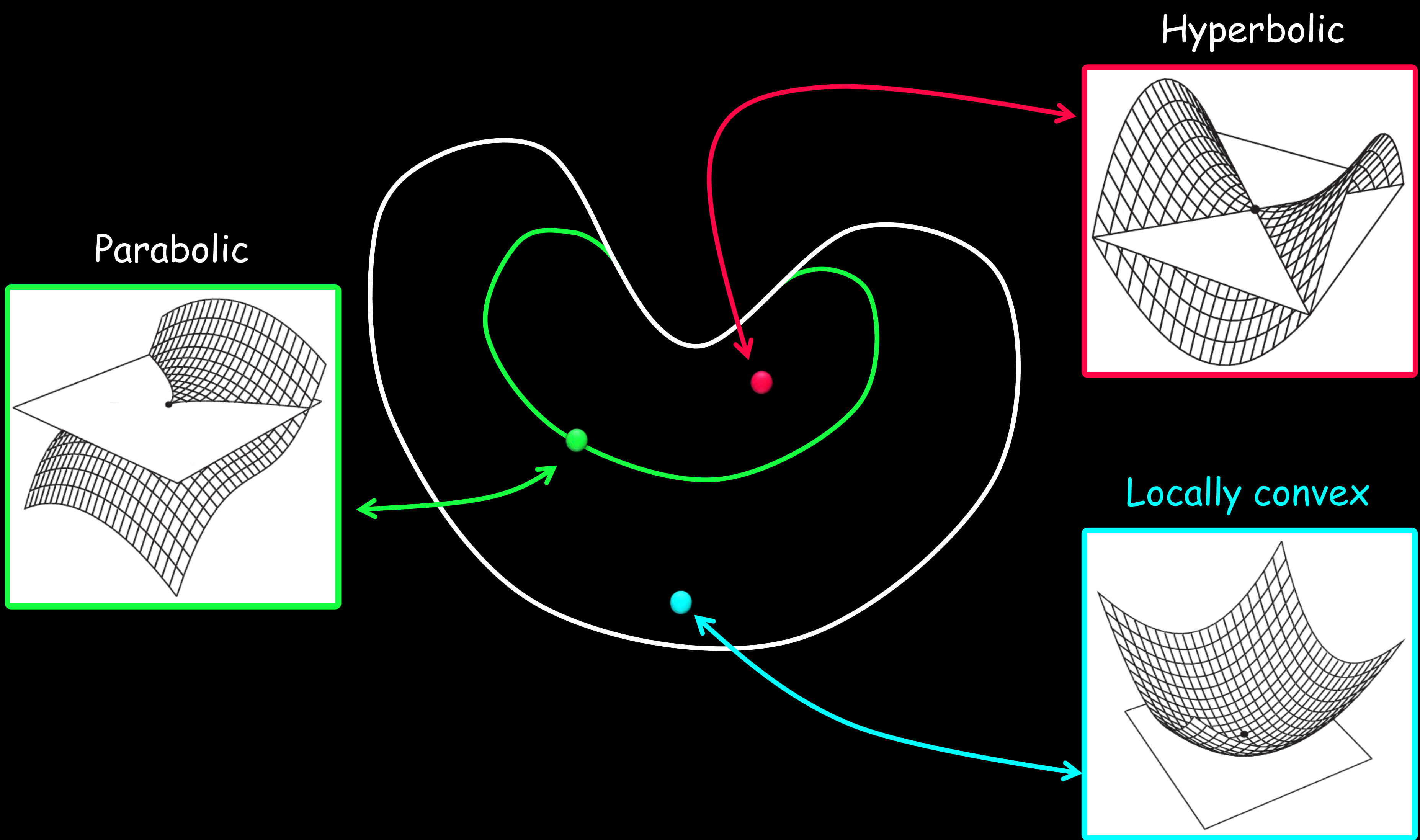
Regular points

- Locally concave
- Locally convex

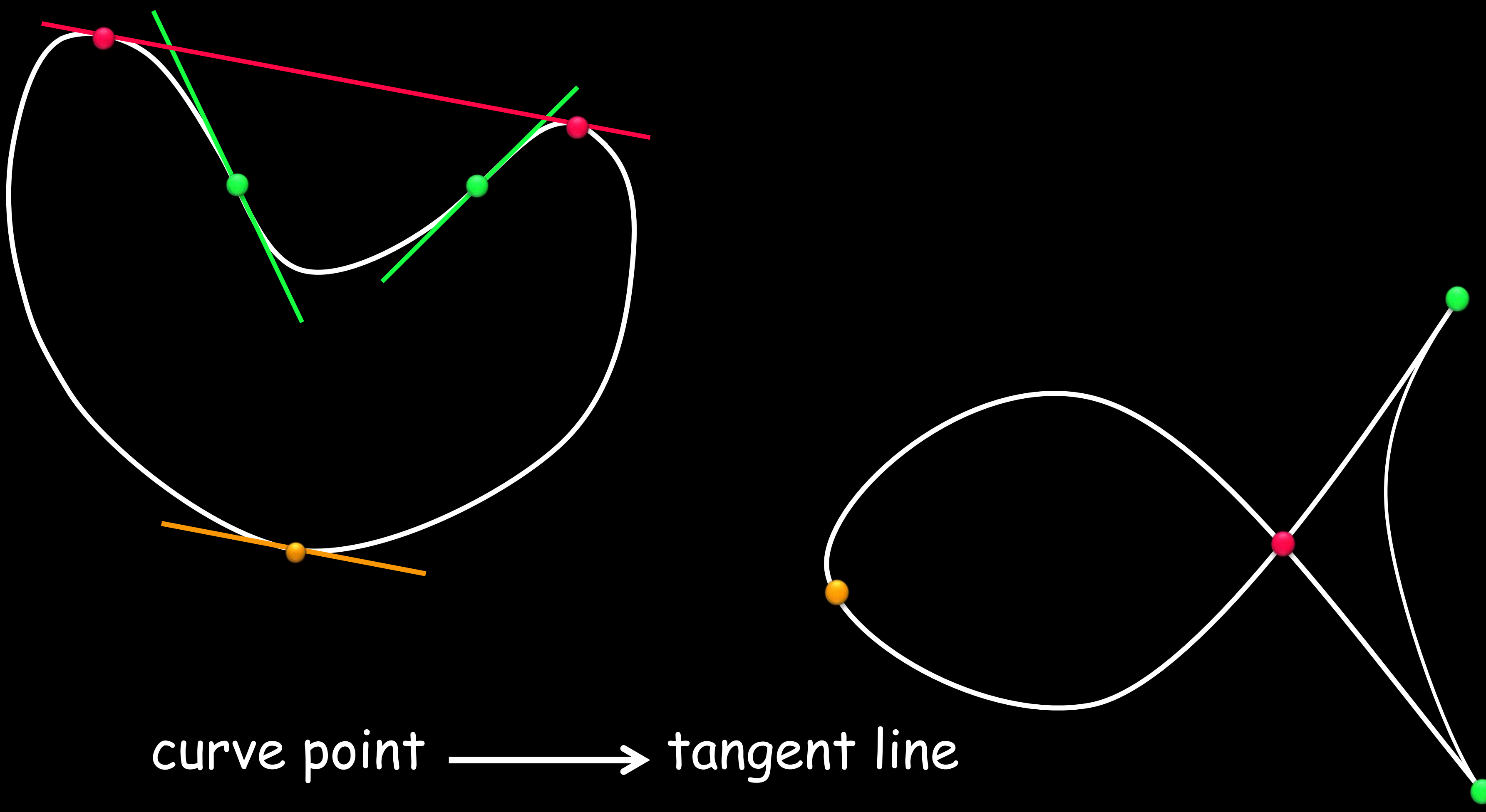
The local projective shape of smooth surfaces



Smooth surfaces bounding solid 3D shapes have a well defined inside and outside

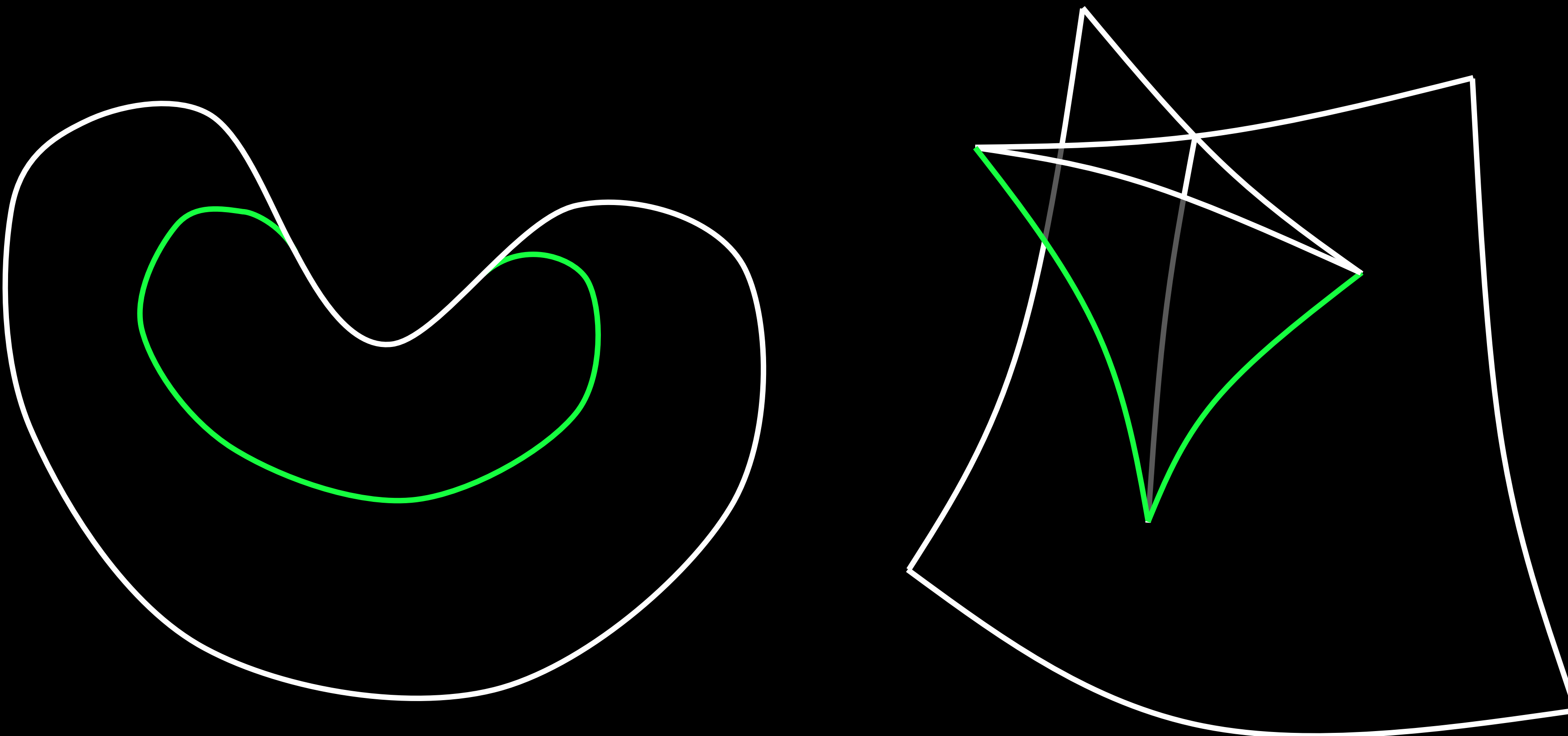


The projective Gauss map
primal → dual



curve point ⟶ tangent line

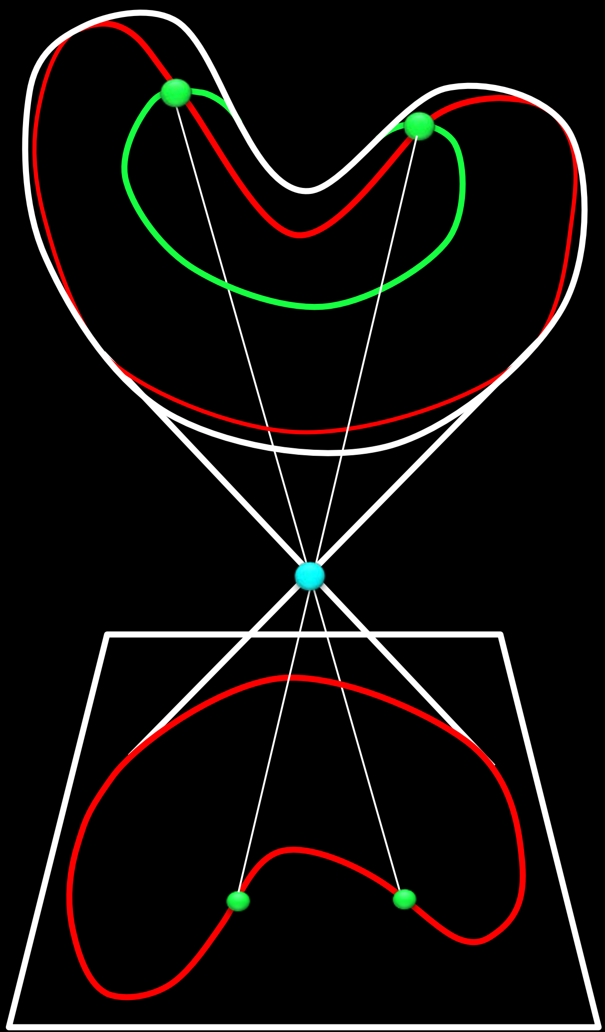
The projective Gauss map
primal → dual



surface point ⟶ tangent plane

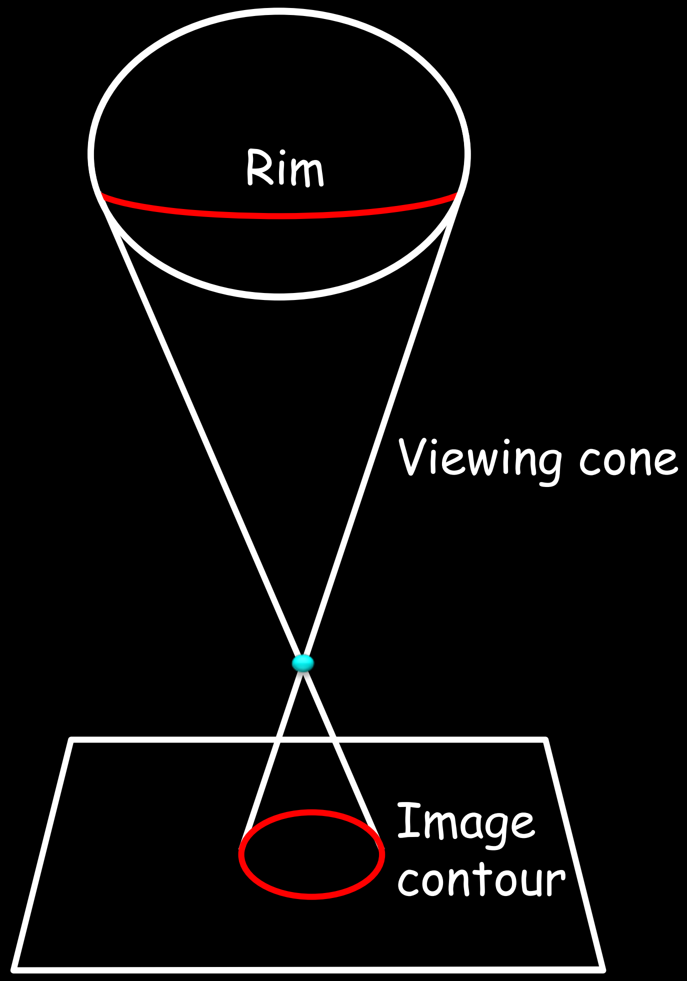
(Bruce'81; McCrory & Shiffrin'84)

Theorem 1. *Under perspective projection, the inflections of the image contour of a smooth surface are the images of parabolic points.* *(Koenderink'84)*



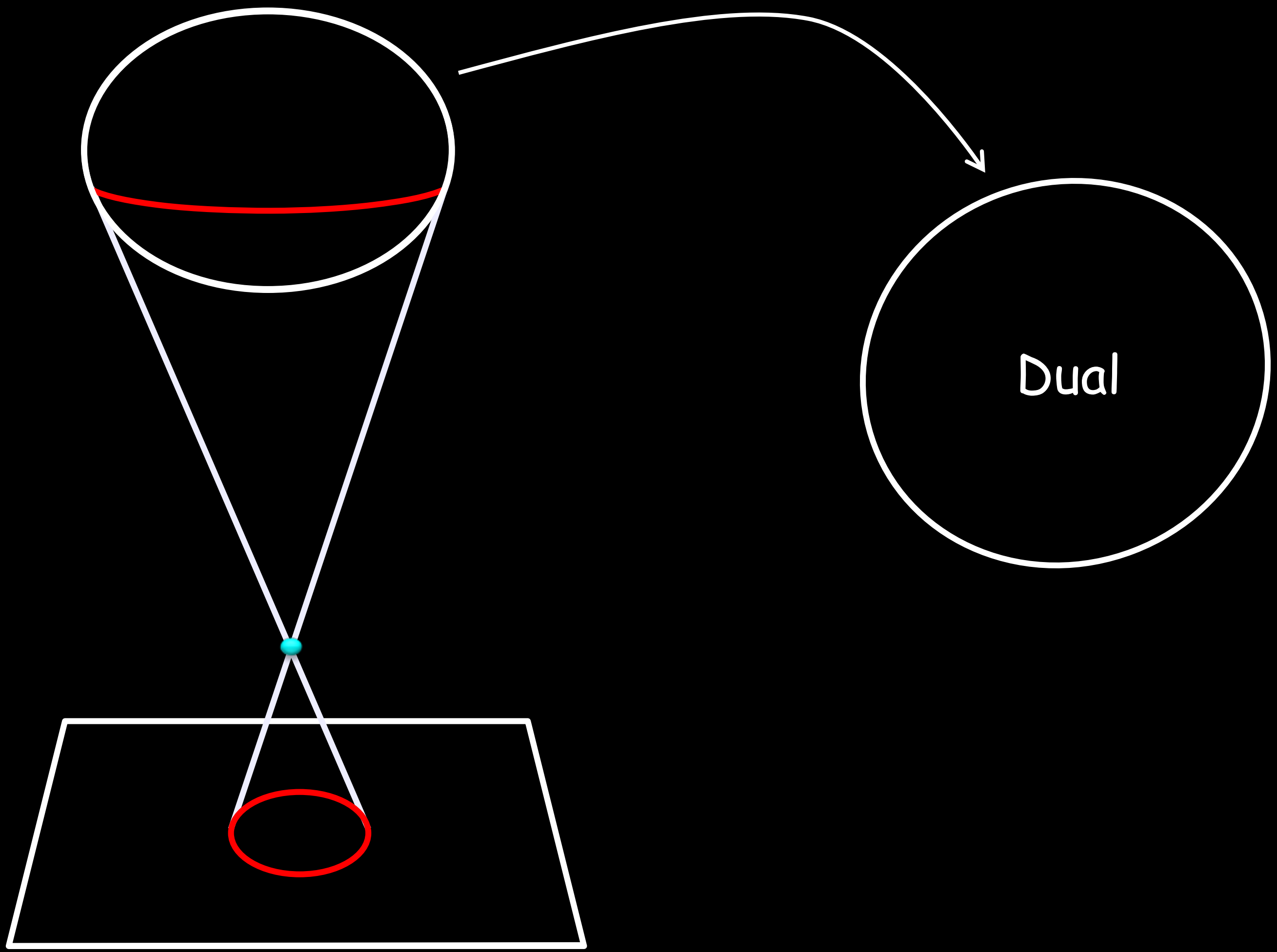
Theorem 1. *Under perspective projection, the inflections of the image contour of a smooth surface are the images of parabolic points.* *(Koenderink'84)*

Proof:



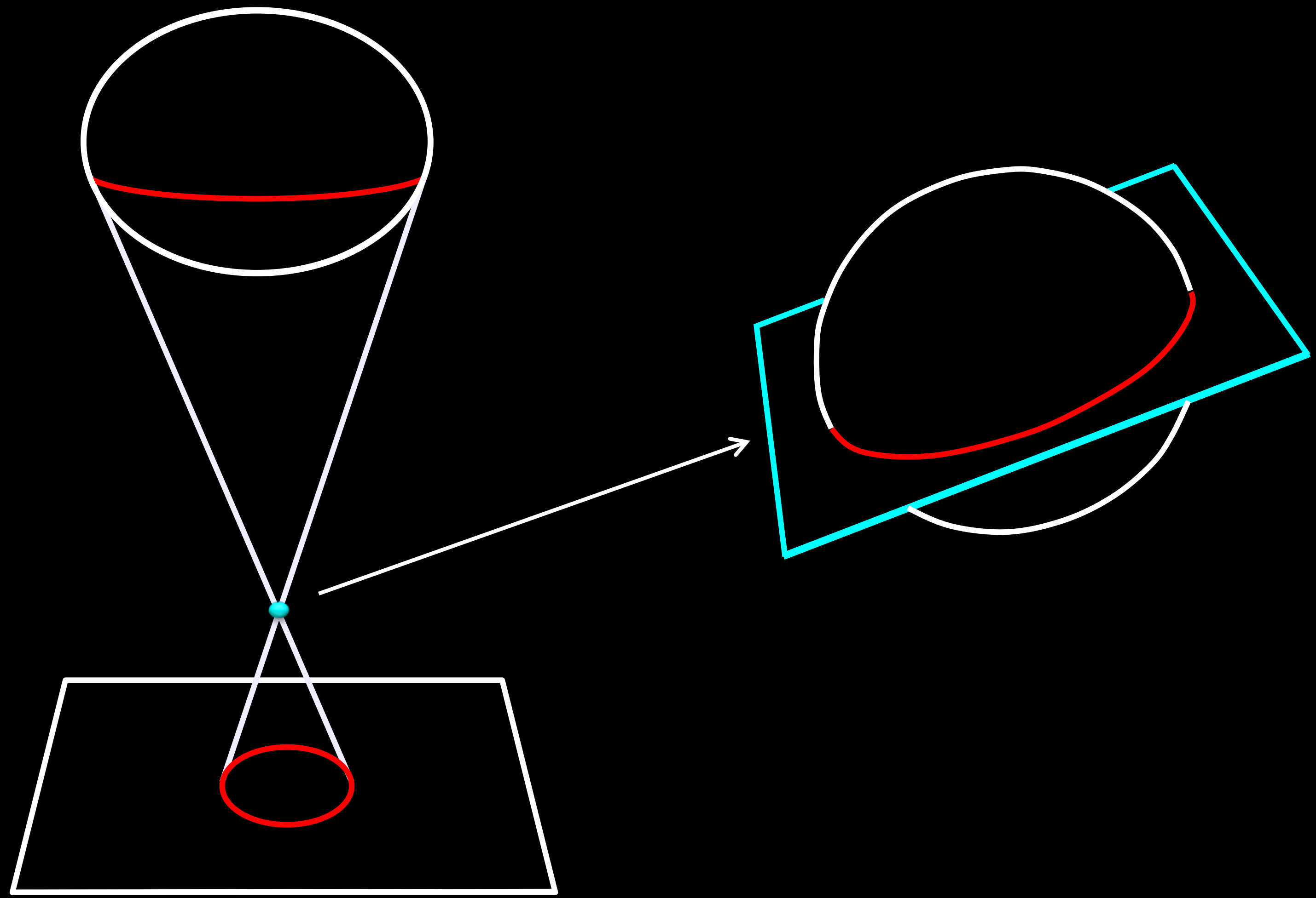
Theorem 1. *Under perspective projection, the inflections of the image contour of a smooth surface are the images of parabolic points.* *(Koenderink'84)*

Proof:



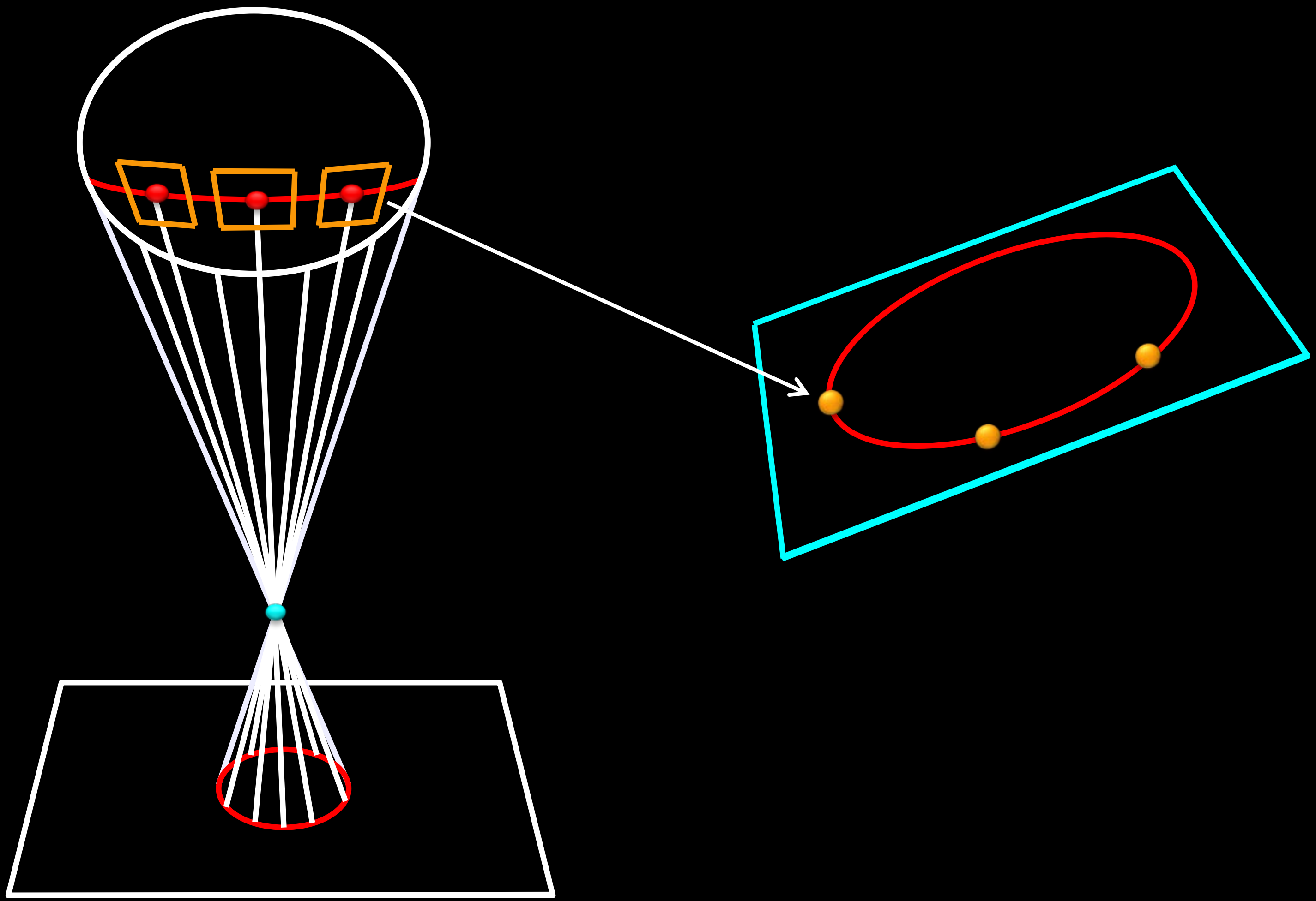
Theorem 1. *Under perspective projection, the inflections of the image contour of a smooth surface are the images of parabolic points.* *(Koenderink'84)*

Proof:



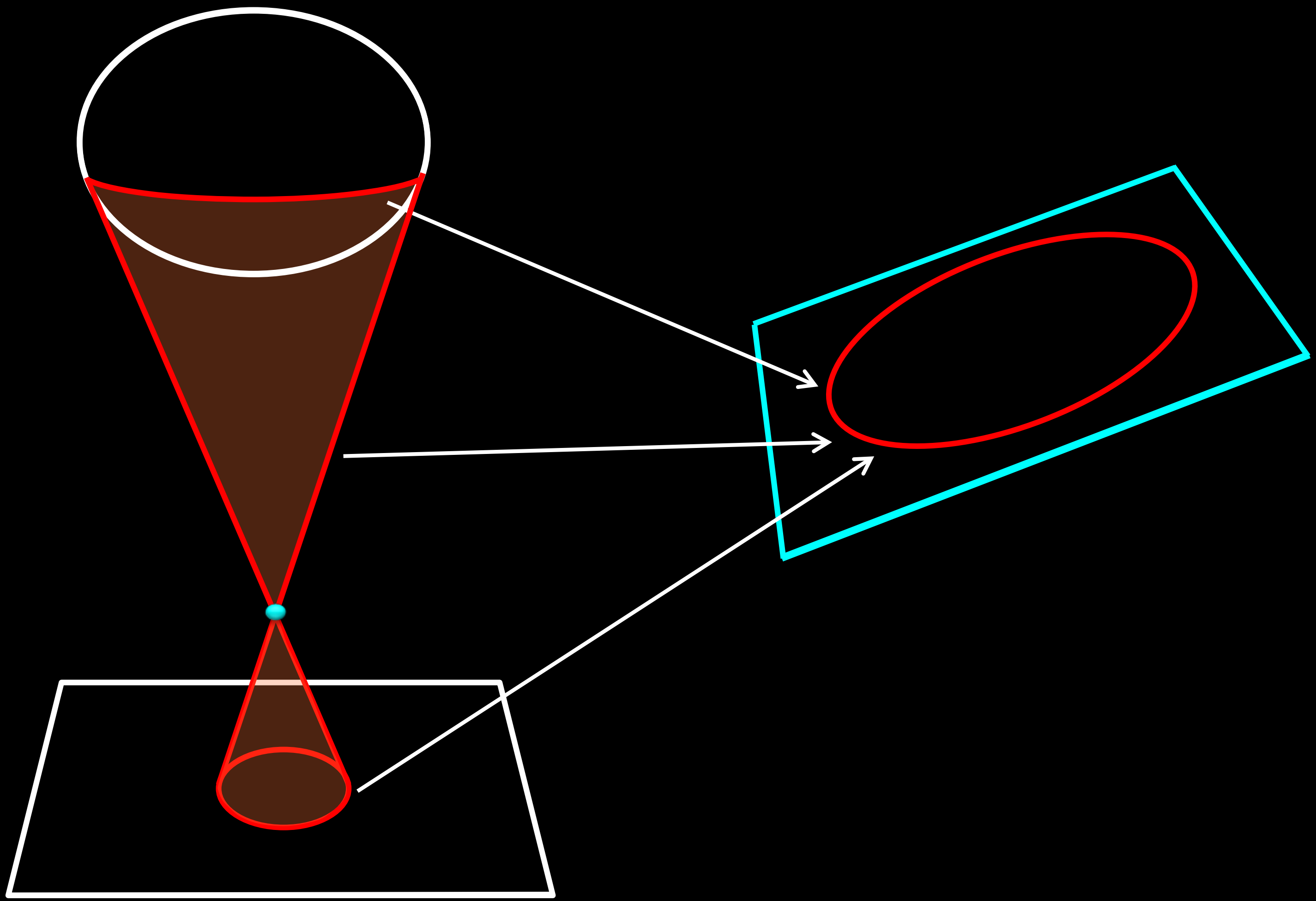
Theorem 1. *Under perspective projection, the inflections of the image contour of a smooth surface are the images of parabolic points.* *(Koenderink'84)*

Proof:



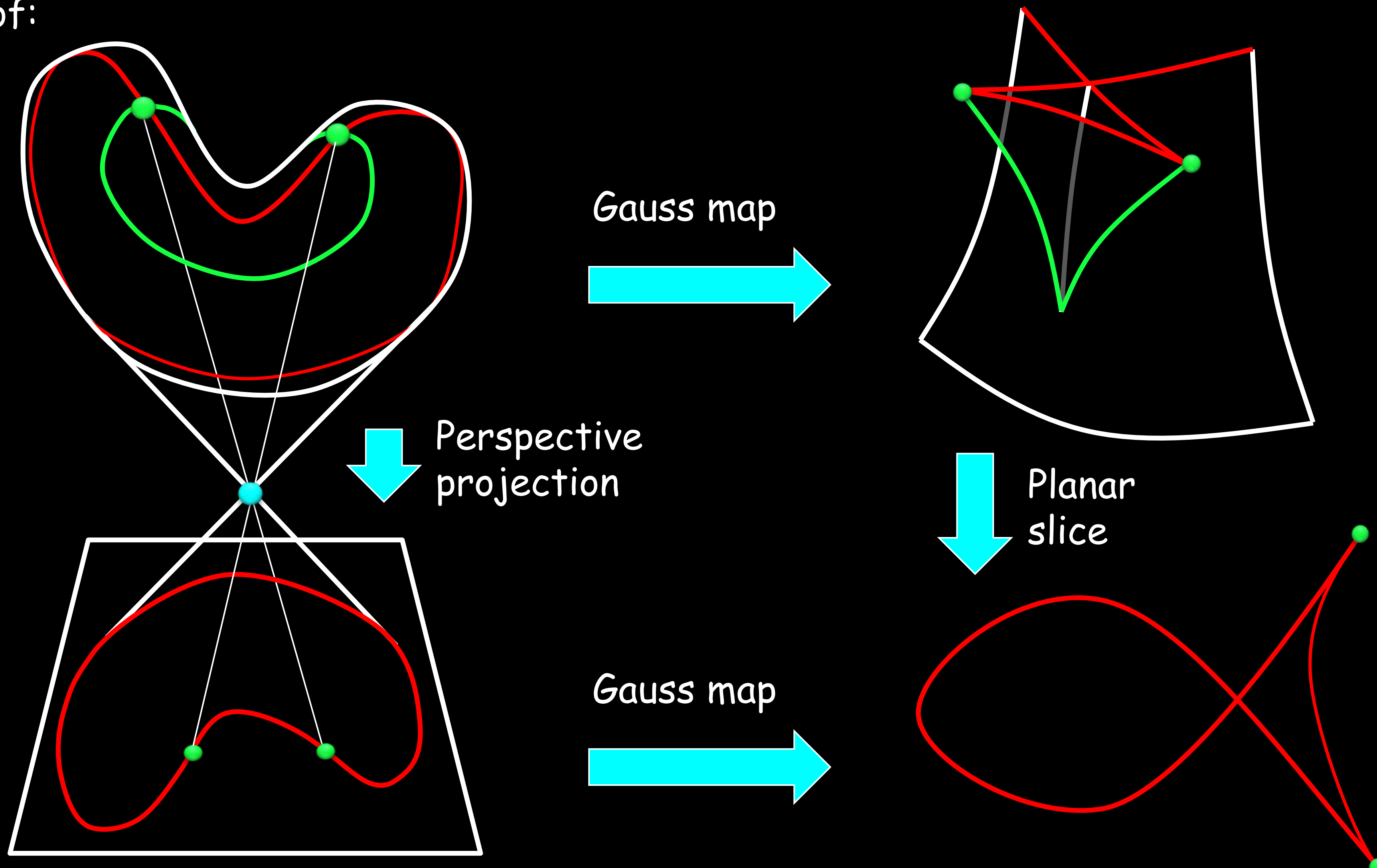
Theorem 1. *Under perspective projection, the inflections of the image contour of a smooth surface are the images of parabolic points.* *(Koenderink'84)*

Proof:

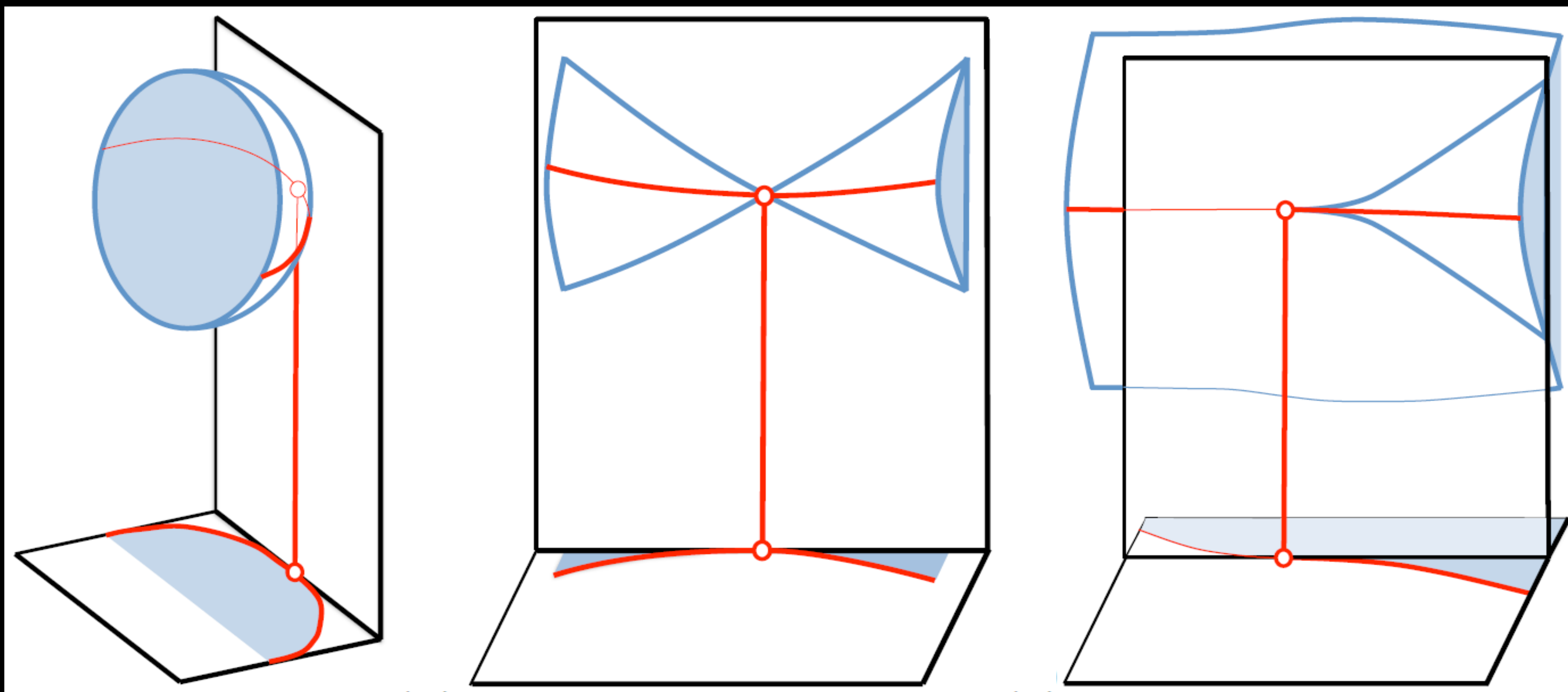


Theorem 1. *Under perspective projection, the inflections of the image contour of a smooth surface are the images of parabolic points.*

Proof:



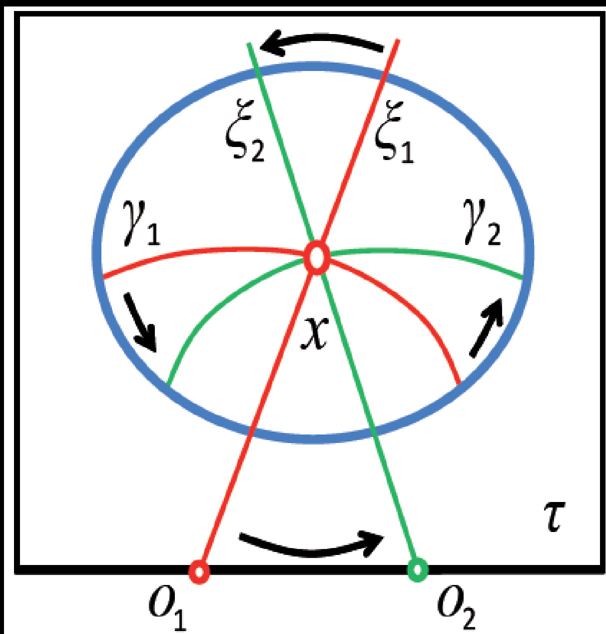
Theorem 2. *Under perspective projection, the convexities of the image contour of an oriented smooth surface are the images of convex points, its concavities are the projections of hyperbolic points, and its inflections are the images of parabolic points.*

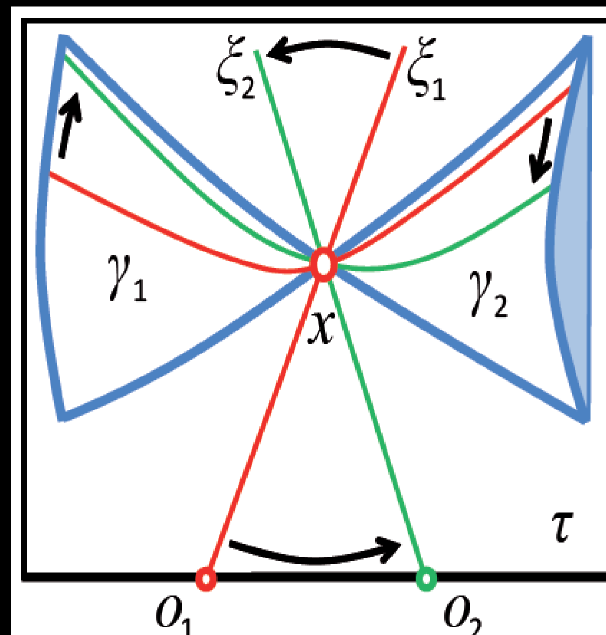


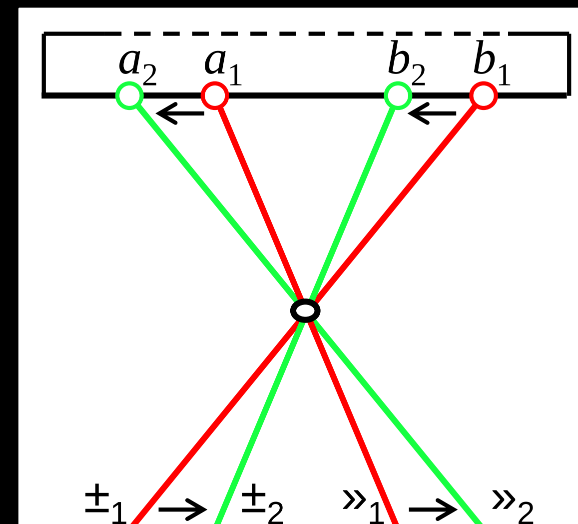
Proof: Reason about the tangent plane/line lying inside/outside the viewing cone.

(Koenderink'84; Lazebnik'02; Lazebnik et al.'05)

Theorem 3. *The rim turns in the same direction as the viewpoint in the tangent plane of a smooth oriented surface at a convex point, and in opposite ones at a hyperbolic point.*



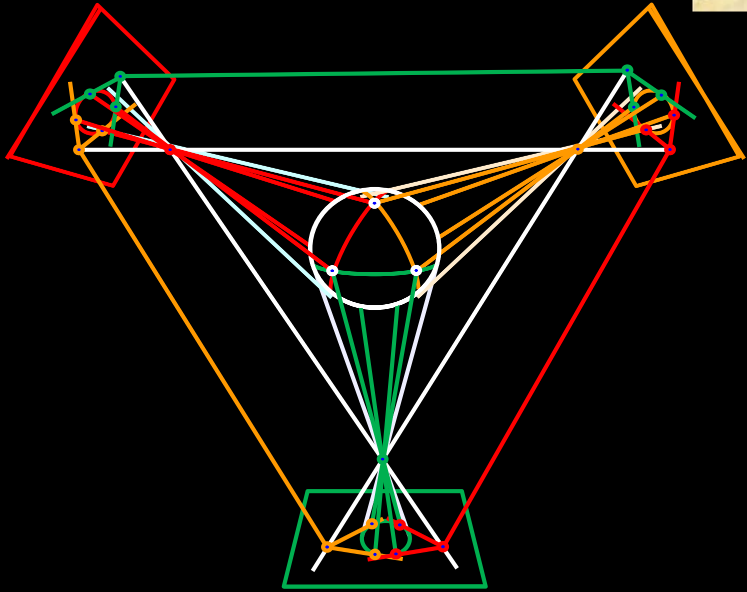




Proof: Reason about order & separability.

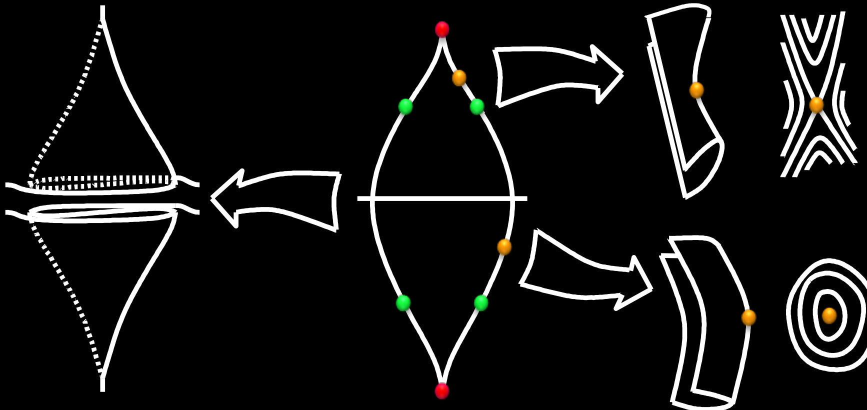
(Classical result for large motions in Euclidean differential geometry.)

Going further



3D models:
- rim meshes
- visual hulls

(Baumgart'74; Matusik et al.'97)
(Lazebnik et al.'01; Franco & Boyer'03)
(Lazebnik et al.'07)



Dynamic contours:
- *the geometry of the Gauss map*
- aspect graphs

(Koenderink & Van Doorn'76)
(Bruce'81; Arnold'83; Platonova'84)
(Banchoff et al.'82; McCrory & Shifrin'84)