An Empirical Study of Reserve Price Optimisation in Display Advertising

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Reserve Price Optimisation

The task:

• To find the optimal reserve prices (hard floors)
Why

- Suppose it is second price auction
  - Normal case: $b_2 \geq \alpha$
  - Preferable case: $b_1 \geq \alpha > b_2$ (it increases the revenue)
  - Undesirable case: $\alpha > b_1$ (but there is risk)
An example

- Suppose: two bidders, private values drawn from Uniform[0, 1]
- Without a reserve price (or \( a = 0 \)), the payoff \( r \) is:
  \[
  r = E[\min(b_1, b_2)] = 0.33
  \]
- With \( a = 0.2 \):
  \[
  r = E[\min(b_1, b_2) | b_1 > 0.2, b_2 > 0.2] + 0.32 \times 0.2 = 0.36
  \]
- With \( a = 0.5 \):
  \[
  r = E[\min(b_1, b_2) | b_1 > 0.5, b_2 > 0.5] + 0.5 \times 0.5 = 0.42
  \]
- With \( a = 0.6 \):
  \[
  r = E[\min(b_1, b_2) | b_1 > 0.6, b_2 > 0.6] + (0.6 \times 0.4) \times 2 \times 0.6 = 0.405
  \]

Paying the second highest price  Paying the reserve price
The optimal auction theory

- In the second price auctions, advertisers bid their private values \([b_1, \ldots, b_K]\)
- Private values -> Bids’ distributions \(F(b) = F_1(b_1) \times \cdots \times F_K(b_K)\)
  - Uniform
  - Log-normal
- The publisher also has a private value \(V_p\)
- The optimal reserve price is given by:
  \[
  \alpha - \frac{1 - F(b)}{F'(b)} - V_p = 0
  \]

Questions:
- How are advertisers bidding?
- Does Uniform/Log-normal fit well?
Bidding is complicated

- They usually use a private regression model (No access to publishers)
- Perhaps they don’t even know it! (Just try to maximise the ROI)

Many advertisers bid at fixed values (Think about a decision tree)
And they come and go (with different lifetime span)
Uniform/Log-normal distributions do NOT fit well

Test at the placement level
(because we usually set reserve prices on placements)

- Chi-squared test for Uniformity
- Anderson-Darling test for Normality

Test at the auction level
Results from a field experiment

- On Yahoo! Sponsored search
- Using the Optimal Auction Theory

Mixed results

Table 7: Restricted sample (optimal reserve price < 20c)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>t-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of keywords (T – treatment group)</td>
<td>222,249</td>
<td>-60.29</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>Number of keywords (C – control group)</td>
<td>11,615</td>
<td>-2.45</td>
<td>0.0144</td>
</tr>
<tr>
<td>(Mean change in depth in T) – (mean change in depth in C)</td>
<td>-0.8612</td>
<td>-11.1</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>(Mean change in revenue in T) – (mean change in revenue in C)</td>
<td>-11.88%</td>
<td>1.79</td>
<td>0.0736</td>
</tr>
</tbody>
</table>

Table 8: Restricted sample (optimal reserve price ≥ 20c)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>t-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of keywords (T – treatment group)</td>
<td>216,383</td>
<td>-55.09</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>Number of keywords (C – control group)</td>
<td>11,401</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Mean change in depth in T) – (mean change in depth in C)</td>
<td>-0.9664</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Mean change in revenue in T) – (mean change in revenue in C)</td>
<td>14.59%</td>
<td>3.80%</td>
<td>&lt; 0.0001</td>
</tr>
</tbody>
</table>
Our solution

• A dynamic and one-shot game between the winner (w) and the publisher (p)

• Extension form representation
  
  – Information nodes:
    • $I_1$: Auction succeeded: the winning bid $b_1$ is higher
    • $I_2$: Auction failed: the reserve price $\alpha$ is higher
  
  – Actions:
    • $a_{w1}$: to increase $b_1$ so that $b_1 \geq \alpha$
    • $a_{w2}$: to increase $b_1$ so that $b_1 < \alpha$
    • $a_{w3}$: to decrease $b_1$ so that $b_1 \geq \alpha$
    • $a_{w4}$: to decrease $b_1$ so that $b_1 < \alpha$
    • $a_{p1}$: to increase $\alpha$ so that $\alpha \geq b_1$
    • $a_{p2}$: to increase $\alpha$ so that $\alpha < b_1$
    • $a_{p3}$: to decrease $\alpha$ so that $\alpha \geq b_1$
    • $a_{p4}$: to decrease $\alpha$ so that $\alpha < b_1$
1) Expected payoff of advertiser, publisher

2) Payoff for the advertiser could be negative if one has been bidding the max price ($a_{w1}$: to increase $b_1$ so that $b_1 \geq \alpha$)

3) One won’t do that, so discounted publisher’s payoff
Heuristics and Modification

• If the reserve price is too high, lower it
• If too low, higher it
• If in the preferable range \((b_1 \geq \alpha \geq b_2)\), slightly higher it
• A parameter \(\lambda\) allowing to converge over time

\[
\Delta \alpha(t) = \begin{cases} 
\lambda^t h\alpha(t) & \alpha > b_1 \\
\lambda^t l\alpha(t) & b_2 > \alpha \\
\lambda^t p\alpha(t) & b_1 \geq \alpha \geq b_2 
\end{cases}
\]
Dataset (it’s online experiment)

- Observed 130m impressions from Dec 2012 to Feb 2013
  (Subsampled 10% due to computing power restraint)
- From 39 placements, 19 websites of different categories
- From a production Supply Side Platform in UK
Competing Algorithms

- Round-robin scheduling
  - Zero (the base line)
  - Weighted average (linear)
  - Optimal auction theory
  - Heuristics (OneShot)
  - Bayesian (univariant and bivariant)

- Reserve prices are set for each (placement, hour_of_day)
Findings

12.3% better than the 2\textsuperscript{nd} best 
28.5% better than the optimal auction theory
Findings

Advertisers are overpaying because of tricky set ups
(They don’t know it could be first price auction!)
(And seems they don’t care!)

Real-time Bidding for Online Advertising: Measurement and Analysis, Yuan et al., 2013
Advertisers need to catch up (at least from 1-year ago’s point of view) and consider cost in bidding algorithms. Weinan Zhang, Shuai Yuan, Jun Wang, Optimal Real-Time Bidding for Display Advertising, KDD 2014.
Future Work

- Reserve price optimisation
  - Audience data integration
    (Because the demand side is doing it!)
  - Finding better fitting distributions

- A unified supply side optimisation framework for big players
  - Enough volume for various online tests
  - Dynamic allocation of inventories
    (programmatic guarantee, private/public exchange, etc.)
    (Bowei Chen, Shuai Yuan, Jun Wang,
    A Dynamic Pricing Model for Unifying Programmatic Guarantee and Real-Time Bidding in
    Display Advertising,
    ADKDD 2014)
  - Joint optimisation
• Thanks for your time!

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