Computational Topology
Computational Geometry
Graph Drawing
Topological Graph Theory

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Kuratowski Theorem:

\[ G \text{ planar} \iff \text{contains no } K_5/K_{3,3} \text{-subdivision.} \]

- Two "obvious" obstructions suffice.
- Easy to argue either way.
- Fast algorithms, powerful theory.
Theorem: Every 3-connected planar graph has a convex (straight-line) drawing in $\mathbb{R}^2$.

\[ G \quad \rightarrow \quad G/e \quad \rightarrow \quad \cdots \quad \rightarrow \quad K_4 \]
Circle Packing Theorem (Andreev-Thurston-Koebe)

Every planar graph has a circle packing representation.

- History
- Riemann Mapping Theorem ($\Omega \to \Delta$ conformally equivalent)
Primal-dual Circle Packing

**Theorem** (Brightwell & Scheinerman): Every planar 3-connected graph admits a primal-dual circle packing. PDCP is unique up to Möbius transformations.

**Corollary:** $G$ and $G^*$ have straight-line simultaneous drawing with dual edges.

**Theorem** (Mohar): PDCP can be determined in polynomial time.

- $\varepsilon$-approximation, long arithmetic
- works on arbitrary surfaces (using spherical/Euclidean/hyperbolic geometry)
A remark on using long arithmetic:

Rectilinear crossing number

\[ \overline{cr}(G) \leq 2 \]
Universal set for graph drawing

Is there a set of \( n \) points in \( \mathbb{R}^2 \) such that every \( n \)-vertex planar graph admits a straight-line drawing on these \( n \) points used as vertices?

**Theorem:** For large \( n \), no such "universal" set exists.

**Theorem:** For every \( n \), there exists a "universal" set with \( O(n^2) \) points.
Straight-line drawings with vertices on $O(n) \times O(n)$ grid are of interest in computer graphics.

While area of GRAPH DRAWING arose from such a simple question.