

# Transductive Inference for Class-Membership Propagation in Web Ontologies

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# Motivation

Knowledge encoded in Semantic Web (SW) standards is increasing rapidly. Purely deductive inference may have some limitations:

- ▶ scalability;
- ▶ uncertainty [da Costa et al., 2008];
- ▶ uses axiomatic prior knowledge, ignoring statistical regularities;
- ▶ requires expensive knowledge engineering processes.

Machine learning can provide scalable solutions to exploit prior knowledge and statistical regularities in data.

# Learning from DL Representations

In SW literature, several learning tasks have been faced:

- ▶ **Concept learning** ([Lehmann et al., 2010, Fanizzi et al., 2008, Esposito et al., 2004]);
- ▶ **Schema** (TBox) induction ([Völker et al., 2011]);
- ▶ **Assertion** (ABox) prediction (survey [Rettinger et al., 2012]).

Focus: **assertion prediction** problem

- ▶ deciding whether an individual belongs to a given concept;
- ▶ often approached using statistical models.

# Generative vs. Discriminative Models

Proposed statistical assertion prediction models classified in:

- ▶ **Generative models** construct a joint probabilistic model of the variables involved in the prediction task;
- ▶ **Discriminative models** only focus on the actual accuracy of the induced predictive model.

Building a generative prediction models can possibly be an harder problem than building a discriminative one.

# Generative vs. Discriminative Models

Real-life ontologies characterized by abundance of unlabelled instances and few labelled ones:

- ▶ Unknown concept-membership relations considered as *unlabelled instances*.

Generative models can be learned with unlabelled instances

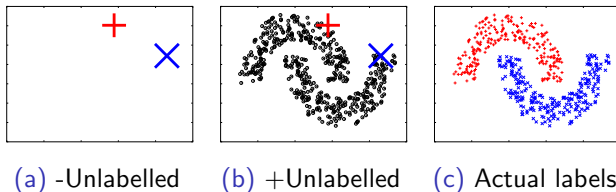
- ▶ By maximizing the likelihood of observable data, e.g. by means of the classic EM framework.

How to build discriminative models accounting for unlabelled instances ?

# Leveraging unlabelled instances

Proposal: adoption of *Semi-Supervised Learning* (SSL) for building discriminative models from SW representations.

- ▶ **Semi-Supervised Smoothness Assumption** – if two points are linked by a path through a high-density area, they are likely to be associated to closely located labels.



**Figure:** The distribution of instances can be informative wrt. the distribution of labels

# Inductive vs. Transductive Semi-Supervised Learning

- ▶ **Inductive SSL** – learn a prediction function expected to be a good predictor on future, unseen instances;
- ▶ **Transductive SSL** – learn a prediction function expected to be a good predictor on available, unlabeled instances.

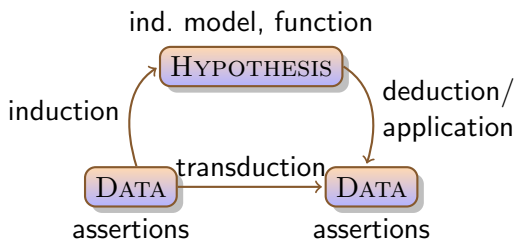


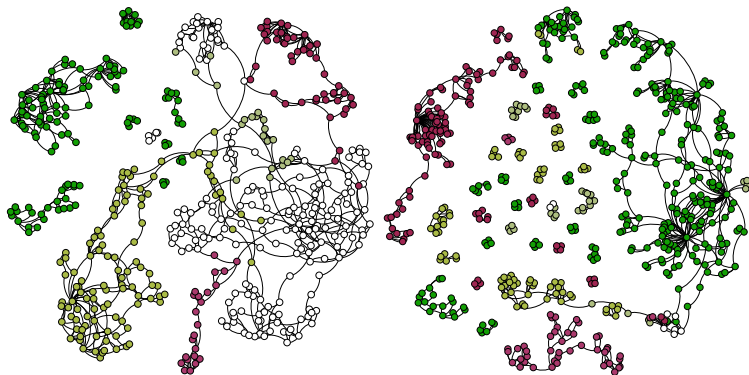
Figure: Transductive and inductive inference

# Performing Transductive Semi-Supervised inference

- ▶ Looking for a *smooth* labelling is a *simpler problem* than finding a smooth prediction function wrt. the instance space.
- ▶ We will thus focus on *Transductive Semi-Supervised inference*:
  - ▶ **Aim:** given a set of labelled and unlabelled instances, find a labelling consistent with labelled instances that *varies smoothly* among similar instances.
  - ▶ **Proposed approach:** *Graph-based regularization* (consisting in *penalizing non-smooth labellings*) allows to:
    - ▶ Scale up to large-size sets of instances;
    - ▶ Provide confidence indicators about inferred knowledge.
- ▶ Algorithm:
  - ▶ Create a *k-NN similarity graph*: each individual linked with top *k* similar individuals ( $\{0, 1\}$  adjacency matrix  $\mathbf{W}$ );
  - ▶ Find labels minimizing a cost function enforcing consistency (wrt. training labels) and smoothness (wrt. the graph).
- ▶ Similarity relations calculated using *graph* and *DL kernels*.



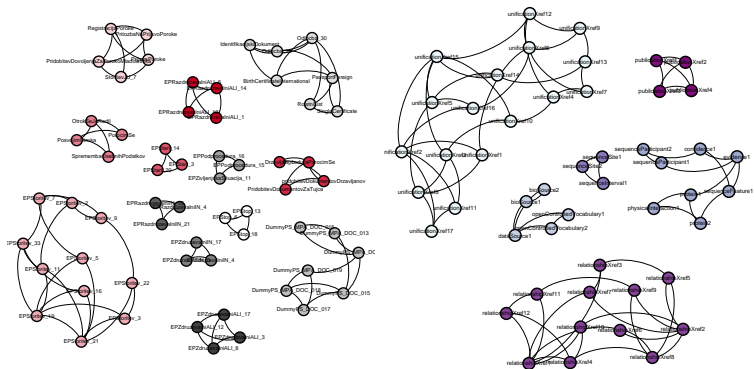
# Clustered structures in Web Ontologies



(a) AIFB Affiliations (Persons)    (b) AIFB Affiliations (Articles)

**Figure:** Neighbourhood relations (wrt. the SubTree kernel [Lösch et al., 2012]) in the AIFB Affiliations ontology

# Clustered structures in Web Ontologies

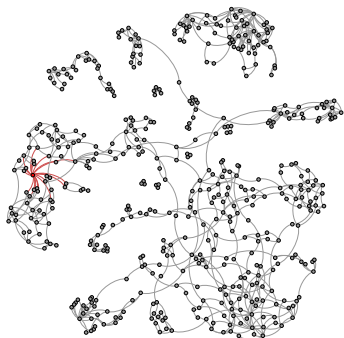


(a) Leo

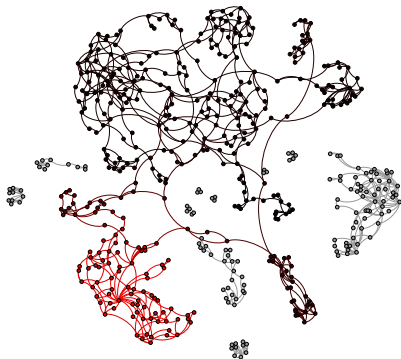
(b) BioPAX

**Figure:** Neighbourhood relations (wrt. a DL kernel) between individuals in two ontologies from the TONES repository

# Propagating knowledge in the AIFB Affiliations ontology



(a) AIFB Affiliations (Persons)



(b) AIFB Affiliations (Articles)

**Figure:** Example of label propagation via graph-based regularization in the AIFB Affiliations ontology.

# Propagating membership knowledge among instances

Assume:

- ▶ predicted labellings encoded in a vector  $\hat{\mathbf{f}} \in (-1, 1)^N$ ;
- ▶ similarity relations encoded in a symmetric  $\{0, 1\}$  adjacency matrix  $\mathbf{W}$ .

Need a *cost function* that both:

1. Enforces consistency with training examples;
2. Enforces smoothness among similar individuals.

Many options to define such costs. By adopting *quadratic cost criteria*, two options were considered:

- ▶  $C(\hat{\mathbf{f}}) = \|\hat{\mathbf{f}}_L - \mathbf{f}_L\|^2 + \mu \hat{\mathbf{f}}^T \mathbf{L} \hat{\mathbf{f}} + \mu \epsilon \|\hat{\mathbf{f}}\|^2$ ;
  - ▶ note that  $\hat{\mathbf{f}}^T \mathbf{L} \hat{\mathbf{f}} = 0.5 \sum_{i,j} W_{ij} (\hat{f}_i - \hat{f}_j)^2$ ;
- ▶  $C'(\hat{\mathbf{f}}) = \|\hat{\mathbf{f}} - \mathbf{f}\|^2 + \mu \hat{\mathbf{f}}^T \mathcal{L} \hat{\mathbf{f}}$ ;

where:

- ▶  $\mathbf{L} = (\mathbf{D} - \mathbf{W})$  *graph Laplacian* ( $\mathbf{D}$  diagonal:  $\mathbf{D}_{ii} = \sum_j \mathbf{W}_{ij}$ );
- ▶  $\mathcal{L} = \mathbf{D}^{-1/2} \mathbf{L} \mathbf{D}^{-1/2}$  *normalized graph Laplacian*.

## Propagating membership knowledge among instances

Finding  $\mathbf{f}^* = \arg \min_{\mathbf{f}} C(\mathbf{f})$  reduces to solving a (large) sparse linear system; e.g. let  $C(\hat{\mathbf{f}}) = \|\hat{\mathbf{f}}_L - \mathbf{f}_L\|^2 + \hat{\mathbf{f}}^T \mathbf{L} \hat{\mathbf{f}} + \mu \epsilon \|\hat{\mathbf{f}}\|^2$ :

$$\begin{aligned} \frac{1}{2} \frac{\partial C(\hat{\mathbf{f}})}{\partial \hat{\mathbf{f}}} &= \mathbf{S}(\hat{\mathbf{f}} - \mathbf{f}) + \mu \mathbf{L} \hat{\mathbf{f}} + \mu \epsilon \hat{\mathbf{f}}; \\ \arg \min_{\hat{\mathbf{f}}} C(\hat{\mathbf{f}}) &= (\mathbf{S} + \mu \mathbf{L} + \mu \epsilon \mathbf{I})^{-1} \mathbf{S} \mathbf{f}; \\ \rightarrow \mathbf{x} &= \mathbf{A}^{-1} \mathbf{b}; \end{aligned}$$

Solving a sparse linear system is a well-known problem whose time complexity is nearly linear in the number of non-zero entries in the coefficient matrix [Spielman and Teng, 2004].

## Working around the irrelevant roles issue

Proposed approach: combine several *heterogeneous* similarity/affinity graphs (encoding e.g. co-authorship, friendship, collaboration relations).

Different graph-based regularizers combined together:

- ▶ **HMRW**: regularizer  $\mu \hat{\mathbf{f}}^T \mathbf{L} \hat{\mathbf{f}}$  becomes  $\sum_{r=1}^R \mu_r \hat{\mathbf{f}}^T \mathbf{L}_r \hat{\mathbf{f}}$ ;

where  $\mathbf{L}_r$  is the graph Laplacian of the  $r$ -th similarity/affinity graph, and  $\mu_r \geq 0$  is a weight associated to each of such graphs.

- ▶ Evaluation setting: predicting research group affiliations in the AIFB Affiliations ontology [Lösch et al., 2012];
- ▶ Evaluated approaches:
  - ▶ **RG**:  $C(\hat{\mathbf{f}}) = \|\hat{\mathbf{f}}_L - \mathbf{f}_L\|^2 + \mu \hat{\mathbf{f}}^T \mathbf{L} \hat{\mathbf{f}} + \mu \epsilon \|\hat{\mathbf{f}}\|^2$ ;
  - ▶ **MRW**: like  $C$ , but  $\mu \rightarrow 0$  (enforced training labels);
  - ▶ **HRMW**: linear comb. of multiple graph-based regularizers;
  - ▶ **SVM**: state of the art discriminative method.
  - ▶  $C'$  defined previously found too conservative.

# Results

Method	F1	AUC-PR	Method	F1	AUC-PR
HMRW	<b><math>0.76 \pm 0.207</math></b>	<b><math>0.928 \pm 0.117</math></b>	HMRW	<b><math>0.738 \pm 0.118</math></b>	<b><math>0.907 \pm 0.073</math></b>
MRW	$0.278 \pm 0.202$	$0.734 \pm 0.166$	MRW	$0.276 \pm 0.163$	$0.665 \pm 0.095$
RG	$0.293 \pm 0.218$	$0.723 \pm 0.166$	RG	$0.271 \pm 0.17$	$0.665 \pm 0.111$
SVM	$0.34 \pm 0.177$	$0.588 \pm 0.16$	SVM	$0.338 \pm 0.167$	$0.488 \pm 0.092$

(a) Training: 9/10 folds

(b) Training: 5/10 folds

Figure: 10-fold CV results for SVM, RG, MRW and HMRW

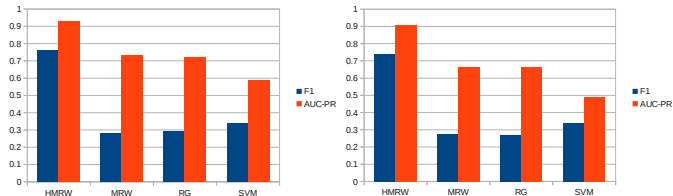


Figure: 10-fold CV results using 9 and 5 training folds

# Conclusions

- ▶ Tackling the problem of (semi-supervised) transductive inference from SW representations.
  - ▶ source: [code.google.com/p/knowledge-propagation/](https://code.google.com/p/knowledge-propagation/)
- ▶ From evaluations, it emerged that single graph (and DL) kernels may tend to be domain dependant:
  - ▶ Sometimes they work well, sometimes they do not;
  - ▶ It gives hints of how well a kernel may suit a problem.
- ▶ Current effort: learning a task-dependent combination of kernels, taking in account different aspects of the domain (from terminology to relational structure);





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