Scale and Rotation Invariant Detection of Singular Patterns in the Vector Flow Fields

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Contribution
We propose a SIFT-like detector that locates singular flow patterns at varying scales and orientations. We model flow fields locally using a linear combination of complex monomials that form an orthogonal basis for analytic flows. We investigate the approximation coefficients’ invariance to rotation and scaling, and show how singular patterns can be compared under rigid transformations.

Model
We model 2-D vector-flow fields as complex-valued functions \( f(z) \) defined on \( \Omega \subset \mathbb{C} \). We adopt the inner product based on cross-correlation:

\[
\langle f(z), g(z) \rangle = \int f(z) \cdot g(z) \, dz,
\]

where \( \cdot \) is the standard inner product on \( \mathbb{C} \) (i.e., dot product between two complex numbers). Thus, a flow field can be approximated as a linear combination of complex (orthogonal) basis \( \phi_k(z) \):

\[
f(z) \approx \sum_{k=0}^{N} a_{k,1} \phi_{k,1}(z).
\]

Cross-correlation between a flow field and basis.

Rotation and Scaling
Rotation is linear, and the rotated coefficients are:

\[
a'_{k,1}(\theta) = \cos((k-1)\theta) a_{k,1} - \sin((k-1)\theta) a_{k,2}
\]
\[
a'_{k,2}(\theta) = \sin((k-1)\theta) a_{k,1} + \cos((k-1)\theta) a_{k,2}.
\]

Excluding the constant flow basis \( \phi_{0,0} \) and \( \phi_{0,1} \), the remaining basis functions are singular. We define a rotation-invariant singular energy term:

\[
E_{s\text{rg}}(z) = \sum_{k \geq 1} \left( \|a_{k,1}\|^2 + \|a_{k,2}\|^2 \right)
\]

(3)

Scaling operator \( \Psi_s(.) \) is also linear, and coincides with scaling of the Gaussian weighting function:

\[
\Psi_s(\phi_{k,0}(z)) = s \phi_{k,0}(s^{-1}z) = s \phi_{k,0}(z) / \|\phi_{k,0}(z)\|.
\]

(4)

Multiscale Detection
As in SIFT, we create octaves of the input flow by smoothing and down-sampling. To avoid destroying singular points due to smoothing, within each octave, we obtain finer intermediate scales by scaling the basis functions (4). The singular patterns are detected as the local maxima of the singular energy (3) along both spatial and scale dimensions. For each detected singular pattern, we align it to a subspace basis \( \phi_{k,0} \), by calculating what we call the flow field’s principle orientations:

\[
\hat{\theta} = \arg \max_{\theta} a'_{k,0}(\theta).
\]

(5)

In contrast to scalar images, flow fields may have multiple principle orientations.

Detection Results
Singular patterns in a weather system.

Multiscale Detection Results
Singular patterns in incompressible fluid flows.

Classification
Cluster centers and examples of singular patterns with no constant background flows.

Background flows
Cluster centers and examples of singular patterns skewed by background flows.

Future work
1. Handling of shear flows.
2. Extension to 3-D flow fields.

Source code and test data
Source code and test data available from:
http://www.cs.fit.edu/~eribeiro/flowdetector/