The Search for the Perfect Language

- I'll tell you how the search for certainty led to incompleteness, uncomputability & randomness,
- and the unexpected result of the search for the perfect language.

Bibliography

- Umberto Eco, *The Search for the Perfect Language*, in Italian, French, English...
- Chaitin, *Meta Math* (USA), *Meta Maths* (UK), *Alla ricerca di omega*, *Hasard et complexité en mathématiques* + Greek + Japanese + Portuguese
- David Malone, *Dangerous Knowledge*, BBC TV, 90 minutes, Google video

Umberto Eco, The Search for the Perfect Language

- Language of creation (Hebrew?)
- How God summoned things into being by naming them
- Expresses inner structure of world
- Kabbalah
- Ramon Llull ~ 1200
  - Combine concepts in all possible ways mechanically
  - Wheels
- Source of all (universal) knowledge!

Leibniz

- Mechanical reasoning
- Characteristica universalis
- Calculus ratiocinator
- Settle disputes: "Gentlemen, let us compute!"
- Computer to multiply
- Binary arithmetic, base-two
- Christian Huygens: "The calculus is too mechanical!"

Georg Cantor

- Mathematical theology!
- Infinite sets, infinite numbers
- Transfinite ordinals: 0, 1, 2, 3 ... ω, ω+1 ... 2ω ... ω^2 ... ω^ω ... $\omega^{\omega\omega\omega\omega\ldots}$
- Transfinite cardinals: Integers = $\aleph_0$, reals = $\aleph_1$, $\aleph_2$, $\aleph_3$ ... $\aleph_\omega$ ... ${\aleph_\omega}^\omega$ ... $\aleph_{\omega}{}^{\omega\omega\omega\omega\ldots}$
- Hilbert/Poincaré: Set theory = heaven/disease!

Bertrand Russell

- Cantor's diagonal argument: Set of all subsets gives next cardinal.
- Russell's paradox: Is the set of all subsets of the universal set bigger than the universal set?!
- Problem: Set of all sets that are not members of (in) themselves.
- Member of itself or not?!

David Hilbert

- Formalize **all** math = TOE = Theory of Everything
- Objective not subjective **truth**
- Absolute truth — **black** or **white**, not **gray**
- Formal axiomatic theory for all mathematics:
  - Has proof-checking algorithm.
  - Has algorithm to generate all the theorems in the theory.
- Meta-mathematics: Study from outside.

Positive Work on Hilbert's Program

- John von Neumann
- Zermelo-Fraenkel

John von Neumann

- Monist ontology (Spinoza!)
- $0 = \{\}$ empty set
- $1 = \{0\} = \{\{\}\}$
- $2 = \{0, 1\} = \{\{\}, \{\{\}\}\}$
- $\omega = \{0, 1, 2, 3, \ldots\}$
- Everything is a set!
- Including ordered pairs, sequences & functions.
- All math built from empty set.

Abraham Fraenkel (Hebrew University)

- Zermelo-Fraenkel set theory = Abstract, formal set theory
- First-order logic
- Axiom of choice
- Avoids sets that are too big.

Negative Work on Hilbert's Program

- Gödel, 1931
- Turing, 1936
- Algorithmic Information Theory (AIT), 1960s → present

Kurt Gödel, 1931

- "This statement is false!"
- True or false?
- "This statement is unprovable!"
- Provable or unprovable?
- No TOE for mathematics!
- Incompleteness!

Alan Turing, 1936

- Universal Turing machine = Universal programming language
- <u>The Halting Problem!</u>
- Uncomputability $\Rightarrow$ Incompleteness

Algorithmic Information Theory

- Program-Size Complexity =
  - minimum size program,
  - minimum number of yes/no decisions God has to make to create something.
- $U(\pi_C\, p) = C(p)$ Simulation!
- Given program p for computer C, concatenate prefix π_C to p.
- Universal machine U simulates the computation C performs given p.
- Most concise, expressive
  - universal Turing machine,
  - universal programming language.
- The Halting Probability Ω.
- Randomness = Irreducible complexity ⇒ Incompleteness!

Negative Conclusions

- Hilbert's search for the perfect language giving all math knowledge
- (= formal axiomatic theory for all of mathematics)
- cannot succeed!
- Every formal axiomatic theory is incomplete, as shown by Gödel, Turing, Ω.
- Search for certainty ⇒ Incompleteness, uncomputability & randomness.
- Nevertheless, mathematicians remain enamored with formal proof:
- See *AMS Notices* special issue on formal proof, December 2008.

Positive Conclusions

- However, perfect languages have emerged for computing, **not** for reasoning.
- Universal Turing machines and universal programming languages can all simulate each other.
- Most expressive, concise languages: $U(\pi_C\ p) = C(p)$.
- AIT's program-size complexity is a measure of conceptual complexity, of complexity of ideas.
- Furthermore, viewed from the perspective of the Middle Ages, programming languages
- give us the God-like power to breathe life into (some) inanimate matter:
- Hardware ≈ **Body**, Software ≈ **Soul**!