Resampling based methods for design and evaluation of neuroimaging models

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OUTLINE

Machine learning – with two agenda
- Aim I: To abstract generalizable relations from data
- Aim II: Robust interpretation / visualization
- The PR-plot for optimization

Unsupervised (explorative)
- Factor models - Linear hidden variable representations
- Generalization in unsupervised models
- Visualization
- Non-linear models, KPCA, PR-plots for tuning
- Network modeling

Supervised models (retrieval)
- Visualization of non-linear kernel machines
- PR-plotting supervised models
- Pattern recognition in network models

Outlook

Do not multiply causes!
Machine learning classifiers and fMRI: A tutorial overview

Francisco Pereira a,*, Tom Mitchell b, Matthew Botvinick a

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Review

Encoding and decoding in fMRI

Thomas Naselaris a, Kendrick N. Kay b, Shinji Nishimoto a, Jack L. Gallant a,b,*
a Helen Wills Neuroscience Institute, University of California, Berkeley, CA 94720, USA
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Decoding fMRI brain states in real-time

Stephen M. LaConte

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Introduction to machine learning for brain imaging

Steven Lemm a,c,*, Benjamin Blankertz a,b,c, Thorsten Dickhaus a, Klaus-Robert Müller a,c
Functional MRI

- Indirect measure of neural activity - hemodynamics

- A cloudy window to the human brain

- Challenges:
  - Signals are multi-dimensional mixtures
  - No simple relation between measures and brain state - "what is signal and what is noise"?

TR = 333 ms
Main message: Models should be predictive and informative

BOLD fMRI: Is hemodynamic de-convolution feasible?

Bayesian Model Comparison in Nonlinear BOLD fMRI
Hemodynamics

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Balloon model: Non-linear relations between stimulus and physiology – described by four non-linear differential eqs.

Bayesian averaging with split-1/2 resampling loop
to establish generalizability and reproducibility

\[
R(M) = \frac{1}{K} \sum_{i=1}^{K} \int p(\theta \mid D_i^1, M) \log \frac{p(\theta \mid D_i^1, M)}{p(\theta \mid D_i^2, M)} d\theta,
\]

Figure 3.1: Overview diagram of hemodynamic models.
BOLD hemodynamics R-Bayes model selection

Model A: Sustained neural input vs Model B: Fading input

Figure 3: Regions of interest, marked with white squares. (A) Data set 1; T2* weighted image slice parallel to the calcarine sulcus. (B) Data set 2; MPRAGE (magnetization prepared rapid gradient echo) horizontal slice.

Figure 4: Prediction of data set 1. (A) Model A. (B) Model B. Note that the confidence interval is an empirical confidence interval for the mean prediction, based on the MCMC samples.

Figure 6: Generalization and reproducibility for real data; crosses mark the mean. (A) Data set 1. (B) Data set 2.
Machine learning for neuroimage data

Neuroimaging aims at extracting the mutual information between stimulus and response.

- Stimulus: Macroscopic variables, "design matrix" ... $s(t)$
- Response: Micro/meso-sopic variables, the neuroimage ... $x(t)$
- Mutual information is stored in the joint distribution ... $p(x,s)$.

Often $s(t)$ is assumed known....unsupervised methods consider $s(t)$ or parts of $s(t)$ "hidden".....
Multivariate neuroimaging models

- **Univariate models** - SPM, fMRI time series models etc.

\[ p(x, s) = p(x \mid s) p(s) = \prod_j p(x_j \mid s) \cdot p(s) \]

- **Multivariate models** - PCA, ICA, SVM, ANN (Lautrup et al., 1994, Mørch et al. 1997)

\[ p(x, s) = p(s \mid x) p(x) \]

- **Modeling from data with parameterized function families** – rather than testing (silly) null hypotheses
Navigational Aid: Do not multiply causes!

**AIM I: Generalizability**

- Generalizability is defined as *the expected performance on a random new sample*
  - A model’s mean performance on a “fresh” data set is an unbiased estimate of generalization

- Typical loss functions:
  
  \[
  \langle -\log p(s \mid x, D) \rangle, \quad \langle -\log p(x \mid D) \rangle, \\
  \langle (s - \hat{s}(D))^2 \rangle, \quad \log \frac{p(s, x \mid D)}{p(s \mid D)p(x \mid D)}
  \]

- Note: No problem to estimate generalization in hidden variable models!

- Results can be presented as “bias-variance trade-off curves” or “learning curves”
Bias-variance trade-off as function of PCA dimension

Learning curves for multivariate brain state decoding

Finger tapping, analysed by PCA dimensional reduction and Fisher LD / ML Perceptron. Mørch et al. IPMI (1997)...Brain state decoding in fMRI
A brain map is a visualization of the information captured by the model:
- The map should take on a high value in voxels/regions involved in the response and a low value in other regions...

Statistical Parametric Maps
Weight maps in linear models
The saliency map (1995)
The sensitivity map (2000)
Consensus maps

Asymptotic theory investigates the sampling fluctuations in the limit $N \to \infty$.

Cross-validation good news: The ensemble average predictor is equivalent to training on all data (Hansen & Larsen, 1996).

Simple asymptotics for parametric and semi-parametric models (Some results available also for non-parametric e.g. kernel machines).

In general: Asymptotic predictive performance has bias and variance components, there is proportionality between parameter fluctuation and the variance component...
The sensitivity map measures the impact of a specific feature/location on the predictive distribution.

\[ m_j = \left\langle \left( \frac{\partial \log p(s|x)}{\partial x_j} \right)^2 \right\rangle \]

**FIG. 3.** Plot of scan/label mutual information versus reproducibility signal/noise for the four data sets, for varying numbers of subjects in the training set. There were 2 labels/4 scans per subject (balanced data set; Setup 1, Table 1) corresponding to the dashed solid line in Fig. 4. We see that both measures indicate improved performance of the model as the number of subjects increases.
NPAIRS: Reproducibility of parameters

- NeuroImage: Hansen et al. (1999), Lange et al. (1999)
- Hansen et al. (2000), Strother et al. (2002), Kjems et al. (2002), LaConte et al. (2003), Strother et al. (2004), Mondrup et al. (2011)
- Brain and Language: Hansen (2007)
Reproducibility of internal representations

Split-half resampling provides unbiased estimate of reproducibility of SPMs


Predicting applied static force with visual feed-back
Unsupervised learning

Explorative modeling
Learning stable structures in data \( p(x,s) \)
Unsupervised learning:
Factor analysis generative model

\[ x = As + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \Sigma) \]

\[
p(x \mid A, \theta) = \int p(x \mid A, s, \Sigma) p(s \mid \theta) ds
\]

\[
p(x \mid A, s, \Sigma) = \sqrt{2\pi\Sigma}^{-1/2} e^{-\frac{1}{2}(x-As)^T \Sigma^{-1}(x-As)}
\]

Source distribution:
PCA: ... normal
ICA: ... other
IFA: ... Gauss. Mixt.
kMeans: .. binary

PCA: \[ \Sigma = \sigma^2 \cdot 1, \]
FA: \[ \Sigma = D \]

S known: GLM
(1-A)^{-1} sparse: SEM
S,A positive: NMF

Højen-Sørensen, Winther, Hansen,
Factor models

- Represent a datamatrix by a low-dimensional approximation
- Identify spatio-temporal networks of activation

\[ X(i,t) \approx \sum_{k=1}^{K} A(i,k)S(k,t) \]
Matrix factorization: SVD/PCA, NMF, Clustering

Figure 1: Non-negative matrix factorization (NMF) learns a parts-based representation of faces, whereas vector quantization (VQ) and principal components analysis (PCA) learn holistic representations. The three learning methods were applied to a database of $m = 2,429$ facial images, each consisting of $n = 19 \times 19$ pixels, and constituting an $n \times m$ matrix $V$. All three find approximate factorizations of the form $V = WH$, but with three different types of constraints on $W$ and $H$, as described more fully in the main text and methods. As shown in the $7 \times 7$ montages, each method has learned a set of $r = 49$ basis images. Positive values are illustrated with black pixels and negative values with red pixels. A particular instance of a face, shown at top right, is approximately represented by a linear superposition of basis images. The coefficients of the linear superposition are shown next to each montage, in a $7 \times 7$ grid, and the resulting superpositions are shown on the other side of the equality sign. Unlike VQ and PCA, NMF learns to represent faces with a set of basis images resembling parts of faces.

Learning the parts of objects by non-negative matrix factorization

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ICA: Assume $S(k,t)$'s statistically independent

(McKeown, Hansen, Sejnowski, Curr. Op. in Neurobiology (2003))
DTU:ICA toolbox

- Infomax/Maximum likelihood

- Dynamic Components

- Mean Field ICA
  Højen-Sørensen et al. (2001,2002)

- Features:
  - Number of components (BIC)
  - Parameter tuning
  - Binary and mixing constraints (A)
  - Demo scripts incl. fMRI data

http://cogsys.imm.dtu.dk/toolbox/ica/
Rich physics literature on “retarded” learning

- **Universality**
  - Generalization for a “single symmetry breaking direction” is a function of ratio of N/D and signal to noise S
  - For subspace models-- a bit more complicated -- depends on the component SNR’s and eigenvalue separation
  - For a single direction, the mean squared overlap \( R^2 = \langle (u_1^T \cdot u_0)^2 \rangle \) is computed for N,D -> \( \infty \)

\[
R^2 = \begin{cases} 
(\alpha S^2 - 1) / S (1 + \alpha S) & \alpha > 1 / S^2 \\
0 & \alpha \leq 1 / S^2
\end{cases}
\]

\[
\alpha = N / D \quad S = 1 / \sigma^2 \quad N_c = D / S^2
\]


N_c = (0.0001, 0.2, 2, 9, 27, 64, 128, 234, 400, 625)
\( \sigma = (0.01, 0.06, 0.12, 0.17, 0.23, 0.28, 0.34, 0.39, 0.45, 0.5) \)
Generalizability – test training misalignment
Now what happens if you are on the slope of generalization, i.e., N/D is just beyond the transition to retarded learning?

The estimated projection is offset, hence, future projections will be too small!

...problem if discriminant is optimized for unbalanced classes in the training data!

Heuristic: Leave-one-out re-scaling of SVD test projections

Kjems, Hansen, Strother: “Generalizable SVD for Ill-posed data sets” NIPS (2001)
Re-scaling the component variances

- Possible to compute the new scales by leave-one-out doing $N$ SVD’s of size $N \ll D$

Compute $U_0 \Lambda_0 V_0^T = \text{svd}(X)$ and $Q_0 = [q_j] = \Lambda_0 V_0^T$

foreach $j = 1 \ldots N$

$$\bar{q}_{-j} = \frac{1}{N-1} \sum_{j' \neq j} q_{j'}$$

Compute $B_{-j} \Lambda_{-j} V_{-j}^T = \text{svd}(Q_{-j} - \bar{Q}_{-j})$

$$z_j = B_{-j} B_{-j}^T (q_j - \bar{q}_{-j})$$

$$\hat{\lambda}_i^2 = \frac{1}{N-1} \sum_j z_{ij}^2$$

Kjems, Hansen, Strother: NIPS (2001)
More challenges for the linear factor model

• Too simple?
  – Non-linear manifolds
  – Temporal structure in networks -> Convolutive ICA

• Too rich and over-parametrized?
  – Multi-dimensional macro and micro variables
    (space/time/frequency, group study, repeat trials)
  – Multiway methods
Beyond the linear model: Motivation

Fig. 4. Test set reference activation time course and activation time courses produced by $k$-nearest neighbor classifiers based on linear and non-linear feature spaces. The non-linear feature based classifier’s errors basically occurs at the onset and at end of stimulation. The KNN model based on the linear feature representation make additional generalization errors in the baseline, where it suggest a few short burst of activation.
Beyond the linear model: De-noising by projection onto non-linear signal manifolds w/ kPCA

- Kernel PCA is based on non-linear mapping of data to

\[ x_n \rightarrow \varphi(x_n) \equiv \varphi_n, \quad n = 1, \ldots, N \]

Aim is to locate maximum variance directions in the feature space, i.e.

\[ l_1 \equiv \arg \max_{\|l\|=1} \left\langle (l^T \cdot \varphi)^2 \right\rangle, \quad \varphi(x_n) = \sum_k l_k s_{k,n} \]

The principal direction is in the span of data:

\[ l_1 = \sum_{n=1}^{N} a_{1,n} \varphi_n \]

\[ a_1 = \arg \max_{\|a\|=1} \left\langle a^T \cdot K \cdot a \right\rangle, \quad K_{n,n'} = \varphi_{n'}^T \cdot \varphi_n = \exp \left( -\frac{\|x_n - x_{n'}\|^2}{2c} \right) \]

Manifold de-noising: The pre-image problem

Now, assume that we have a point of interest in feature space, e.g. a certain projection on to a principal direction “Φ”, can we find its position “z” in measurement space?

\[ z = \varphi^{-1}(\phi) \]

Problems: (i) Such a point need not exist, and (ii) if it does there is no reason that it should be unique!

Mika et al. (1999): Find the closest match.

Figure 1: The pre-image problem in kernel PCA denoising concerns estimating \( z \) from \( x_0 \), through the projection of the image onto the principal subspace in feature space, \( \mathcal{F} \).
INPUT SPACE REGULARIZATION STABILIZES PRE-IMAGES FOR KERNEL PCA DE-NOISING

Trine Julie Abrahamsen   Lars Kai Hansen

Fig. 4. Top: Example of de-noised digits using a very non-linear kernel ($c = 50$) and 100 principal components. (b) Mika et al. and (d) our approach, note the visual improvement of the recovered pre-images in the red box. The colormap has been adjusted for better visualization. Bottom: The image intensity along the red line indicated above. Note the improved SNR in the result of the new method.

Fig. 2. Experiment to illustrate the stability of pre-image based de-noising of USPS digits. A training set of 400 digits (10060,2,4,9) is used to define the signal manifold. We show the confidence intervals (5th and the 95th percentile) for the mean square error (MSE) in different combinations of KPCA subspace dimension and non-linearity. MSE computed for 400 de-noised test samples for (a) Kwok-Tang, (b) Mika et al., (c) Dambreville et al., and (d) the new input space distance regularization approach. The previous schemes are seen to deteriorate in the non-linear regime (small $c$).

Regularization mechanisms for pre-image estimation in fMRI denoising

L2 regularization on denoising distance

Figure 4.10: The pre-image problem in kernel PCA denoising concerns estimating $z$ from $x_0$, through the projection of the image onto the principal subspace. Presently available methods for pre-image estimation lead to unstable pre-images because the inverse is ill-posed. We show that simple input space regularization, with a penalty based on the distance $||z - x_0||$ leads to a stable pre-image.

L1 regularization on pre-image

Figure 1: The pre-image problem in kernel PCA denoising concerns estimating $z$ from $x_0$, through the projection of the image onto the principal subspace in feature space, $\mathcal{F}$. 
Figure 2: Prediction/reproducibility plots using all scans for the single slice fMRI visual block activation experiment. The GPS estimate when using a non-linear kernel are seen to outperform all other estimates in terms of combined prediction and reproducibility measures. Location in the upper right corner is preferred.

\[ z = \operatorname{argmin}_{z \in \mathcal{X}} \left\| \varphi(z) - P_q \varphi(x_0) \right\|^2 + \lambda \| z \|_{\ell_1}. \]

GPS = General Path Seeking, generalization of the Lasso method
Does denoising by kernel PCA help fMRI decoding?

(A) Comparison of resampling z-score = z-kPCA – z-Raw

(B) FDR corrected:
   yellow: consensus,
   blue: only kPCA,
   red: only raw

Variance inflation in kernel PCA!

Handwritten digits:

fMRI data
single slice rest/visual stim (TR= 333 ms)

Challenges from more complex data structures
...Data represented as multiway arrays

A full linear factor model may be overparametrized

Example
EEG visual response to meaningful vs non-meaningful drawings (N=11).

3-way analysis:
Channel*freq*time

5-way analysis:
Channel*freq*time*subject*condition

ERPWAVELAB

- Interfaced with EEGLAB

  - Single subject analysis
    Artifact rejection in the time/freq domain
    NMF decomposition
    Cross coherence tracking

- Multi subject analysis
  Clustering
  Analysis of Variance (ANOVA)
  Tensor decomposition

Toolbox download from www.erpwavelab.com

Supervised learning

Retrieval of relevant patterns  $p(s|x)$
Generalizable supervised models - ‘mind reading’

- Non-linear kernel machines, SVM

\[ s(n') \approx \sum_{n=1}^{N} \alpha(n) K(x_n, x_{n'}) \]

\[ K(x_n, x_{n'}) = \exp \left\{ - \frac{||x_n - x_{n'}||^2}{2c} \right\} \]

Local voting +/-
Visualization of SVM learning from fMRI

- Visualization of kernel machines
  - How to create an SPM for a kernel machine
  - The sensitivity map for kernels
  - Example:

\[
\begin{align*}
s(n') & \approx \sum_{n=1}^{N} \alpha(n) K(x_n, x_{n'}) \\
K(x_n, x_{n'}) & = \exp \left\{ - \frac{||x_n - x_{n'}||^2}{2c} \right\}
\end{align*}
\]
Input Space Versus Feature Space in Kernel-Based Methods
Bernhard Schölkopf, Sebastian Mika, Chris J. C. Burges, Philipp Knirr,
Klaus-Robert Müller, Gunnar Rätsch, and Alexander J. Smola

The Pre-Image Problem in Kernel Methods

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Hong Kong

Existing visualization methods

- Pre-image (Mika et al., NIPS 1998, Schölkopf et al., 1999)
  Basically an ill-defined objective, useful for denoising

- Multi-dimensional scaling (Kwok & Tsang, ICML 2003)
  Interpolates nearest neighbors, suffers in high dimensions

Problem: Existing methods provide local visualization, which point should be visualized? Algorithms are reported unstable (may be fixed though!).
The sensitivity map measures the impact of a specific feature/location on the predictive distribution.
Consistency across models (left-right finger tapping)

LogReg  SVM  RVM

Figure 7: fMRI finger-tapping experiment - consensus analysis. The plots show the extend of consensus in the average rSPI among the three models. The rSPI for LogReg was scaled by its maximum value. Hereafter the rSPIs from the SVM and RVM were transformed to match the histogram of that of LogReg. Correlation coefficients between histograms are found on top of the plots.
Sensitivity maps for non-linear kernel regression

Fig. 1. XOR-image set define by four activated regions (A,B,C,D). Initially we let regions (A,B,D) be activated by random sequence taking values ±1, as shown in example in the bottom panel (full curve). The target signal, also taking values $t_n = pm1$, and is also indicated in the bottom panel (dashed line). The region (C) is activated with an XOR-sequence relative to (A) and $t_n$, so that $C_n = A_n \oplus t_n$, hence, in the active state the two regions (A,C) are randomly, but identically activated, while in the resting condition, they are random, but opposite.

Fig. 2. XOR-image set define by four activated regions. The results of analyzing a image set with $N = 400$ examples. The image signal-to-noise ratio is $SNR = 1$, i.e., the additive noise is unit variance. The target function has in addition been contaminated by $10\%$ random label noise. The four subplots show: the sensitivity map (upper left), the near-perfect receiver operating curve (ROC, upper right), the true activation map (lower left), and a random example of the simulated brain images. We modeled the data set using the kernel regression method. The linear model was estimated using the so-called least angle elastic net method (LARSEN) with a degree of sparsity of 0.2, i.e., using $N = 0.2 \times 400 = 80$ support vectors.

Fig. 4. XOR-image set define by four activated regions. Similar to figure 2, however the image signal-to-noise ratio is $SNR = 0.1$. 
Initial dip data: Visual stimulus (TR 0.33s)

- Gaussian kernel, sparse kernel regression
- Sensitivity map computed for whole slice
- Error rates about 0.03
- How to set
  - Kernel width?
  - Sparsity?
Initial dip data: Visual stimulus (TR 0.33s)

- Select hyperparameters of kernel machine using NPAIRS resampling
  - Degree of sparsity
  - Kernel width, localization of map
Non-linearity in fMRI?  

Visual stimulus: half checker board no/left/right/both

Figure 1: PCA analysis of the fMRI data set. An example of the three first PCs estimated from the training set in a NPAIRS split. The scatter plots show both training (filled markers) and test examples projected onto the PCs. The blue and red voxels on the brain slices corresponds to negative and positive PC loadings respectively. The maps are thresholded to show the 5 upper positive and negative percentiles.
A: Easy problem - (Left vs Right) and RBF kernel is wide ... i.e. similar to linear kernel

B: Easy problem - Pars optimized to yield the best P-R

C: Hard XOR problem - Pars optimized to yield The best P-R
Conclusions

Machine learning in brain imaging has two equally important aims
- Generalizability
- Reproducible interpretation

Can visualize general brain state decoders maps with perturbation based methods (saliency maps, sensitivity maps etc)

NPAIRS split-half based framework for optimization of generalizability and robust visualizations

More complex mechanisms may be revealed with non-linear detectors

Numerous challenges remaining in neuroimage modelling

=> outlook
Outlook – future of mind reading?

- More ecological valid imaging conditions
- Long time observations in the “wild”
- EEG real time 3D imaging for bio-feedback
- 24/7 monitoring

Fig. 1. Handheld brain scanner components. Emotiv EPOC wireless EEG headset (1), Emotiv Receiver module with USB connector (2), USB connector and adapter (3+4), and Nokia N900 mobile phone. The total cost of the system is less than USD1000.

Fig. 3. A user interacting with a 3D model of the brain using the handheld brain scanner device with touch-based interaction.
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www.imm.dtu.dk/~lkh
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