Coherent Inference on Optimal Play in Game Trees

inference in exponentially big trees, in linear time

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Overview

the main idea:

Games evolve smoothly. This insight allows approximate inference on optimal play, using data from nonoptimal play.

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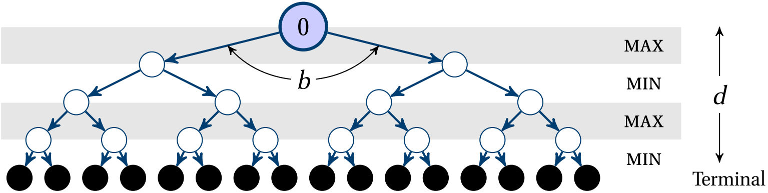
- Reinforcement Learning
- Probabilistic Optimization
- Graphical Models
- Bandit Problems
- Tree Search
- Approximate Inference

Exact Game Tree Search is Hard

games are AND/OR Trees



- real game trees are very big (*Go* has $\sim 10^{400}$ nodes)

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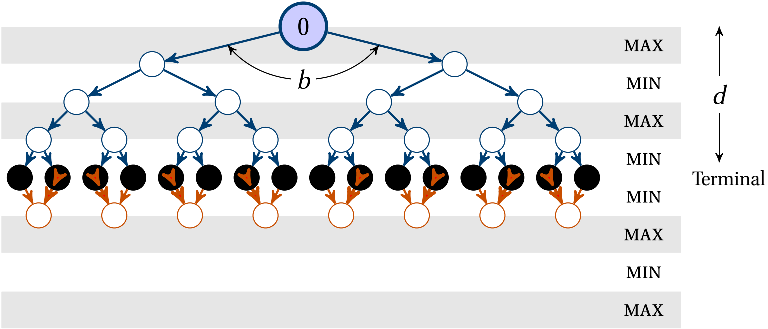

- real game trees are very big (*Go* has $\sim 10^{400}$ nodes)
- a min/max (AND/OR) optimization problem
- solving exact is $\mathcal{O}(b^d)$ (with good heuristics, $\mathcal{O}(b^{d/2})$ at best)

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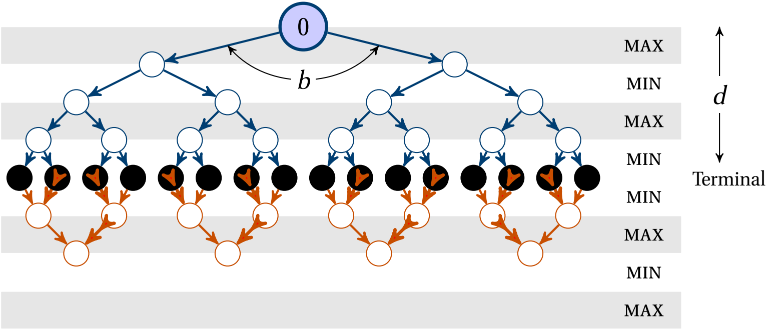

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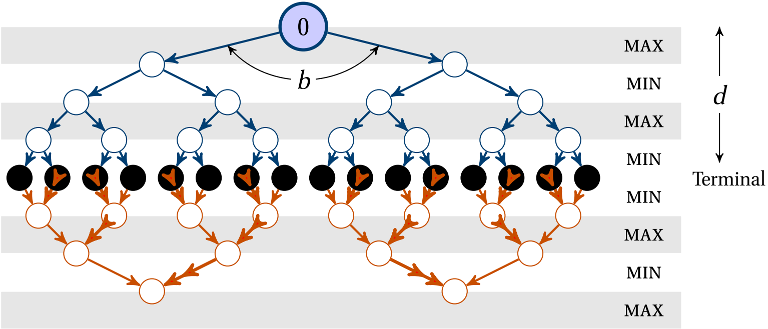

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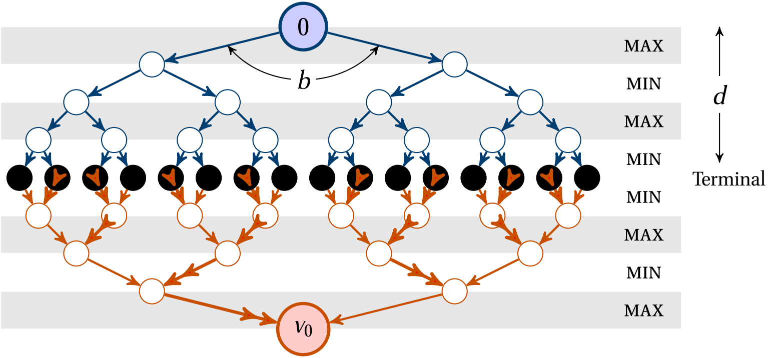

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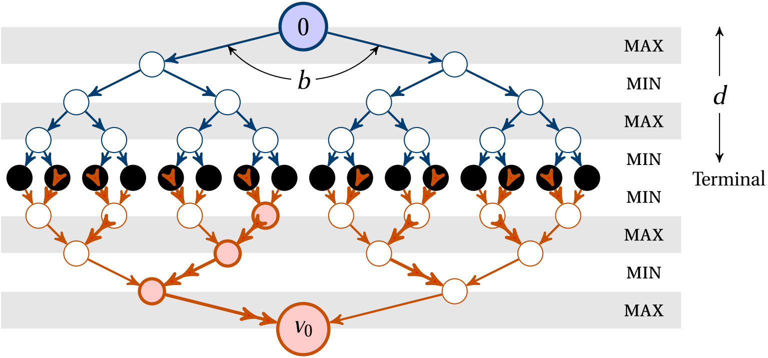

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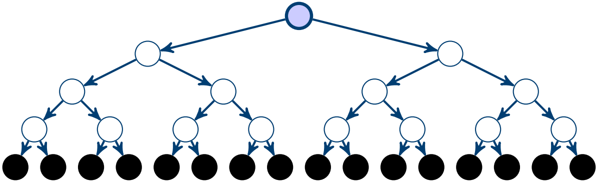
State of the Art: Monte Carlo Tree Search

stochastically explore tree, focus on promising parts



- assymetric part of tree stored below the root
- repeated descents through the stored part, roll-outs at end
- performance depends crucially on descent policy
- one such policy: Upper Confidence Bound for Trees (UCT) [Kocsis and Szepesvári, 2006]
- works surprisingly well on Go [Gelly & Silver, 2008]

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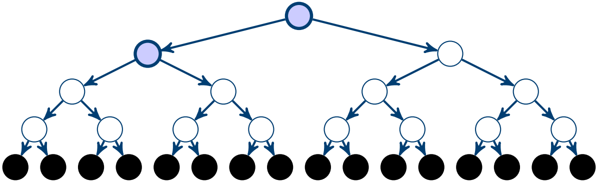

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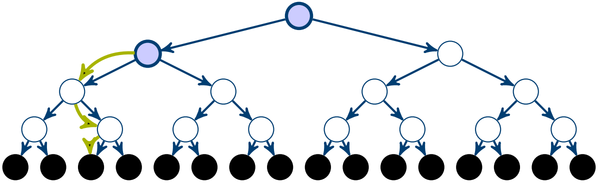

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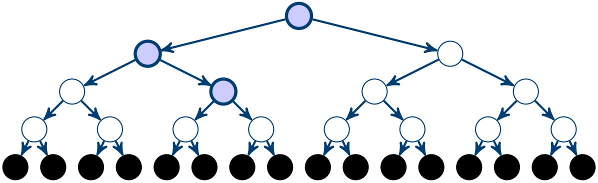

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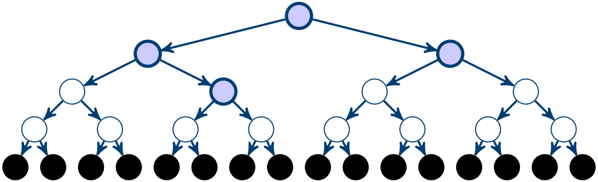

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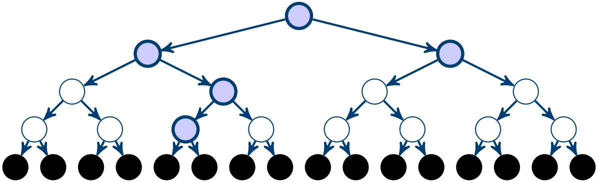

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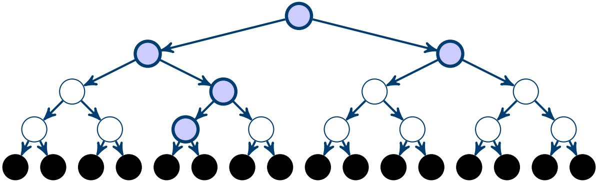

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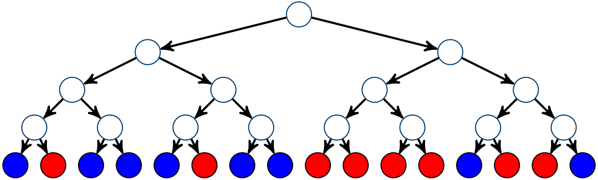
- **why?**

Games Have Structure

wins and losses are clustered among leaves



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- implies a latent score g for non-terminal nodes
- each roll-out contains information about entire tree
- vanilla MC tree search passes information upwards only

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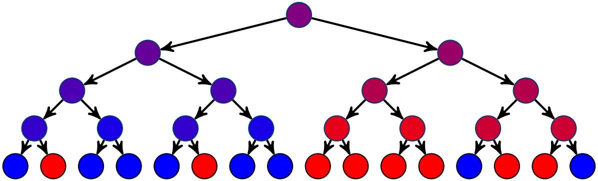

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Building a Bayesian Tree Searcher

rest of this talk

- define generative model for latent scores g
- establish model match with Go
- approximate inference on g
- approximate inference on optimal values v

A Generative Model for On-Policy and Off-Policy Values

random play follows Brownian random walks

- let $G = \{g_i\}$ the values under the *roll-out policy*

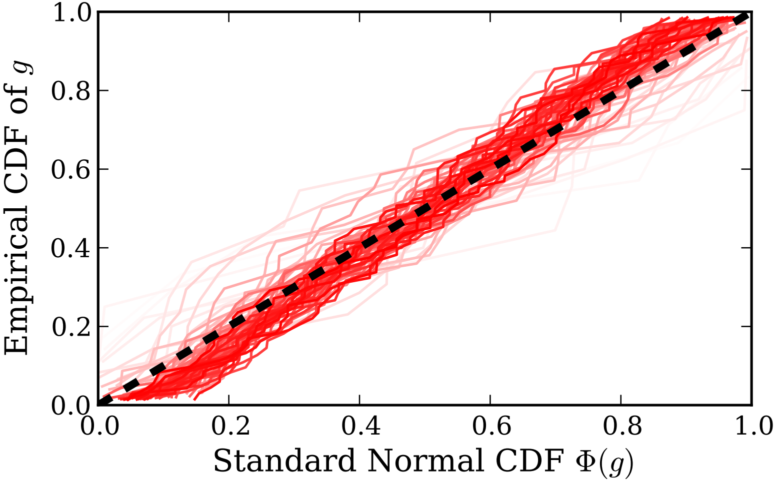
$$p(g_0) = \mathcal{N}(g_0; \mu_0, \sigma_0^2)$$
$$p(g_i | g_{\text{parent}(i)}) = \mathcal{N}(g_i; g_{\text{parent}(i)}, 1)$$

- let $V = \{v_i\}$ the values under *optimal play*

$$v_i = \begin{cases} g_i & \text{if } i \text{ is terminal} \\ \max_{\text{children}(i)}\{v_c\} & \text{if MAX plays at } i \\ \min_{\text{children}(i)}\{v_c\} & \text{if MIN plays at } i \end{cases}$$

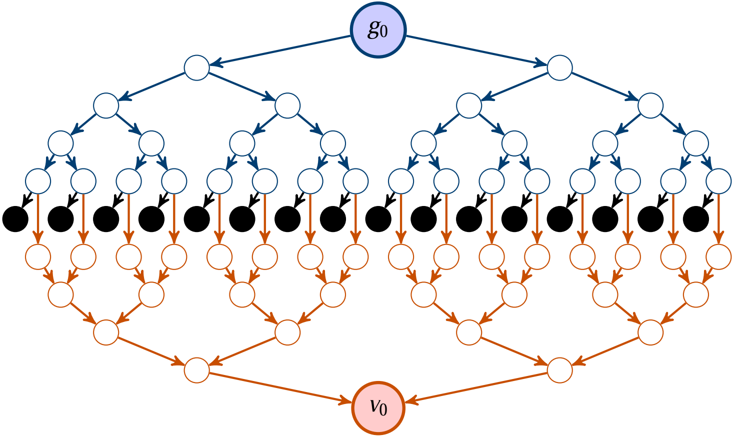
Model Matches Well With Go

empirical distributions of g values relative parents



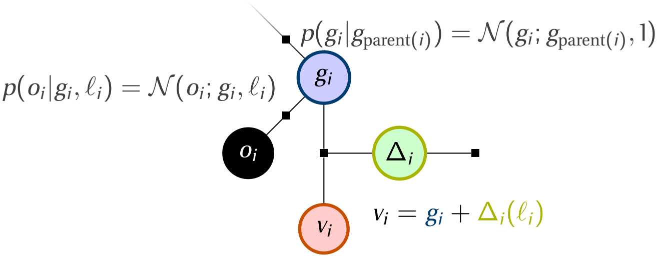
How to Obtain Marginals Over Optimal Values

generative model of optimal values



How to Obtain Marginals Over Optimal Values

inference separates into *inductive* and *deductive* parts



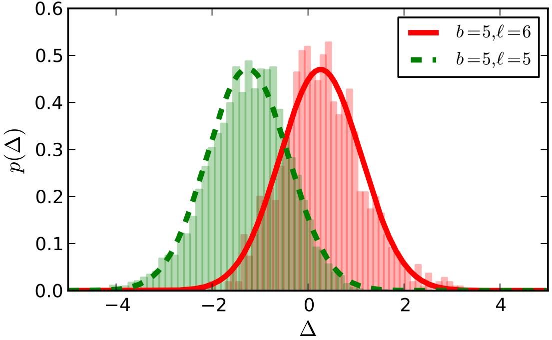
- inference separates into deductive and inductive parts

Inductive Inference by Approximate Density Evolution

Expectation Propagation yields recursive relationship on Δ

$$\Delta_i(\ell_i, b) = \begin{cases} 0 & \text{if } i \text{ terminal} \\ \max_{\text{children } j(i)}\{\Delta_j(\ell_i - 1, b) + \xi_j\} & \text{if } i \mod 2 = 0 \\ \min_{\text{children } j(i)}\{\Delta_j(\ell_i - 1, b) + \xi_j\} & \text{if } i \mod 2 = 1 \end{cases}$$

where $\xi_j \sim \mathcal{N}(0, 1)$



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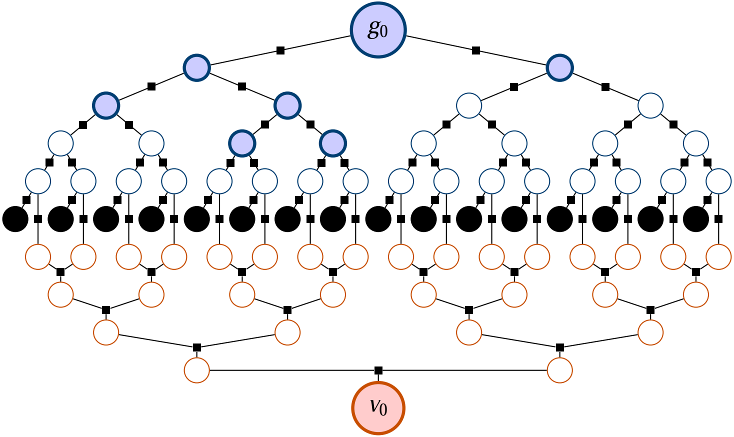
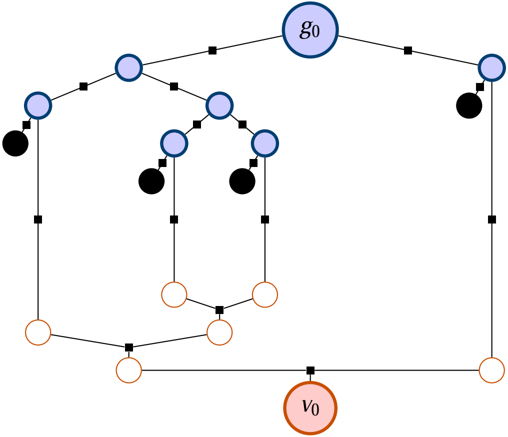
factor graph representation



How to Obtain Marginals Over Optimal Values

replace unobserved parts of the tree with a probabilistic model



Notes

- an instance of Bayesian off-policy reinforcement learning
- can use any policy for stored tree (e.g. UCT) and roll-outs
- inference separates into deductive and inductive parts
- can evaluate belief over optimal value of any node in the tree
- Expectation Propagation keeps inference tractable
- marginals are coherent throughout the tree (i.e. can be used to train global evaluation function)

Conclusion

- game trees have structure
- this allows (approximate) inference on the solution of the min/max optimization problem
- see paper #113 for details