Deterministic Chaos

Blaž Krese
Edvard Govekar

University of Ljubljana
Faculty of Mechanical Engineering
Laboratory of Synergetics
1. Introduction

2. Informal introduction to chaos

3. Double pendulum chaos demonstration

4. Take home messages
Introduction

Dynamical system

High dimensionality, Stochasticity

\[ \dot{x}(t) = F(x(t)), \quad x \in \mathbb{R}^d, \quad t \in \mathbb{R} \]

Low dimensionality, Determinism, Chaos

\[ m\ddot{x} + r_y \dot{x} + k_y x = F_y \]
In science:

Long-term irregular behaviour of deterministic dynamical system as a consequence of sensitive dependence on initial conditions.

Determinism & irregularity

Deterministic chaos
Sensitivity on initial conditions

Deterministic description of complexity

Long term states of deterministic system are unpredictable
Conditions for chaos to emerge

- determinism
- nonlinearity
- dimensionality > 2
- instabilities
- sensitivity on initial conditions

Initial conditions problem in practice

- final precision of measurements
The butterfly effect
Areas of chaos

Scientific disciplines
- Mathematics
- Physics
- Informatics
- Biology
- Economics
  ...
- Psychology
- Chemistry
- Medicine
- Engineering

Systems
- Mechanical
- Production
- Electronic
- Fluid
- Laser
  ...
- Sociological
- Chemical
- Population dynamics
Take home messages

• The world is not linear but mostly nonlinear

• Chaos inherent to many phenomena

• Significant contribution of chaos theory to theoretical and empirical modeling, prediction and control

• But most importantly:
  - high-dimensional dynamical system
  - stochastic system
  - low-dimensional chaotic dynamical system
Engineering system or process

Nonlinearity, Nonstationarity, Instability, Chaos

High dimensionality, noise

Synergetics of complex systems and processes

Courses
- Physics
- Random Phenomena
- Chaotic Dynamics
- Neural Networks
- Evolutionary Computation

Goals:
- model development
- deeper understanding
- diagnostics
- prediction
- optimization and control
- new systems development