Reasoning with Inconsistent Knowledge

This material with Zhisheng Huang & Annette ten Teije
Knowledge will be inconsistent

Because of:

- mistreatment of defaults
- polysemy
- migration from another formalism
- integration of multiple sources

(“Semantic Web as a wake-up call for KR”)
New formal notions are needed

New notions:

- **Accepted:** \( T \models \phi \) and \( T \not\models \neg \phi \)
- **Rejected:** \( T \not\models \phi \) and \( T \models \neg \phi \)
- **Overdetermined:** \( T \models \phi \) and \( T \models \neg \phi \)
- **Undetermined:** \( T \not\models \phi \) and \( T \not\models \neg \phi \)

**Soundness:** (only classically justified results)

\[ T \models \phi \Rightarrow (\exists T' \subseteq T)(T' \not\models \bot \text{ and } T' \models \phi) \]

**Meaningful:** (sound & never overdetermined)

soundness + \( T \models \phi \Rightarrow T \not\models \neg \phi \)
General framework

Use selection function \( s(T,\phi,k) \), with \( s(T,\phi,k) \subseteq s(T,\phi,k+1) \)

1. Start with \( k=0 \):
   \( s(T,\phi,0) \models \phi \) or \( s(T,\phi,0) \models \neg \phi \) ?

2. Increase \( k \), until
   \( s(T,\phi,k) \models \phi \) or \( s(T,\phi,k) \models \neg \phi \)

3. Abort when
   - undetermined at maximal \( k \)
   - overdetermined at some \( k \)
General Framework

\[ T \models \phi \]
\[ T \models \neg \phi \]

\[ s(T, \phi, 2) \]
Nice general framework, but...

- which selection function $s(T,\phi,k)$ to use?
- Simple option: **syntactic distance**
  - put all formulae in clausal form:
  
  \[
  a_1 \land a_2 \land \ldots \land a_n
  \]
  
  - **distance $k=1$** if some clausal letters overlap
  
  \[
  a_1 \land X \land \ldots \land a_n, \quad b_1 \land \ldots \land X \land b_n
  \]
  
  - **distance $k$** if chain of $k$ overlapping clauses are needed
  
  \[
  a_1 \land X \land \ldots \land X_1 \land a_n \\
  b_1 \land X_1 \land \ldots \land X_2 \land b_n \\
  \ldots \\
  \tilde{c}_1 \land X_k \land \ldots \land \ldots \land \land \land \land c_n
  \]
Works surprisingly well

Almost all answers are “intuitive”

- Not well understood why
- Hypothesis:
  - due to local structure of knowledge
- Currently experimenting with more informed selection function $s(T,\phi,k)$
Other approaches:

- Debugging a knowledge base ("don’t live with it, but find the cause")
  - finding the “cause” of the inconsistency
  - = find the smallest set of axioms that, when removed, fix the inconsistency

- Applying belief revision ("don’t just find the cause, but repair it")