Incompleteness as a metaphor

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Dedication

To the Russian school of Mathematics of the 60's, the generation of my parents, a phenomenon that will never be repeated.

References

1. Yu.I. Manin. Mathematics as a metaphor. (Selected essays with a foreword by F. Dyson.) AMS, 2007.
2. Yu.I. Manin. Truth as value and duty: lessons of mathematics. 2008.
3. F. Dyson. Birds and Frogs in Mathematics and Physics. Notices AMS, 2009.
4. V.I. Arnold. What is Mathematics? МЦНМО, 2002, 104 p. (in Russian).
5. V.A. Uspensky. Apology of Mathematics. Amphora, 2009, 560 p. (in Russian).
6. G. Lakoff, R. Núñez. Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being. Basic Books, 2000, 493 p.

Themes

1. Arnold, Bourbakists and "Axiomatizers".
   (Is Mathematical Logic part of Mathematics?)
2. Manin: Birds and Metaphors.
3. Incompleteness as a metaphor.

A test: two questions

Question 1: Is 0 a natural number? (Yes or No, please.)

A test: two questions

Question 2: Is Question 1 relevant? (Does it seriously matter, or it is just a matter of convention.)

Outcome

- If you think it is not relevant, you are a bourbakist.
- If you think it matters, you are a follower of Arnold.

(Accidentally, Arnold believed natural numbers begin with 1, but this is not the main point.)

Some of Arnold's beliefs

- Mathematics is part of Physics, in fact, the cheapest one.
- It is an experimental science, like any other natural science.
- Discovering theorems is more important than proving them.
- Examples are important. Cf. I. Gelfand:
  'Theories come and go, the examples remain'.
- Abstract notions (such as groups or manifolds) are irrelevant: e.g., any group is just a group of transformations.
- Formalization and axiomatization kills the spirit of mathematics. Hence: the 'axiomatizers' such as Leibnitz, Cauchi, and Hilbert are evil. The good are: Newton, Riemann, and especially Poincaré.
- Logic is not mathematics.

Why (I think) mathematical logic is mathematics?

- Not because it is applicable to problems in mathematics;
- Not because it studies formal systems or provides foundations for mathematics;
- Only because it uses mathematical method: we work like mathematicians, give definitions, prove theorems, etc.;
- Even though a peculiar and fairly isolated kind of mathematics is created in doing it.
- One can be a logician and share Arnold's beliefs (I share some but not all of them). Provided one considers formal systems as a natural phenomenon (part of Physics).

Question: Why do we want logic to be part of mathematics?

Metaphors 1

Ordinary (poetic) metaphors:

'A proof is a route, which might be a desert track boring and unimpressive until one finally reaches the oasis of one's destination, or a foot path in green hills, exciting and energizing, opening great vistas of unexplored lands and seductive offshoots, leading far away even after the initial destination point has been reached.'
(Yu. I. Manin)

Metaphors 2

Conceptual metaphors

Lakoff & Johnson, 'Metaphors we live by'.

E.g. the words we use:

'Foundations of Mathematics'
'Incompleteness'

Metaphors 3

Mathematical metaphors (after Manin)

Theory – model – metaphor

‘A mathematical metaphor, when it aspires to be a cognitive tool, postulates that some complex range of phenomena might be compared to a mathematical construction’