How to quantify the influence of correlations on investment diversification

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Investor’s story

Coca-Cola
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DIVERSIFICATION

Diversification?

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Mean-Variance portfolio (Markowitz, 1952)

- \( M \) stocks:
  - average returns \( \mu_i \)
  - return variances \( V_i \)
  - return correlations \( C_{ij} \) (matrix \( M \times M \))

portfolio return:
\[
R_P = \sum_{i=1}^{M} f_i \mu_i
\]

portfolio variance:
\[
V_P = \sum_{i=1}^{M}, j=1 \ldots M f_i f_j C_{ij} \sqrt{V_i V_j}
\]

mean-variance portfolio:
minimizes \( V_P \) for a given \( R_P \)
Mean-Variance portfolio (Markowitz, 1952)

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- **our portfolio**: fractions of wealth \( f_i \) invested in individual stocks

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Mean-Variance portfolio (Markowitz, 1952)

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  \]

- **mean-variance portfolio**: minimizes $V_P$ for a given $R_P$
the optimal portfolio variance

$$V_P^*(R_P, M, C) = \ldots$$

let’s focus purely on correlations: $$\mu_i = \mu, \ V_i = V$$
the optimal portfolio variance

\[ V^*_P(R_P, M, C) = \ldots \]

let's focus purely on correlations: \( \mu_i = \mu, \ V_i = V \)

**effective portfolio size** \( m_{ef} \)

optimal portfolio constructed from \( M \) correlated assets \( \iff \) optimal portfolio constructed from ??? uncorrelated assets
the optimal portfolio variance

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effective portfolio size $m_{ef}$

optimal portfolio constructed from $M$ correlated assets $\iff$ optimal portfolio constructed from $\implies m_{ef}$ uncorrelated assets

$V_P^*(R_P, M, C) = V_P^*(R_P, m_{ef}, 1) \implies m_{ef}$
Effective portfolio size: properties

\[ m_{\text{ef}} = \sum_{i,j=1}^{M} (C^{-1})_{ij} \]
Effective portfolio size: properties

\[ m_{ef} = \sum_{i,j=1}^{M} (c^{-1})_{ij} \]

- no correlations:
  \[ m_{ef} = M \]
Effective portfolio size: properties

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- perfect correlations:
  \[ m_{ef} = 1 \]
Effective portfolio size: properties

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- no correlations:
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- perfect correlations:
  \[ m_{ef} = 1 \]

- \( N \) groups of stocks with no inter-group correlations:
  \[ m_{ef} = m_{ef}(1) + \cdots + m_{ef}(N) \]
Effective portfolio size: saturation

- all correlations identical:

\[ m_{\text{ef}} = \frac{M}{1 + (M - 1)C} \]
Effective portfolio size: saturation

- all correlations identical:

\[ m_{ef} = \frac{M}{1 + (M - 1)C} \rightarrow \frac{1}{C} \]
Effective portfolio size: saturation

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Effective portfolio size: evolution

20 current stocks from the DJIA (Jan 1973—Apr 2008)
The end

Thank you for your attention