Spiraling toward complete markets and financial instability

Matteo Marsili
Abdus Salam ICTP, Trieste

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Bad guys or bad theories?

- “... very frequently the “world images” that have been created by “ideas” have, like switchmen, determined the tracks along which action has pushed the dynamic of interest.” (M. Weber)
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• The game:
  a) consumers in a risky world
  b) the financial industry: engineer new trading instruments

General Equilibrium Theory: optimality with complete markets
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• The game:
  a) consumers in a risky world
  b) the financial industry: engineer new trading instruments
  General Equilibrium Theory: optimality with complete markets

• Results:
  • in an ideal world: i) completeness = instability
    ii) trading volumes in interbank market diverges
  • in non-ideal world: i) derivative markets destabilize underlying markets
    ii) from supply limited to demand limited equilibria
Outline

- The General Equilibrium Theory perspective: What is the role of financial markets?
- A simple model of a complex market
- Spiraling toward market completeness in ideal markets
- Non-ideal markets: some preliminary results
- Conclusions
The perspective of General Equilibrium Theory:

- Tomorrow: rain or sun?
  - wait and buy sunglasses or umbrella
  - Inefficient, if e.g. tomorrow
  - price of sunglasses > price of umbrella
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- **Contingent commodity markets:**
  markets and prices, open today for
  (sunglasses if rain), (sunglasses if sun), (umbrella if rain), (umbrella if sun)
  Today: shopping in contingency commodity markets
  Tomorrow: delivery and consumption
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- **Optimal allocation under perfect competition**
What if contingent commodity markets do not exist?
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- Financial market: 1 riskless $B_t$ and 1 risky $S_t$ assets
  - Today $B_0 = S_0 = 1$
  - Tomorrow $B_1 = 1, S_1 = 1 + u$ if sun, $S_1 = 1 - d$ if rain
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- I want to have $C^{\text{rain}}$ euros to buy an umbrella if it rains and $C^{\text{sun}}$ euros to buy sunglasses if it is sunny. Can I do that? How much does it cost?
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  \[
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  \]
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- How much does it cost?

  \[ C_0 = z_B + z_S = \frac{d}{u + d}C^{\text{sun}} + \frac{u}{u + d}C^{\text{rain}} = E_q[C_{t=1}] \]
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- This can be done for any contingent claim $C^w$. Independent of probability!
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  \]

• This can be done for any contingent claim \( C^w \). Independent of probability!

• Assumptions:
  i) perfect competition
  ii) full information
  iii) no-arbitrage: \( ud > 0 \)
  iv) complete market: what if there are three states? (e.g. sun, cloud, rain)
The financial innovation spiral
(Merton and Bodie 2005)

"As products such as futures, options, swaps, and securitized loans become standardized [...] the producers (typically, financial intermediaries) trade in these new markets and volume expands; increased volume reduces marginal transaction costs and thereby makes possible further implementation of more new products and trading strategies by intermediaries, which in turn leads to still more volume [...] and so on it goes, spiraling toward the theoretically limiting case of zero marginal transactions costs and dynamically complete markets."
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“When particular transaction costs or behavioral patterns produce large departures from the predictions of the ideal frictionless neoclassical equilibrium for a given institutional structure, new institutions tend to develop that partially offset the resulting inefficiencies. In the longer run, after institutional structures have had time to fully develop, the predictions of the neoclassical model will be approximately valid for asset prices and resource allocations.”

(see also R. J. Shiller, “The Subprime Solution” 2008)
A simple model of a complex financial market

<table>
<thead>
<tr>
<th></th>
<th>consumers</th>
<th>market</th>
<th>banks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Today</strong></td>
<td>max E[u(c)] buy assets</td>
<td>portfolio ⇒</td>
<td>sell financial instruments</td>
</tr>
<tr>
<td><strong>Tomorrow (?)</strong></td>
<td>buy and consume</td>
<td>payoff ⇐</td>
<td>state dependent return</td>
</tr>
</tbody>
</table>
The game: $N$ assets, $\Omega$ states

The market

\[
\begin{array}{cccc}
  r_1^1 & \ldots & r_1^\omega & \ldots & r_1^\Omega \\
  \vdots & \ddots & \vdots & \ddots & \vdots \\
  r_k^1 & \ldots & r_k^\omega & \ldots & r_k^\Omega \\
  \vdots & \ddots & \vdots & \ddots & \vdots \\
  r_N^1 & \ldots & r_N^\omega & \ldots & r_N^\Omega \\
\end{array}
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  r_N^1 & \cdots & r_N^\omega & \cdots & r_N^\Omega \\
\end{array}
\]

Financial industry (banks)
The game: \( N \) assets, \( \Omega \) states

\[
\begin{array}{cccccc}
\cdots & r_1^1 & \cdots & r_1^\omega & \cdots & r_1^\Omega \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
z_N & r_N^1 & \cdots & r_N^\omega & \cdots & r_N^\Omega \\
\end{array}
\]

\[
\max_{\bar{z} \geq 0} E \left[ u \left( c(\bar{z}) \right) \right]
\]
The game: \( N \) assets, \( \Omega \) states

<table>
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<tr>
<th>Investors</th>
<th>demand</th>
<th>The market</th>
<th>Financial industry</th>
</tr>
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<tbody>
<tr>
<td>( z_1 )</td>
<td>( r_1^1 ) \ldots ( r_1^\omega ) \ldots ( r_1^\Omega )</td>
<td>( r_k^1 ) \ldots ( r_k^\omega ) \ldots ( r_k^\Omega )</td>
<td>( z_1 ) \ldots ( z_N )</td>
</tr>
<tr>
<td>\vdots</td>
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</tr>
<tr>
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<td>( r_N^1 ) \ldots ( r_N^\omega ) \ldots ( r_N^\Omega )</td>
<td>( r_{N+1}^1 ) \ldots ( r_{N+1}^\omega ) \ldots ( r_{N+1}^\Omega )</td>
<td>(banks)</td>
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\max_{\vec{z} \geq 0} E \left[ u \left( c(\vec{z}) \right) \right]
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The game: \( N \) assets, \( \Omega \) states

\[
\begin{array}{cccc}
& \text{demand} & & \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\ddots & \ddots & \ddots & \ddots & \ddots \\
\text{Investors} & z_1 & r_1^1 & \cdots & r_1^\Omega \\
& \vdots & \vdots & \ddots & \vdots & \vdots \\
& \vdots & \vdots & \ddots & \ddots & \ddots \\
& z_N & r_N^1 & \cdots & r_N^\Omega \\
\end{array}
\]

\[
\begin{array}{cccc}
& \text{The market} & & \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\ddots & \ddots & \ddots & \ddots & \ddots \\
\text{financial industry} & r_1^1 & \cdots & r_1^\Omega & r_1^1 \\
& \vdots & \vdots & \ddots & \vdots & \vdots \\
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& r_N^1 & \cdots & r_N^\Omega & r_N^1 \\
\end{array}
\]

Financial industry

\[
\max_{\bar{z} \geq 0} E [u (c(\bar{z}))]
\]

\[N, \Omega \to \infty, \quad n = \frac{N}{\Omega}\]
Optimizing consumers

Solution of optimal consumption

\[ \frac{\partial}{\partial z_i} E_{\pi} [u(c^\omega)] = \sum_{\omega} \pi^\omega \frac{u'(c^\omega)}{p^\omega} r^\omega \ \begin{cases} = 0 & \Leftrightarrow \ z_i > 0 \\ < 0 & \Leftrightarrow \ z_i = 0 \end{cases} \]

i) investors select the assets which are traded \[ z_i > 0 \]

ii) they determine the Equivalent Martingale Measure (EMM)

\[ q^\omega = \pi^\omega \frac{u'(c^\omega)}{Qp^\omega}, \quad Q = \sum_{\omega} \pi^\omega \frac{u'(c^\omega)}{p^\omega} \]
A creative financial sector

- Financial instruments are drawn at random from a probability distribution with

\[ E_\pi [r_i] = \sum_\omega \pi^\omega r_i^\omega = -\frac{\epsilon}{\Omega}, \quad \text{Var} [r_i] = \frac{1}{\Omega}, \quad i = 1, \ldots, N \]

- Successful innovations \((z_i > 0)\) are not independent draws!
Theory: statistical mechanics

Typical behavior of self-averaging quantities

\[
\lim_{\Omega \to \infty} \left\langle \max_{\vec{z} \geq 0} E[u(c^\omega)] \right\rangle_{\vec{p}, \hat{a}} = \lim_{\beta \to \infty} \lim_{\Omega \to \infty} \frac{1}{\beta} \left\langle \log Z(\beta) \right\rangle_{\vec{p}, \hat{a}}
\]

1- The partition function
\[
Z(\beta) = \sum_{\{\vec{z} \geq 0\}} e^{\beta u[c^\omega(\vec{z})]}
\]

2- The replica trick
\[
\left\langle \log Z \right\rangle_{\vec{p}, \hat{a}} = \lim_{r \to 0} \frac{1}{r} \log \left\langle Z^r \right\rangle_{\vec{p}, \hat{a}}
\]

3- For integer \(r\)
\[
\left\langle Z^r \right\rangle_{\vec{p}, \hat{a}} = \sum_{\{\vec{z}_1 \geq 0\}} \cdots \sum_{\{\vec{z}_r \geq 0\}} \left\langle e^{\beta \sum_{a=1}^r u[c^\omega(\vec{z}_a)]} \right\rangle_{\vec{p}, \hat{a}}
\]
\[
= \int d\hat{\Phi} e^{r\beta \nu(r, \beta, \hat{\Phi})} \quad \hat{\Phi} = \text{order parameters}
\]

4- Saddle point:
\[
\lim_{\Omega \to \infty} \left\langle \max_{\vec{z} \geq 0} E[u(c^\omega)] \right\rangle_{\vec{p}, \hat{a}} = \lim_{\beta \to \infty} \lim_{r \to 0} \max \nu(r, \beta, \hat{\Phi})
\]

(De Martino et al. Macroecon. Dyn. 2007)
The typical behavior

- **Observables:**
  - response function
  - EMM dispersion
  - market completeness
  - volume (or revenue)

\[
\chi = \lim_{\beta \to \infty} \frac{\beta}{2N} \sum_{i=1}^{N} (z_{i,a} - z_{i,b})^2 = \frac{1}{N} \sum \frac{\delta z_i}{\delta p_i^0}
\]

\[
\sigma = |q - \pi|
\]

\[
\phi = |\{ i : z_i > 0 \}| / \Omega
\]

\[
V = \sum z_i
\]

- **Consistency relations**
  - Conservation
  - no-arbitrage

\[
1 = \langle c^* p \rangle_{t,p} + \epsilon n \langle z^* \rangle_t
\]

\[
E_q[c^\omega p^\omega] = E_q[1] = 1
\]
Independent of \( u(c) \) & \( p \)

- \( \chi \to \infty \ \forall \ \varepsilon \)
- \( \sigma \to 0 \) for \( \varepsilon > 0 \)
- \( \sigma \to \infty \) for \( \varepsilon < 0 \)
- For \( \varepsilon > 0 \)
  singularity = complete market (\( \varepsilon = 0, \ n > 2 \))
- For \( \varepsilon < 0 \)
  singularity < complete market


**Distribution of c**

\[ \sigma \to \infty \text{ for } \varepsilon < 0 \]

\[ \varepsilon = -0.01 \]

\[ \sigma \to 0 \text{ for } \varepsilon > 0 \]

\[ n=10 \]
Increasing financial complexity

$\epsilon = 0.01, 0.05, 0.10$
Learning to invest

$\epsilon = 0.01, \quad \gamma = 0.5, \quad \Omega = 32$

Hard to learn when market is nearly complete

(cfr Brock, Hommes, Wagener, 2006)

\[
\sigma^2 = \frac{1}{\Omega} \sum_{\omega} (q^\omega - \bar{q})^2
\]

\[
\langle z \rangle = \frac{1}{N} \sum_{i} z_i
\]
A competitive Financial Industry

- Part of the risk of a new instrument can be hedged buying existing instruments

- Residual risk
  \[
  \Sigma = \min_{\bar{u}} \text{Var} \left[ r^\omega_{\text{new}} - \sum_i v_i r^\omega_i \right] = 1 - \phi
  \]

- Risk premium vanishes as markets become complete e.g. Mean Variance profit function
  \[
  \Rightarrow \epsilon = \frac{\gamma}{2} (1 - \phi)
  \]

- The weights of portfolios used to hedge each instrument diverges as \( \phi \to 1 \)
  \[
  \sum_i v_i^2 = \frac{\phi}{1 - \phi}
  \]

- Susceptibility in the interbank market also diverges
A competitive Financial Industry

- Part of the risk of a new instrument can be hedged by buying existing instruments.

- Residual risk

\[ \sum = \min_{\vec{u}} \text{Var} \left[ r_{\text{new}} - \sum_i v_i r_i^\omega \right] = 1 - \phi \]

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\[ \sum_i v_i^2 = \frac{\phi}{1 - \phi} \]

- Susceptibility in the interbank market also diverges.
the original problem for vanishing optimization problem. To define this quantity, one first introduces a small variation in the utility that captures sensitivity with respect to the definition of the utility function. Likewise, it is possible to define another quantity which turns out to play a key role: the risk premium.

Figure 1: Phase diagram of the economy. The equilibrium is unstable in the shaded region. The dashed and dotted lines correspond to “trajectories” where the susceptibility diverges. The two trajectories are depicted in Figure 1, and at the optimum one finds

\[ \epsilon = \frac{\gamma}{2} \sum \]

Mean variance banks

Consumer market: infinite susceptibility, finite volume.

Consumer market: unstable

Interbank market: both susceptibility and volumes diverge as \( \phi \to 1 \).
Conclusions I

• The proliferation of financial instruments, even in an ideal world (perfect competition and full information), is problematic

• Complete markets lie on a critical line with infinite susceptibility

• A competitive financial sector is expected to converge to this singularity

• The volume generated by banks to hedge financial instruments they sell diverges as market approaches completeness

• Learning to invest optimally is hard (as in Brock, Hommes, Wagener 2006)

• Market imperfections amplified close to complete markets: institution size grows with financial complexity
Illiquid markets: underlying and derivatives

<table>
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Derivatives:

\[ f_h^\omega = F_h(r_1^{\omega}, \ldots, r_N^{\omega}) - f_h^0 \]

Return of underlying:

\[ r_k^{\omega} = \rho(z_k, \zeta_1, \ldots, \zeta_H) \]

Price of derivatives:

\[ f_h^0(z_1, \ldots, z_N, \zeta_1, \ldots, \zeta_H) \]
Illiquid markets:
N derivatives on 1 underlying

- derivative:
  pay $c$ today $\Rightarrow a^\omega$ units of asset in state $\omega=1,\ldots,\Omega$ tomorrow

- don’t sell

- sell!
Illiquid markets: 
N derivatives on 1 underlying

- derivative: 
  pay \( c \) today \( \Rightarrow a^\omega \) units of asset in state \( \omega = 1, \ldots, \Omega \) tomorrow

\[ N, \Omega \to \infty, \quad n = \frac{N}{\Omega} \]
The price of the underlying

\[ p^\omega(t = 1) \equiv 1 + r^\omega = D^\omega + \sum_{i=1}^{N} s_i a_i^\omega \]

\( s_i = \text{supply of derivative } i \)

\( > 0 \text{ if } E[\text{profit}] > \text{risk premium} \)
Competitive equilibria

- For general demand functions
- Banks supply a quantity of derivative contracts \( \{s_i, i=1,\ldots,N\} \) which is given by the minima of the function

\[
H = \frac{1}{2} \sum_{\omega=1}^{\Omega} \pi^\omega \left( d^\omega + \sum_{i=1}^{N} s_i a_i^\omega \right)^2 + \sum_{i=1}^{N} g(s_i)
\]

return\(^2\) \hspace{1cm} g \text{ related to inverse demand function}

= GC Minority Game
Phase diagram

\[ \chi \to \infty \]
on phase boundary
Increasing financial complexity

$$\epsilon = c_i - c_i^{(0)} - \rho_i \sim \text{risk premium}$$

Derivative markets destabilize underlying markets
Conclusions

- System-wide picture of complex markets as large random economies
- Quantifying financial stability
  \[ \chi = \frac{\delta \text{equilibrium}}{\delta \text{parameters}} \]
  fragility when repertoire of instruments expands
- Asset Pricing Theory for illiquid markets
Thanks

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- Pierpaolo Vivo (ICTP)
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