Multiview Fisher Discriminant Analysis

Tom Diethe, David R. Hardoon, John Shawe-Taylor

{t.diethe,d.hardoon,j.shawe-taylor}@cs.ucl.ac.uk

Department of Computer Science
University College London

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Outline

Classification Function

- FDA is Bayes optimal for two normal distributions with equal covariance (KFDA is Bayes optimal in the feature space)
- Outputs of the classifier can be seen as probabilities
- If we simply sum the outputs of KFDA trained on two different view of the same data, the output will always be that of the most confident classifier

$$f(x_i) = \operatorname{sgn}\left(\sum_{j=1}^{k} \langle \mathbf{w}_j, \phi_j(x_i)\rangle + b\right)$$

Convex Formulation

- We follow the approach outlined by [Mika et al., 2001] and provide a disciplined convex form of Multiview Fisher Discriminant Analysis.
- Since we are trying to minimise the variance of the data along the projection whilst maximising the distance between the average outputs for each class over all of the views.
- This can be done it two ways, which we have denoted as the block method and concatenation method.

Block Method

$$\begin{aligned} \min_{\boldsymbol{\alpha}, b, \boldsymbol{\xi}} \quad & H(\boldsymbol{\xi}) + \lambda P(\boldsymbol{\alpha}, \boldsymbol{\beta}) & \\ \text{s.t.} \quad & \mathbf{K}_j \boldsymbol{\alpha}_j + \mathbf{1} b = \mathbf{y} + \boldsymbol{\xi} & j = 1, \ldots, k \\ & \mathbf{1}_i' \boldsymbol{\xi} = 0 & i = 1, 2 \end{aligned}$$

where,

$H(\cdot)$ is the Loss function
$P(\cdot)$ is the Regularisation function

Concatenation Method

$$\min_{\boldsymbol{\alpha}_j, b, \boldsymbol{\xi}} \quad H(\boldsymbol{\xi}) + \lambda P(\boldsymbol{\alpha}_j), \qquad\qquad j = 1, \ldots, k$$

$$\text{s.t.} \quad \sum_{j=1}^{k} \mathbf{K}_j \boldsymbol{\alpha}_j + \mathbf{1} b = \mathbf{y} + \boldsymbol{\xi}$$

- Concatenation method more efficient (scales linearly)
- Block method closer to spirit of KCCA

Regularisation

- The natural choices for the regularisation function $P(\boldsymbol{\alpha}, \boldsymbol{\beta})$ would either be the L_2-norm of the dual weight vectors

$$P(\boldsymbol{\alpha}) = \sum_{j=1}^{k} \|\boldsymbol{\alpha}_j\|_2^2$$

- or the L_2-norm of the primal weight vector

$$P(\boldsymbol{\alpha}) = \sum_{j=1}^{k} \boldsymbol{\alpha}_j' \mathbf{K}_i \boldsymbol{\alpha}_j$$

- More interesting is the L_1-norm of the dual weight vector, as this choice leads to sparse solutions

$$P(\boldsymbol{\alpha}) = \sum_{j=1}^{k} \|\boldsymbol{\alpha}_j\|_1$$

Loss Functions

- Standard loss function assumes Gaussian Densities:
  - $H(\xi) = \|\xi\|_2^2 \quad \Longrightarrow \quad \frac{1}{\sqrt{2\pi}} \exp(-\frac{\xi^2}{2})$
- Can remove Gaussian noise model, resulting in different loss functions, e.g. Laplacian noise model:
  - $H(\xi) = \|\xi\|_1 \quad \Longrightarrow \quad \frac{1}{2} \exp(-|\xi|)$
- Using L_1 regulariser with Laplacian Loss results in a linear programme
- Other loss functions: ϵ-insensitive loss, Huber robust loss etc.

EEG & Music Dataset

- EEG experiment analysed in [Diethe et al., 2008]. The principal hypothesis was that neural patterns should reflect relative changes in the key of music that a listener is attending to.

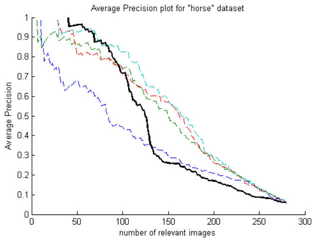
Table: Test errors for within-subject classification for Tonal vs Atonal. The SVM classification results are on the EEG data alone. The KCCA + SVM, and SMFDA classification used a kernel on the music as a second view. ** denotes significance at the $p < 0.001$ level

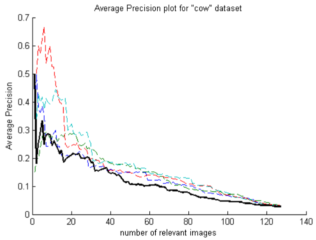
Classifier	# Train	# Test	Linear
SVM (Linear)	1152	383	0.2298**
SVM (RBF)	1152	383	0.1175**
KCCA + SVM (linear)	1152	383	0.0157**
SMFDA	1152	393	0.0000**

VOC 2007 Dataset

- Dataset from the PASCAL VOC2007 challenge [1]
- Features from [Viitaniemi and Laaksonen, 2008], with an extra feature extraction method (SIFT)

Figure: Average precision recall curves for 3 VOC 2007 datasets for Multiview Fisher Discriminant Analysis using the Block method plotted against SOM results





Selected References

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