Modeling Deformable Surfaces from Single Videos

CVLab, EPFL, Switzerland
http://cvlab.epfl.ch/

Ambiguity

• 3D Shape or deformation models are needed.

How can we design models that do not make unwarranted assumptions?

Problem Definition

Given:
- Reference image.
- Corresponding 3D surface.
- Projection matrix P.
- 3D-to-2D correspondences between reference configuration and input image.

Unknowns:
- Mesh vertex coordinates corresponding to input image:
  \[ X = [x_1, y_1, z_1, \ldots, x_n, y_n, z_n]^T \]

Talk Outline

• Very textured surfaces:
  - Linear Formulation;
  - SOCP Formulation.

• Poorly textured surfaces:
  - Smooth deformations;
  - Generic deformations.

Linear Formulation

• Calibrated camera.
• Coordinates expressed in the camera referential.
• Unknown mesh vertex coordinates: \( Y = (y_1^T, \ldots, y_m^T) \), \( y_i = (x_i, y_i, z_i) \)
• Correspondences \( (u_i, v_i)^T \)
  - Barycentric coordinates from reference configuration: \( (a_i, b_i, c_i) \)
  - Current image location:
  - Correspondence equation:
    \[
    \begin{bmatrix}
    u_i \\
    v_i \\
    1
    \end{bmatrix} = \frac{1}{k_i} \begin{bmatrix}
    a_i y_i + b_i z_i + c_i v_i \\
    b_i y_i + a_i z_i + c_i u_i \\
    c_i y_i + a_i x_i + b_i z_i
    \end{bmatrix}
    \]

M. Salzmann, V. Lepetit, and P. Fua, CVPR 2007
Linear System and Singular Values

Single frame: \( \mathbf{M} \mathbf{Y} = \mathbf{b} \)

Multiple frames:

\[
\begin{bmatrix}
\mathbf{M}_1 \\
\mathbf{M}_2 \\
\mathbf{M}_3 \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{Y}_1 \\
\mathbf{Y}_2 \\
\mathbf{Y}_3 \\
\end{bmatrix} = \mathbf{b}
\]

Constraints are required:

- Penalize vertex displacement along the line of sight between consecutive frames.
- Minimal assumption on material properties.

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Second Order Cone Programming

Find \( \mathbf{Y} \) that satisfies the \( m \) constraints:

\[
\left\| \mathbf{A}_i \mathbf{Y} + \mathbf{b}_i \right\|^2 \leq \mathbf{c}_i^T \mathbf{Y} + \mathbf{d}_i, \quad \text{for } i = 1, \ldots, m
\]

→ A convex optimization problem.

Linear Formulation

- Calibrated camera.
- Coordinates expressed in the camera referential.
- Unknown mesh vertex coordinates: \( \mathbf{Y} = (\mathbf{y}_1^T, \ldots, \mathbf{y}_n^T), \mathbf{v}_i = (x_i, y_i, z_i)^T \)
- Correspondences \( (a_i, b_i, c_i)^T \)
  - Barycentric coordinates from reference configuration: \( (a_i, b_i, c_i)^T \)
  - Current image location:
- Correspondence equation:

\[
\begin{bmatrix}
\mathbf{b}_i \\
\mathbf{v}_i \\
1
\end{bmatrix} = \begin{bmatrix}
a_i \\
b_i \\
c_i
\end{bmatrix} (\mathbf{y}_1^T, \ldots, \mathbf{y}_n^T)
\]

Reprojection Error

\[
\left| \mathbf{P}_j \mathbf{h}_i - \mathbf{b}_i \right| \leq |\mathbf{P}_j \mathbf{P}_h | \mathbf{h}_i | = \gamma |\mathbf{b}_i |, \quad \text{for } j = 1, \ldots, m
\]

\[
\min \gamma \text{ subject to } \gamma \geq 0 \text{ and}
\]

For a fixed \( \gamma \), this is a SOCP feasibility problem; \( \gamma \) can be determined by bisection.
Outlier Rejection

Problem:
• Minimal \( \gamma \) takes the value of the worst correspondence.
• Other reprojection errors can be worse than they should.

Solution:
• The set of points with reprojection error = \( \gamma \) contains outliers.
• Remove them.
• Re-optimize.
• Iterate until \( \gamma \) is less than 2 pixels.

Correspondences are not enough!

--> Ambiguities remain.

Frame to Frame Constraints

Between consecutive frames, neither the orientation nor the length of an edge can change excessively.

\[
\begin{align*}
\begin{bmatrix} v_{t+1}^i \\ v_{t+1}^j \\ v_{t+1}^k \\ v_{t+1}^{l+1} \\ \vdots \\ v_{t+1}^m \end{bmatrix} &= \begin{bmatrix} v_{t}^i \\ v_{t}^j \\ v_{t}^k \\ v_{t}^{l+1} \\ \vdots \\ v_{t}^m \end{bmatrix} + L_i \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ \vdots \\ -1 \end{bmatrix}
\end{align*}
\]

Waves and Creases

Cloth

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Statistical Learning Approach

For a given object or material
1. Create database of possible deformations.
2. Learn a low dimensional model from it.
3. Fit by minimizing with respect to the model and pose parameters.

Creating the database is non-trivial because the space of possible deformations is huge.

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Deformation Modes

\[ Y = Y_0 + \sum \alpha_i S_i \]

\[ = Y_0 + SA \]

with \( A = [\alpha_1 \ldots \alpha_N]^T \)

Real-Time 3D Shape

Database of Feasible Shapes

PCA

Alinghi

Racing Spinnakers
Inextensible Meshes

Mesh whose coordinates are expressed in the camera referential:

\[ MY = 0 \Rightarrow Y = \sum p_i \]

where the \( p_i \) are the eigenvectors corresponding to small eigenvalues.

Inextensible mesh:

\[ \| \sum p_i \| \cdot \| \sum p_i \| = \text{cte} \]

for all neighboring vertices \( j \) and \( k \).

\[ \rightarrow \text{A system a quadratic equations that could be solved in closed form using extended linearization, but with too many variables for existing solvers.} \]

Closed Form Solution

\[ MY = 0 \]
\[ Y = Y_0 + SA \]
\[ \begin{bmatrix} MS & MY \end{bmatrix} \begin{bmatrix} A \end{bmatrix} = 0 \]

where the \( W \) is a diagonal matrix of modal penalty terms.

\[ \begin{cases} \text{A can be expressed a weighted sum of eigenvectors of the extended matrix.} \\
\text{The inextensibility constraints give rise to a smaller set of quadratic equations than can now be solved.} \end{cases} \]

M. Salzmann et al., ECCV 2008

Detection in Every Frame

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From Local to Global

- Learn a local deformation model.
- Form a global model using a PoE framework.

\[ \rightarrow \text{Requires far less training data that learning a global model.} \]

Very Little Texture

M. Salzmann, R. Urtasun, and P. Fua, CVPR 2008
Different Topology

Wing Deformation

- Compare predicted and observed values.
- Improve simulation software until the two match.
  → Virtual wind tunnel.

Intelligence Gathering

A Generic Paradigm

Automated 3D deformable surface detection:

- Obtain training data using textured surfaces.
- Learn a deformation model.
- Apply such model to detect less textured surfaces.

→ A robust method that is easy to deploy.

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