Broken symmetries in financial markets

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Exchange goods without a market
Exchange goods without a market

- Find something to buy/sell
- Find somebody interested
- Think of a fair price
- Agree on a price
- Exchange goods
Internet era

- Find something to sell/buy
- Go to auction site
- Auctioneers determine price
- Goods travel by post

MARKET!!!!

The more bidders, the better the price
Role of financial markets

Centralised markets
- Exchange goods
- Speed
- Good prices
- Low transaction costs

The more traders, the better the prices
Financial markets

Real goods:
- Stocks
- Commodities (oil, gold, ...)
- Foreign exchange
Financial markets

Real goods:
- Stocks
- Commodities (oil, gold, ...)
- Foreign exchange

Risk-related goods:
- Bonds
- Futures
- Options
- Insurance
- Structured products
- ...
Double auction

eBay

- One item
- Several buyers: bids
- Several sellers: asks
1 year = 1 Tb of data

- Daily CERN-like experiment
- Much noise
- What to measure?
- How many computers?
Symmetries

- buy ≡ sell
- buy, sell ≡ sell/buy

Efficient markets

- $E[p(t + 1)] = p(t)$
- $r_\tau(t) = \log p(t + \tau) - \log p(t)$: price return

\[
E[r(t)r(t + T)] = 0
\]

(necessary condition)
Why symmetry in markets?

Because markets hate asymmetry. Any asymmetry is detected and corrected.
Why symmetry in markets?

*Because markets hate asymmetry*

Any asymmetry is detected and corrected

Quality $\rightarrow$ reputation
Why markets
Symmetric markets?
Broken symmetries

Modelling

Simplest
- Bachelier: Gaussian uncorrelated random walk
- Mandelbrodt: Levy uncorrelated random walk
- Now: power-law random walks with long memory (stay awake)
Typical agent-based model

- $N$ agents
- $S_i(t)$ state of agent $i \in \{-1, 0, 1\}$
- $S_i(t + 1) = \text{sign} \left[ \sum_j J_{i,j} S_j(t) + h_i S_i(t) + F \right]$
- $\log p(t + 1) = \log p(t) + \sum_i S_i(t)$
Buy/sell

Long position

- Time $t$, buy $N$ shares
- Time $t'$, sell $N$ shares
- Capital gain:

$$N[p(t' + 1) - p(t + 1)] - T(N)$$

where $T(N)$ transaction costs
Buy/sell

Long position
- Time $t$, buy $N$ shares
- Time $t'$, sell $N$ shares
- Capital gain:

$$N[p(t' + 1) - p(t + 1)] - T(N)$$

where $T(N)$ transaction costs

Short position
- Time $t$, sell $N$ shares
- Time $t'$, buy $N$ shares
- Capital gain:

$$-N[p(t' + 1) - p(t + 1)] - T(N)$$

where $T(N)$ transaction costs

Broken symmetries: $T(N)$, maximum gain
Limit/market order

- Market order: BUY NOW!
- Limit order: buy at price $p$

Bouchaud et al:
- Cost limit order = Cost market order electronic markets
- Cost limit order < Cost market order markets w/ human
Microscopic asymmetry

- More buy limit orders than sell orders
- Limit order markets have a long memory:
Microscopic asymmetry

- More buy limit orders than sell orders
- Limit order markets have a long memory:
  - Buy 100, Sell 100 \(\not\equiv\) Sell 100, Buy 100
Microscopic asymmetry

- More buy limit orders than sell orders
- Limit order markets have a long memory:
  - Buy 100, Sell 100 NOT ≡ Sell 100, Buy 100
  - Buy 100, Buy 10 NOT ≡ Buy 10, Buy 100
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Winning ratios

Measures of success:
- % of successful trades
- won/(won + lost)

Distribution of gains (trend followers) (Bouchaud Potters):
Heterogeneity

- information cost
- trading frequency
- wealth
- risk profile
- annual turnover

trading frequency $\leftrightarrow$ wealth, risk profile, turnover
Asymmetric information

information cost $\rightarrow$ heterogeneity
- processing power
- sources of information
- education
- honesty (insider information)
(In-)efficient markets

- $E[r(t)] = 0$ TRUE
- $E[r(t) - T|X] \neq 0$?
- What is $X$?
- How many of them?

Symmetry breaking, phase transition:

$$H = \sum_X E[r(t) - T|X]^2 > 0$$
In real markets?

- $H > 0$ ALWAYS
- Risk?
- $\frac{H}{(\delta H)^2}$ significant?
- $H > 0$ stabilizing?
Agent-based models

Usual aim: reproduce market behavior

Most important ingredient
- predictability $H$
- central to real life
- *how does it disappear?*
Why markets
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Trading frequency

- Who knows more about the market?
- Information spread?

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Time reversal asymmetry

\[ X(p(t)) = X(p(-t)) \]

- Humans not time reversal
- News not time reversal
- Markets?
  - who cares?
Measures of time reversal asymmetry

- Visually: Omori law
- autocorrelation NO
- asymmetry past/future.
Zumbach volatility asymmetry

- price returns noisy
- most likely asymmetric

Historical volatility over $T$ units of time of $\tau$ seconds:
$$\sigma_h(T, \tau)$$

Realized volatility over $T$ units of time of $\tau$ seconds:
$$\sigma_r(T, \tau)$$
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\( P(\delta\sigma) \)

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The mathematics of time reversal asymmetry

- 100s of models
- Only models with heterogeneous time scales OK

\[ \sigma = \sum_n 2^{-\alpha n} \sigma_h(T_0 2^n, \tau) \]

- Information flows between time scales
- Put by hand
New route: Baldovin and Stella

Unit of time \( \tau = 1 \)

\[
 r_{t,T} = \ln p(t + T) - \ln p(t)
\]

\( t \neq t' \)

\[
 E(r_{t,1}, r_{t',1}) = 0
\]

(no linear arbitrage)

price follows random walk

\[
 E[r^2] \propto T^{2D}
\]

\[
 P_T(r) = \frac{1}{T^D} g \left( \frac{r}{T^D} \right)
\]
“Renormalisation”: $T \rightarrow 2T$

\[ P_{2T}(r) = \frac{1}{2^D} P_T \left( \frac{r}{2^D} \right) \]

\[ r = r_1 + r_2 \]

\[ P_{2T}(r) = \int dr_1 dr_2 \ p^{(2)}_{2T}(r_1, r_2) \delta(r - r_1 - r_2) \]

\[ P_T(r_1) = \int dr_2 \ p^{(2)}_{2T}(r_1, r_2) \]

\[ P_T(r_2) = \int dr_1 \ p^{(2)}_{2T}(r_1, r_2) \]
Why markets
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Consequences

Scaling:

\[ E[(r_1 + r_2)^2] = E[(r_1)^2] + E[(r_2)^2] \]

\[ (2T)^{2D} = 2(T^{2D}) \]

→ \( D = 1/2 \)
Consequences 2

Characteristic function

\[ \tilde{p}_{2T}^{(2)}(k_1, k_2) \leftrightarrow p_{2T}^{(2)}(r_1, r_2) \]

\[
P_{2T}(r) = \int dr_1 dr_2 \ p_{2T}^{(2)}(r_1, r_2) \delta(r - r_1 - r_2)
\]

\[
\tilde{p}_{2T}^{(2)}(k, k) = \tilde{g}(\sqrt{2T}k)
\]

\[
P_T(r_1) = \int dr_2 \ p_{2T}^{(2)}(r_1, r_2)
\]

\[
\tilde{p}_{2T}^{(2)}(k, 0) = \tilde{g}(\sqrt{T}k)
\]
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Consequences 2

\[ \tilde{p}_{2T}^{(2)}(k_1, k_2) = \tilde{g} \left( \sqrt{Tk_1^2 + Tk_2^2} \right) \]

- Kind of multiplication in characteristic function space
- \text{Sqrt} introduces a dependency between } r_1 \text{ and } r_2.
- Generalisation to } n \text{ returns:

\[ \tilde{p}_{2T}^{(2)}(k_1, \cdots, k_n) = \tilde{g} \left( \sqrt{Tk_1^2 + \cdots + Tk_n^2} \right) \]

→ Multivariate distribution from univariate distribution
Problems

- Constant volatility
- Time reversal invariant

Solution: dilute time so that
\[ E[r^2(t)r(t + \tau)^2] \propto \tau^{-0.2} \]

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**Process simulation**

**Volatility decay**
Asymmetry of time

(with P.P. Peirano)

Omori’s Law

\[ N(t) = l = 2.4 \sigma \]
\[ l = 2.1 \sigma \]
\[ l = 1.8 \sigma \]
\[ \alpha = 0.44 \]
\[ \alpha = 0.35 \]
\[ \alpha = 0.25 \]
Symmetric asymmetry of time

Zumbach-Lynch mugshot: B-S and USD/CHF (from ZL)

Visible time scale
No information cascade between time scales
External shocks only
Conclusions

- Markets are asymmetric
- Time reversal invariance breaking sorts out models
- Time scales: crucial ingredient
- Time scales and efficiency: open problem