1. (a) The characteristic polynomial \( p(s) = s^2 + 6s + k \) has roots \(-3 \pm \sqrt{9-k}\). The equation is underdamped if the roots are not real, and this happens for \( k > 9 \).

(b) This must be an underdamped equation, with a decaying sinusoidal solution. The zeros happen every \( P/2 = \pi/\omega_d \) time units, so \( \omega_d = 2 \). But \( \omega_d = \sqrt{k - 9} \), so \( k - 9 = 4 \) and \( k = 13 \).

2. (a) This happens only when one of the roots of the characteristic polynomial has a positive real part (or is repeated with nonnegative real part). Thus \( k \leq 9 \) to make the roots real. The term \( \sqrt{9-k} \) must be at least \( 3 \) to make one of the roots positive: so \( k \leq 0 \). \( k = 0 \) leads to roots \( 0, -6; 0 \) is not repeated so the solutions do not grow. We must have \( k < 0 \).

(b) A particular solution is \( x_p = 1 \) and since the roots of the characteristic polynomial are \(-3 \pm 2i\) the general homogeneous solution is \( x_h = e^{-3t}(a \cos(2t) + b \sin(2t)) \). The initial condition requires \( x_h(0) = 0, \dot{x}_h(0) = 1 \). The first gives \( a = 0 \), and then \( \dot{x}_h = be^{-3t}(2 \cos(2t) - 3 \sin(2t)) \), so \( 1 = \dot{x}_h(0) = 2b \) and \( b = 1/2 \): \( x = 1 + (1/2)e^{-3t} \sin(2t) \).

3. (a) \( p(i\omega) = (13 - \omega^2) + 6i\omega \) so the amplitude of the sinusoidal solution is \( 1/|p(i\omega)| = 1/\sqrt{(13-\omega^2)^2 + 36\omega^2} \).

(b) The phase lag is the argument of \( p(i\omega) \). It’s \( 90^\circ \) when \( p(i\omega) \) is purely imaginary (with positive imaginary part). This happens when \( \omega = \sqrt{13} \).

4. (a) \( p(-1) = 1 - 6 + 13 = 8 \) so \( x_p = e^{-t}/8 \).

(b) \[
\begin{array}{c|ccc}
13 & x & = & at & + & b \\
 6 & \dot{x} & = & 0 & + & a \\
 1 & \ddot{x} & = & 0 & + & 0 \\
\hline
13t + 19 & = & 13at & + & (6a + 13b)
\end{array}
\]
so \( a = 1 \) and \( 13b = 19 - 6 = 13 \) or \( b = 1 \): \( x_p = t + 1 \).

5. This is the real part of \( \ddot{z} + 6\dot{z} + 13z = e^{(-3+2i)t} \). The roots of the characteristic polynomial are \(-3 \pm 2i\), so the Exponential Response Formula fails and we must use the Resonant Response formula: \( p'(s) = 2s + 6, p'(-3 + 2i) = 2(-3 + 2i) + 6 = 4i \), so \( z_p = te^{(-3+2i)t}/(4i) = -(it/4)e^{-3t}e^{2it} \) and \( x_p = (t/4)e^{-3t} \sin(2t) \).