Op Amps Positive Feedback
Consider this circuit — *negative feedback*

\[
\frac{v_{IN}}{R_1} + R_2 v_{OUT} = -\frac{R_2}{R_1} v_{IN}
\]

and this — *positive feedback*

\[
\frac{v_{IN}}{R_1} + R_2 v_{OUT} = -\frac{R_2}{R_1} v_{IN}^{*}
\]

What's the difference?

Consider what happens when there is a perturbation...

Positive feedback drives op amp into saturation:

\[v_{OUT} \rightarrow \pm V_S\]
Static Analysis of Positive Feedback Ckt

\[ v_{\text{OUT}} = A(v^+ - v^-) \]

\[ = Av^+ \]

\[ = A \left[ \frac{v_{\text{OUT}} - v_{\text{IN}}}{R_1 + R_2} \cdot R_1 + v_{\text{IN}} \right] \]

\[ = \frac{AR_1}{R_1 + R_2} v_{\text{OUT}} - \frac{AR_1 v_{\text{IN}}}{R_1 + R_2} + A v_{\text{IN}} \]

\[ v_{\text{OUT}} \left[ 1 - \frac{AR_1}{R_1 + R_2} \right] = v_{\text{IN}} A \left[ 1 - \frac{R_1}{R_1 + R_2} \right] \]

\[ v_{\text{OUT}} = \left[ 1 - \frac{R_1}{R_1 + R_2} \right] \cdot A v_{\text{IN}} = -\frac{R_2}{R_1} v_{\text{IN}} \]
Representing dynamics of op amp...

\[ (v^+ - v^-) \]

\[ R \quad C \quad v^* \quad + \quad v_o \]

\[ + \quad - \quad (v^+ - v^-) \quad + \quad - \quad Av^* \]

\[ v^+ \quad v^- \]
Representing dynamics of op amp...

Consider this circuit and let's analyze its dynamics to build insight.

Let's develop equation representing time behavior of $v_o$.
Dynamics of op amp...

\[ v_o = A v^* \quad \text{or} \quad v^* = \frac{v_o}{A} \]

\[ RC \frac{dv^*}{dt} + v^* = v^+ - v^- \]

\[ RC \frac{dv_o}{dt} + \frac{v_o}{A} = v^+ - v^- \]

\[ = (\gamma - \bar{\gamma}) v_o \]

\[ v^+ = \frac{v_o R_1}{R_1 + R_2} = \gamma v_o \]

\[ v^- = \frac{v_o R_3}{R_3 + R_4} = \bar{\gamma} v_o \]

or

\[ \frac{dv_o}{dt} + \left[ \frac{1}{RC} + \frac{A}{RC} (\bar{\gamma} - \gamma) \right] v_o = 0 \]

\[ \frac{dv_o}{dt} + \frac{A}{RC} (\bar{\gamma} - \gamma) v_o = 0 \]

\[ \text{time}^{-1} \]

or

\[ \frac{dv_o}{dt} + \frac{v_o}{T} = 0 \quad \text{where} \quad T = \frac{RC}{A(\bar{\gamma} - \gamma)} \]

\[ v_o(0) = 0 \]

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Consider a small disturbance to $v_o$ (noise).

if $\bar{\gamma} > \underline{\gamma}$

$T$ is positive

$$v_o = Ke^{\frac{-t}{T}}$$ stable

if $\underline{\gamma} > \bar{\gamma}$

$T$ is negative

$$v_o = Ke^{\frac{t}{|T|}}$$ unstable

if $\underline{\gamma} = \bar{\gamma}$

$T$ is very large

$$v_o = K$$ neutral

Now, let’s build some useful circuits with positive feedback.
One use for instability: Build on the basic op amp as a comparator

\[ +V_S \]
\[ v^+ \]
\[ v^- \]
\[ v_o \]
\[ -V_S \]

\[ v^- \rightarrow 0 \]
\[ t \]

\[ v^+ - v^- \]

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Now, use positive feedback

\[ v^+ = \frac{v_o R_1}{R_1 + R_2} \]

### Circuit Diagram:

- Input voltage: \( v_i \)
- Output voltage: \( v_o \)
- Feedback resistors: \( R_1, R_2 \)

### Example Conditions:

- \( v^+ = 7.5 \)
- \( v_o = 15 \)
- \( v_i = v^- \) > 7.5
- \( v^- > 7.5 \)
- \( v^- < v^+ \)
- \( v^- < -7.5 \)
- \( v^- = -7.5 \)

**Note:** The diagram illustrates the positive feedback condition with the operational amplifier's inverting and non-inverting inputs. The feedback loop is closed through the resistors, affecting the output voltage in relation to the input voltage and the feedback resistances.

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Now, use positive feedback

\[ v^+ = \frac{v_o R_1}{R_1 + R_2} \]

\[ v^- = \frac{-V_s R_1}{R_1 + R_2} \]

\[ (v_i = v^-) > v^+ \]

\[ v^- > 7.5 \]

\[ v^- < -7.5 \]

\[ v_o = +V_s \]

\[ v_o = -V_s \]

\[ v^- = \frac{-V_s R_1}{R_1 + R_2} \]
Why is hysteresis useful?

*e.g., analog to digital*
Without hysteresis

analog to digital

\[ v_i, v_o \]

\[ 7.5 \]

\[ -7.5 \]
Oscillator — can create a clock

Assume \( v_o = V_S \) at \( t = 0 \)
\( v_C = 0 \)
Clocks in Digital Systems

- We built an oscillator using an op amp.
  
  \[ \text{can use as a clock} \]

- Why do we use a clock in a digital system? (See page 735 of A & L)

  \[ \text{sender} \rightarrow 1, 1, 0 \rightarrow \text{receiver} \]

  \[ \text{clock} \]

  \( \text{a} \)  1,1,0?

  \( \text{b} \)  When is the signal valid?

  - common timebase -- when to “look” at a signal
    (e.g. whenever the clock is high)

  \[ \rightarrow \text{Discretization of time} \]
  
  one bit of information associated with
  an interval of time (cycle)