N. Ay et al. proposed a vector valued complexity measure

\[ I := (I_1, \ldots I_N), \]

which is computed from a discrete time series.

\[ I_k \] quantifies the dependencies between \( k \) units, that cannot be explained by dependencies of any \( k - 1 \) nodes.

- The exponential family \( \mathcal{E}_k \) contains only distributions with interactions between at most \( k \) units.
- Components \( I_k \) are defined as Kullback-Leibler distances between projections to \( \mathcal{E}_k \) and \( \mathcal{E}_{k-1} \).
- Theoretical result: \( I_2 \) equals the multi-information and is maximal in synchronization.

We call a dynamics \textit{complex}, if it has high values of \( I_k \) for \( k \geq 3 \).

Our aim: To identify \textit{complex} dynamics in a coupled map lattice.
coupled tent maps on a graph with adjacency matrix \((g_{ij})\)

discrete time \(t = 0, 1, 2, \ldots\) and real values \(x_i(t) \in [0, 1]\).

simultaneous updates according to

\[
x_i(t + 1) = \epsilon \sum_j g_{ij} \frac{f(x_j(t))}{k_i} + (1 - \epsilon) f(x_i(t))
\]

where \(f\) is the tent map.

Main Example: Circle of 10 Nodes
Symbolic Dynamics of 10-circle

Assign to each node a symbol
\[
\begin{cases} 
0 & \text{if } x_i(t) \leq \frac{1}{2} \\
1 & \text{otherwise}
\end{cases}
\]

\[\epsilon = 0.05\]

\[\epsilon = 0.464\]

\[\epsilon = 0.47: \text{partial synchronization}\]

\[\epsilon = 0.84\]
Special Regime of 10 Node Circle at $\epsilon = 0.47$

Partial synchronization of 10 nodes

- 2 nodes **constant**
- 4 node almost **quasiperiodic** with large amplitude
- 4 nodes almost **quasiperiodic** with smaller amplitude
Results

- Vector $I$ detects the synchronization
- “Complex Dynamics” on the edge of synchronization

Poster (#301): $I_4, I_5, I_6$, full graph, ...