Multi-objective Evolutionary Optimization under Uncertainty

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MOP under uncertainty - Motivations

- MOPs under uncertainty (MOP-U) have received considerable and increasing interest in recent years
  - Discrete and Continuous Optimization
- MOP-U arise in many important decision making contexts in various sciences and industries
  - Risk in Finance
  - Reliability in engineering design
  - Smart grids, cloud, logistics and transportation, ...
- MOP-U pose challenges for both practitioners and researchers.
- Need an efficient modelling and resolution approaches which are robust and non sensitive to those uncertainties.
  - Performance results remain relatively unchanged when exposed to uncertain data.
Outline

• **Background**
  - Multi-objective optimization problems (MOPs)
  - Evolutionary Multi-objective Optimization (EMO)

• **A unified view of EMO under uncertainty**
  - Uncertainty types in MOPs
    - Scheduling, routing, ...
  - Models of uncertainty
  - Aggregation of uncertainty sets
  - Resolution methods

• **Multi-objective Optimization under Random uncertainty**

• **Multi-objective Optimization under Epistemic uncertainty**

• **Conclusion and perspectives**
Multi-objective optimization

Main concepts
Multi-objective Optimization

Multiobjective Optimization Problem (MOP)

\[
(MOP) = \begin{cases}
\min f(x) = (f_1(x), f_2(x), \ldots, f_n(x)) \\
\text{s. t. } x \in X
\end{cases}
\]

- \( n \geq 2 \) objective functions \((f_1, f_2, \ldots, f_n)\)
- \( x \in X \) is a decision vector
- \( X \) is the feasible set in the decision space
- \( Z \) is the feasible set in the objective space
Multi-objective Optimization

Definitions:

- \( z \in Z \) dominates \( z' \in Z \) iff
  - \( \forall i \in \{1, \ldots, n\}, \ z_i \leq z_i' \)
  - \( \exists j \in \{1, \ldots, n\}, \ z_j < z_j' \)

- \( z \in Z \) is a non-dominated vector if there does not exist another \( z' \in Z \) such that \( z' \) dominates \( z \)

- \( x \in X \) is an efficient solution if \( f(x) \) is non-dominated

- The Pareto frontier is the set of all non-dominated points in the decision space

- The efficient / Pareto set is the set of all efficient solutions in the objective space
Multi-objective Optimization

Multi-objective optimization as a part of the decision making (DM) process:

- A priori
  - DM before the search process
- A posteriori
  - DM after the search process
- Interactive
  - DM during the search process

Goal:

- Finding (or approximating) the (minimal) efficient set
- Evolutionary Algorithms well-suited to find multiple efficient solutions in a single run (EMO)
Evolutionary multi-objective optimization

A unified view for design
Evolutionary Multi-Objective Algorithms

Fitness assignment

- Guide the search towards efficient solutions for a good convergence
  - 4 classes: scalar, criterion-based, dominance-based, indicator-based approaches

Diversity preservation

- Generate a diverse set of efficient solutions in the objective space
  - 3 classes: kernel, nearest neighbor, histogram approaches

Elitism

- Preservation and use of elite (non-dominated) solutions
  - Main population (elitist replacement)
  - Archive (unbounded, bounded, fixed-size), elitist selection
## Classical Evolutionary Multi-Objective Algorithms

### SPEA2 (Strength Pareto Evolutionary Algorithm) [Zitzler, 2001]

1. **Fitness Assignment**: Pareto dominance approach.
2. **Diversity Preservation**: Nearest neighbor technique.
3. **Elitism**: Fixed size archive.

### NSGAII (Non-dominated Sorting Genetic Algorithm) [Deb, 2002]

1. **Fitness Assignment**: Pareto dominance approach.
2. **Diversity Preservation**: Crowded comparison method.
3. **Elitism**: Active population as repository.

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**How extending these two algorithms to fuzzy context?**
Performance Analysis Methodology

- 30 runs per algorithm and per problem instance
- Quality indicators (convergence + diversity)

Reference set: non-dominated points extracted from the union of all the executions (per problem instance)

According to Indicator I, is Algorithm A better than Algorithm B?

Statistical testing
What is a good efficient set approximation?

- Strongly depends of the DM preferences
- Given in terms of a binary quality indicator \( I^\Omega : \Omega \times \Omega \to \mathbb{IR} \), where \( \Omega \) stands for the set of all efficient set approximations
- \( I^\Omega(A,B) \) quantifies the difference in quality between \( A \) and \( B \in \Omega \)

Optimization goal:

\[
\arg \min_{A \in \Omega} I^\Omega(A, S)
\]

(S: optimal efficient set)

- \( I^{X^N} : X \times X^N \to \mathbb{IR} \), to compare a single decision vector with a set of decision vectors (i.e. a population \( P \))
- Minimum value by which a point \( z \) can or has to be translated to weakly dominate \( z' \)
- Example: summing approach
  - \( I^{X^N}(x, P\{x\}) = \sum_{x' \in P\{x\}} I^X(x', x) \)
  - (\( P \): current population)
Uncertainty in MOPs

- Source of uncertainty
- Uncertainty models (knowledge)
- Aggregation of uncertainty sets
- Solving optimization methods
Source of uncertainty

- The sources of uncertainty are due to many factors:
  - Environment parameters
  - Decision variables
  - Objective function
Uncertainty and Multiobjective Optimization

TYPE I and III

- Objective functions (noise)
- Environmental parameters (robustness)

⇒ Uncertainty-free evaluation
  unknown

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a possible evaluation of \( x \)

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potential evaluation space

decision space

objective space

TYPE II

- Decision variables (robustness)

⇒ Uncertainty-free evaluation
  known

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uncertainty-free evaluation of \( x \)

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potential evaluation space

decision space

objective space

perturbation area
Models of uncertainty

• Random uncertainty
  • Random (or aleatory) uncertainty is the intrinsic randomness and stochasticity of a phenomenon
  • Random uncertainty is commonly modeled by probability distributions
    • The probability distribution can take the form of classical distribution from a parametric family (ex. normal, lognormal, exponential, Weibull, uniform) or can be empirically constructed (eg. from Monte Carlo simulation).

• Epistemic uncertainty
  • An epistemic uncertainty is caused by lack of knowledge (or data)
  • The uncertainties may come from imprecision and ambiguity of the parameters
  • The main possibilistic approaches are: fuzzy set theory, possibility theory, and evidence theory (i.e. Dempster-Schafer theory).

The model builder has to make the choice according to the general state of scientific knowledge
Aggregation versus non Aggregation methods

- **Aggregation methods**: reduce the set into a single scalar value
  - Probabilistic approaches:
    - Expected value, Deviation, Min-max regret, Hurwicz, .......
  - Epistemic approaches
    - Fuzzy mean value, fuzzy deviation, ...

- **Non aggregation methods**: comparison between sets
  - Probabilistic approaches
    - Stochastic dominance
    - Probabilistic dominance: Gaussian noise, uniform noise, ...
  - Epistemic approaches
    - Fuzzy dominance
    - Possibility dominance, ...
Solving methods (fitness assignment)

• **Scalar approaches**
  • Transformation to mono-objective problem(s)
  • *Weighted metrics, Goal programming, $\varepsilon$-constraint approach, Achievement functions, Goal attainment*

• **Criterion-based approaches**
  • Each objective is treated separately

• **Dominance-based approaches**
  • Concept of dominance is used
  • *Dominance depth, rank, count, ...*

• **Indicator-based approaches**
  • Use performance indicators to guide the search
  • *Hypervolume, additive epsilon, ...*
Illustrative examples of uncertain MOPs

*Scheduling:* Random uncertainty

*Routing:* Epistemic uncertainty

Cloud, smart grids, ...
Permutation Flow-shop Scheduling

Permutation Flowshop Scheduling Problem
- **N jobs** to schedule on **M machines**
- Machines are critical resources
- **n = 2 or 3 objective functions** to optimize (minimize)
  - \( f_1 \) Makespan \( C_{\text{max}} \)
  - \( f_2 \) Total tardiness \( T \)
  - \( f_3 \) Maximum tardiness \( T_{\text{max}} \)
Flow-shop scheduling: Sources of Uncertainty

Uncertainty considered on environmental parameters

- decision variables
- objective functions
- Due dates ($d_j$)
  - Interval [$d_j^1, d_j^2$]
  - Dynamic variations

Processing times ($p_{ij}$)

- Breakdowns
- Human factors
- Unknown / uncontrollable parameters

Proactive stochastic approach where processing times are represented by random variables
Stochastic Processing Times

- **Probabilistic knowledge**
- **Uniform** distribution
  - $p_{ij}$ uniformly included between 2 values
- **Exponential** distribution
  - Breakdown, repair, maintenance...
- **Normal** distribution
  - Human factors
  - Unknown or uncontrollable factors
  - Parameters described in a vague ambiguous way
- **Log-normal** distribution
  - The uncertainties are all taken account simultaneously
Fuzzy vehicle routing problem

- Epistemic knowledge
- Triangular fuzzy demand

Min. Traveled distance \( D = [D, \hat{D}, D] \) + Tardiness time \( T = [T, \hat{T}, T] \)

Triangular Fuzzy Number (TFN)
denoted with triplet of values \([a, \hat{a}, \overline{a}]\)
defined by a linear membership function \(\mu\).

interpreted as possibility distribution
\(\implies\) two dual measures \(\Pi(A)\) and \(N(A)\) can be used!
Multi-objective optimization under Random Uncertainty

- Probabilistic knowledge
- Aggregation
- Indicator-based solving method
Scenario-based Uncertainty Handling

- Let $S = \{s_1, s_2, ..., s_p\}$ be a finite set of independent and equally probable scenarios.
- $x \in X$ is now associated a sample of objective vectors $\{z^{(1)}, z^{(2)}, ..., z^{(p)}\}$, where $z^{(i)}$ represents the outcome vector of $x$ if scenario $s_i$ occurs.

The comparison of solutions induces the comparison of objective vector sets.
Aggregation based approaches

- Aggregation methods
  - Objective vector-level approaches
    - Deterministic approaches
  - Indicator-level approaches
    - Uncertainty-handling fitness assignment

- Best case
- Worst case
- Average case
- Median case
Objective Vector-level Approaches

- convert the objective vector sample set into a single point: best case, worst case, average case, median case
Indicator-level aggregation approach

→ convert indicator-values into a single one
(indicator-based metaheuristic - IBEA)

best case, worst case, average case, median case
Application to flow-shop scheduling

Approximation set reevaluated on a new set of scenarios

Results

Uncertainty-handling approaches $>>$ deterministic approaches

- Objective vector-level performance assessment:
  - indicator-level $>$ objective vector-level (noise = 10%)
  - indicator-level $\approx$ objective vector-level (noise = 20%)

- Indicator-level performance assessment:
  - indicator-level $\approx$ objective vector-level (noise = 10%)
  - indicator-level $>$ objective vector-level (noise = 20%)
Multi-objective optimization under Epistemic uncertainty

- Epistemic knowledge
- Non aggregation
  - Fitness assignment and elitism: Fuzzy Pareto dominance
  - Diversity: Fuzzy distance measures
- Dominance based solving method
Fuzzy data representation

**Triangular Fuzzy Number**

A normal fuzzy set represented by

\[ = \]

A triplet of values \([a, \hat{a}, \overline{a}]\).

**Membership function:**

\[
\mu_A(x) = \begin{cases} 
\frac{x-a}{\hat{a}-a}, & a \leq x \leq \hat{a} \\
1, & x = \hat{a} \\
\frac{\overline{a}-x}{\overline{a}-\hat{a}}, & \hat{a} \leq x \leq \overline{a} \\
0, & \text{otherwise.}
\end{cases}
\]

(1)

**Kernel value:** most plausible value
Fuzzy data representation

Knowing that...

- Propagating uncertainty via the resolution model $\Rightarrow$ Uncertain formulation of objective functions!

Consequently, we obtain...

- Triangular-valued objectives:

$$f(x): \begin{pmatrix} f_1(x) \\ \vdots \\ f_n(x) \end{pmatrix} = y = \begin{pmatrix} [y_1, \hat{y}_1, \bar{y}_1] \\ \vdots \\ [y_n, \hat{y}_n, \bar{y}_n] \end{pmatrix}$$ (2)

How ranking these results in the sense of Pareto Approach?
Fuzzy topological relationships

All possible topological relationship between 2 TFNs → 4 different situations [Boukezzoula et al., 2007].

- **Fuzzy Disjoint**
- **Fuzzy Weak-Overlapping**
- **Fuzzy Overlapping**
- **Fuzzy Inclusion**
Fuzzy Pareto approach: Single objective case

1- Total dominance: $y \prec_t y'$
Let $y$ and $y'$ two TFNs: $y$ totally dominates $y'$ iff:

$$\bar{y} < y'$$
Fuzzy Pareto approach: Single objective case

2- Partial strong-dominance: $y <_s y'$

Let $y$ and $y'$ two TFNs: $y$ partially strong dominates $y'$ iff:

$$(\bar{y} \geq \bar{y'}) \land (\bar{y} \leq \bar{y'}) \land (\bar{y} \leq \bar{y'})$$
Fuzzy Pareto approach: Single objective case

3- Partial weak-dominance: $y \prec_w y'$

Let $y$ and $y'$ two TFNs: $y$ partially weak dominates $y'$ iff:

\[ \begin{align*}
&1. (y < y') \land (\bar{y} \geq \bar{y'}) \\
&2. [y < y' \land \bar{y} < \bar{y'}] \land [(\bar{y} \leq y' \land \bar{y} > \bar{y'}) \lor ..]
\end{align*} \]
Fuzzy Pareto approach: Single objective case

But, \( y \) and \( y' \) can be incomparable if: \( \tilde{y} - \tilde{y'} \geq 0 \Rightarrow \tilde{y} \geq \tilde{y'} \).

New comparison criterion based on kernel-discard values:

If \( (\tilde{y} \; \tilde{y'}) \leq (\tilde{y'} \; \tilde{y}) \Rightarrow y \text{ partial weak dominates } y' \)
Fuzzy Pareto approach: Multiple objective case

1- Strong Pareto dominance: \( \vec{y} <_{SP} \vec{y}' \)

A fuzzy solution \( \vec{y} \) strong Pareto dominates another solution \( \vec{y}' \) iff:

\[
\forall i \in 1..n : (y_i <_{t} y'_i) \lor (y_i <_{s} y'_i) \lor ...
\]
Fuzzy Pareto approach: Multiple objective case

2- Weak Pareto dominance: $\vec{y} <_{WP} \vec{y}'$

A fuzzy solution $\vec{y}$ weak Pareto dominates another solution $\vec{y}'$ iff:

$$\forall i \in 1..n : y_i <_w y'_i$$
Fuzzy Pareto approach: Multiple objective case

3- Indifference: $\vec{y} \parallel \vec{y}'$

Two fuzzy solutions $\vec{y}$ and $\vec{y}'$ are indifferent/incomparable iff:

$\forall i \in 1..n : y_i \subseteq y_i'$
EMOAs under Epistemic Uncertainty

Our Idea is to refine our previously proposed algorithms:

**E-SPEA2 (Extended Strength Pareto Evolutionary Algo)**

1. **Fitness Assignment:** Fuzzy Pareto dominance.
2. **Diversity Preservation:** Fuzzy nearest neighbor technique.
3. **Elitism:** Triangular fixed size archive.

**E-NSGAII (Extended Non dominated Sorting Genetic Algo)**

1. **Fitness Assignment:** Fuzzy Pareto dominance.
2. **Diversity Preservation:** Fuzzy crowded comparison method.
3. **Elitism:** Active population as repository.

How enable them achieving robust optimal solutions?
SPEA2 Diversity strategy

Density Estimation Method of SPEA2

- Nearest Neighbor Method:
  - calculate for each solution the distance to its k-nearest neighbor.
  - add the reciprocal value to the fitness vector.
- based on Euclidean Distance.

Idea...

Given two vectors \( \vec{y} = (y_1, \ldots, y_n) \) and \( \vec{y}' = (y'_1, \ldots, y'_n) \):

- measure distances between each pair of fuzzy values \( d(y_i, y'_i) \) using **Bertoluzza fuzzy distance**
- compute \( D(\vec{y}, \vec{y}') \) as a weighted mean of the values \( d(y_i, y'_i) \)
SPEA2 diversity strategy

Given 2 vectors of TFNs \( \overrightarrow{y} = (y_1, ..., y_n) \) and \( \overrightarrow{y}' = (y'_1, ..., y'_n) \), we compute:

- **Bertoluzza distance** \( d(y_i, y'_i) \) between each pair of fuzzy numbers!
- **Weighted distance mean** \( D(\overrightarrow{y}, \overrightarrow{y}') \) of the set of distances \( d(y_i, y'_i) \)!

\[
d_a(y_i, y'_i) = \sqrt{\int_0^1 (\text{mid}(y_{\alpha}) - \text{mid}(y'_{\alpha}))^2 + \theta (\text{spr}(y_{\alpha}) - \text{spr}(y'_{\alpha}))^2 \, d_{\alpha}}
\]
## Extended Fuzzy SPEA2

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<th>SPEA2 [Zitzler, 2001]</th>
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<td>Nearest Neighbor technique</td>
<td>Nearest Neighbor technique based on Bertoluzza distance</td>
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<td><strong>Elitism</strong></td>
<td>Fixed size archive</td>
<td>Triangular fixed size archive</td>
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Extended Fuzzy SPEA2

E-SPEA2 Algorithm

1: Initialization. Create a random initial P.

2: Evaluation. Rank individuals in P using the fuzzy Pareto dominance.

3: Environmental Selection. Copy all non-dominated solutions in P to the triangular archive A with fixed size N:
   a- If size of A exceeds N, then reduce A by means of truncation operator based on fuzzy nearest neighbor technique to keep non-dominated sol.
   b- Else if size of A is less than N, then fill A with best dominated solutions.
   c- Otherwise (size(A)=N), the environmental selection is completed.

4: Mating Selection. Perform binary tournament selection with replacement on A to fill the mating pool.

5: Variation. Apply job order crossover (JOX) and simple swap mutation.

6: Stopping Condition. If it is satisfied return A, else go to Step 2.
NSGA-II diversity strategy

Density Estimation Method of NSGAII

- Crowded-comparison Method: to get a density estimation of individuals surrounding a particular individual in the population.
- Compute Crowding distance of each individual: sum of its individual objectives’ distances.

Idea...

- Approximate the triangular fuzzy vector (objective) using Expected Values $E(y_i) = (\overline{y_i} + 2 \times \hat{y}_i + \overline{y}_i)/4$.
- Compute the crowded distance between fitness values of the expected vector $CD(i) = \text{sum}(f_{yi}(i+1) - f_{yi}(i-1))/(f_{y_i}^{\text{max}} - f_{y_i}^{\text{min}})$.

How integrating these extensions in its search process?
## Extended Fuzzy NSGA-II

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<td>Crowded comparison based on Expected distance</td>
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<td><strong>Elitism</strong></td>
<td>Active population</td>
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Extended Fuzzy NSGA-II

E-NSGAII Algorithm

1: Initialization. Create a random initial P.
2: Evaluation. Rank individuals in P using the fuzzy Pareto dominance.
3: Environmental Selection. Calculate crowding distance of the expected values of non-dominated solutions and create an external population P’ as follows:
   a- If size of P’ exceeds N, then add the least crowded solutions to P’.
   b- Else if size of P’ is less than N, then set P’ with best dominated sols.
   c- Otherwise (size(P’)=N), the environmental selection is completed.
4: Mating Selection. Apply crowded tournament selection to select parents from P’.
5: Variation. Use job order crossover (JOX) and simple swap mutation.
6: Stopping Condition. If it is satisfied return P’, else go to Step 2.
Performance assessment

Benchmarking

- Adaptation of the Solomon’s benchmark with 56 problem instances.
  ⇒ All the data in Solomon’s instances are crisp values.

⇒ Randomly generate for each crisp instance its fuzzy sampled version.
Performance assessment

Benchmarking
- Adaptation of the Solomon's benchmark with 56 problem instances.
⇒ We obtain: 56 sampled fuzzy instances labeled as Fuzz-C101, ...!

Implementation
- Algorithms developed using ParadisEO-MOEO platform:
  C-SPEA2, E-SPEA2, E-NSGAII and E-NSGAII.
- Parameters: Pop=100, Max Gen=1000, Crossover=0.8, Mutation=0.1
- Runs: × 30 per every fuzzy instance.
Performance assessment SPEA2

Example of E-SPEA2 solutions for Fuzz-C101
Performance assessment NSGA-II

Example of E-NSGAII solutions for Fuzz-C101
Performance evaluation indicators

1. Examining the ability of all algorithms to tolerate fuzziness:
   - Comparison with crisp versions,
   - Use of unary Hypervolume Indicator $I_H$ ⇒ to be maximized.

2. Assessing the performance of both extended algorithms:
   - Evaluation of the solutions quality,
   - Use of binary Hypervolume Difference $I_H^-$ and Additive $\epsilon$-indicator $I_{\epsilon+}$ ⇒ to be minimized.

Quality indicators cannot be applied on fuzzy solutions!

- defuzzify them using Expected values before applying indicators.
Comparison SPEA2 and NSGA-II

Comparison between C-SPEA2 and E-SPEA2
Comparison SPEA2 and NSGA-II

Comparison between C-NSGAII and E-NSGAII

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## Comparison SPEA2 and NSGA-II

### Performance Assessment

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**Symbols:**
- `>` worse
- `<` better
- `≡` No difference

**HYP I_H**

**WILCOXON Test with a P-value = 0.5%**
Conclusion and Perspectives

Conclusions
• Generic and unified view of EMO under uncertainty
• Investigation of some EMOs under uncertainty
  • Probabilistic uncertainty $\rightarrow$ Indicator based EMOs
  • Epistemic uncertainty $\rightarrow$ Fuzzy dominance based EMOs
• Application to scheduling and routing

Perspectives
• Preparing a survey paper “Taxonomy of metaheuristics for multi-objective optimization under uncertainty”
• Parallel computing aspects in MOP under uncertainty
• Uncertainty and meta-modelling
• Performance assessment: new indicators of robustness
• Applications:
  • Smart grids (demand, production, price, ...)
  • Cloud computing (demand, load, price, ...)