Outline

• Motivation
• Point Processes
• Atomic Spatial Processes
• Tractable Inference Strategy
• Data Analysis Results
• Model Comparisons
Urban Spatial Data

- Often possesses rich and non-standard structure
Urban Spatial Data

- Often possesses rich and non-standard structure

Vancouver Graffiti Locations
Urban Spatial Data

- Often possesses rich and non-standard structure

Chicago Pothole Locations

Urban Spatial Data

- Often possesses rich and non-standard structure

Manhattan Noise Complaints

http://www.citylab.com/tech/2013/04/
Urban Spatial Data

- Often possesses rich and non-standard structure
  
  - Non-Euclidean
  - Non-Continuous
  - Inhomogeneous
  - Clustered
Urban Spatial Data

- Often possesses rich and non-standard structure

  - Non-Euclidean
  - Non-Continuous (Atomic)
  - Inhomogeneous
  - Clustered
Motivation

Spatial Distribution of Graffiti in Vancouver

7675 pieces of graffiti occurring at 1982 unique locations
Atomic Data

Multiple observations with identical locations

Can arise:
  Naturally
Multiple Observations

Neil Rules!
Multiple Observations

SEAN

DOC: Rules!!
Multiple Observations

A Mean Doctor Rules!!
Multiple Observations

3 Tags one One Coordinate = Atomicity
Atomic Data

Multiple observations with identical locations

Can arise:

Naturally

During the data collection stages:
Atomic Data

Multiple observations with identical locations

Can arise:

Naturally

During the data collection stages:

- Forced by collection mechanism
- To simplify data collection
- To preserve anonymity
Atomic Data

Multiple observations with identical locations

Can arise:

Naturally

During the data collection stages:

- Forced by collection mechanism
- To simplify data collection
- To preserve anonymity

Other approaches available
Atomic Data

Multiple observations with identical locations

Can arise:
- Naturally
- Focus Today

During the data collection stages:
- Forced by collection mechanism
- To simplify data collection
- To preserve anonymity

Other approaches available
Atomic Data

Multiple observations with identical locations

Can arise:
- Naturally
- Focus Today
  - Graffiti
  - Potholes
  - Parking Tickets
  - Police Speeding Checks
Desired Results

Vancouver Posterior Predictive

- Probability Masses at graffitied Locations
- Probability Density at clean Locations
Spatial Modeling Using Point Processes

(Brief Review)
Spatial Modeling Using Point Processes

Spatial Poisson Process

\[ \Omega = [0, 1] \times [0, 1] \]
Spatial Modeling Using Point Processes

Spatial Poisson Process

Observed Data $\sim$ PP($\mu$)

$\Omega = [0, 1] \times [0, 1]$
Spatial Modeling Using Point Processes

Spatial Poisson Process

\[
\Omega = [0, 1] \times [0, 1]
\]

Observed Data \sim PP(\mu)

\Rightarrow \text{Observed Data} \sim Poi(\mu(\Omega))

![Observed Data](image)
Spatial Modeling Using Point Processes

Spatial Poisson Process

Observed Data \sim \text{PP}(\mu)

\Rightarrow |\text{Observed Data}| \sim \text{Poi}(\mu(\Omega))

Each point \overset{iid}{\sim} \bar{\mu}

\Omega = [0, 1] \times [0, 1]
Spatial Modeling Using Point Processes

**Spatial Poisson Process**

- **Rate Measure:** \( \mu \)
- **Observed Data** \( \sim \) **PP**(\( \mu \))
- \( \Rightarrow \) \(|\text{Observed Data}| \sim \text{Poi}(\mu(\Omega))\)
- Each point \( iid \) \( \sim \bar{\mu} \)

**Rate Measure:** \( \mu \)

**Constant**
Spatial Modeling Using Point Processes

Spatial Poisson Process

Observe Data $\sim \text{PP}(\mu)$

$\Rightarrow |\text{Observed Data}| \sim \text{Poi}(\mu(\Omega))$

Each point $\overset{iid}{\sim} \bar{\mu}$

Rate Measure: $\mu$

Constant

Random (e.g. Bayesian Nonparametric)

$$\Omega = [0, 1] \times [0, 1]$$
Spatial Poisson Process for Atomic Data

Colours signify multiple observations

Each point $\overset{iid}{\sim} \bar{\mu}$

$\Rightarrow$ $\mu$ possesses discrete points (atoms).
Spatial Poisson Process for Atomic Data

Colours signify multiple observations

Each point $\overset{iid}{\sim} \bar{\mu}$

$\Rightarrow \mu$ possesses discrete points (atoms).

Purely Atomic

Rate Measure: $\mu$

Mixed
Gamma Process

- A Poisson Process
- Augmented Space $\Omega \times \mathbb{R}^+$

(weights drawn from positive reals)
Gamma Process

- A Poisson Process
- Augmented Space $\Omega \times \mathbb{R}^+$
- Infinite number of atoms (with probability one)
- Expected sum of weights is finite.
Gamma Process

• A Poisson Process

• Augmented Space \( \Omega \times \mathbb{R}^+ \)

• Infinite number of atoms (with probability one)

• Expected sum of weights is finite.

• Three parameters,
  • \( G_0 \) controls locations of atoms.
  • \( c, \alpha_0 \) parameterize the distribution of atom heights.
Gamma Process

• A Poisson Process

• Augmented Space \( \Omega \times \mathbb{R}^+ \)

• Infinite number of atoms (with probability one)

• Expected sum of weights is finite.

• Three parameters,
  • \( G_0 \) controls locations of atoms.
  • \( c, \alpha_0 \) parameterize the distribution of atom heights.

• Normalizes to Dirichlet Process
Atomic Spatial Processes

Poisson Process

Observed Data $|\mu \sim PP(\mu)$
Atomic Spatial Processes

\[
\mu \mid G_0 \sim \text{GaP}(c^{-1}, \alpha_0, G_0)
\]

Poisson Process

\[
\mu \sim PP(\mu)
\]

Gamma Process

Observed Data \(\mid \mu \sim PP(\mu)\)
Atomic Spatial Processes

Dirichlet Process Mixture

\[ G_0 \sim \text{DPM}(\alpha_0, H_0) \]

\[ \mu | G_0 \sim \text{GaP}(c^{-1}, \alpha_0, G_0) \]

Gamma Process

Poisson Process

Observed Data \( | \mu \sim \text{PP}(\mu) \)
Joint Distribution

Dirichlet Process Mixture

\[
\prod_{j=1}^{\infty} \text{Mult}(dz_j | \pi) L(da_j | \theta_{z_j}) \times \text{Gamma}(d\gamma | \alpha_0, c^{-1}) \prod_{j=1}^{\infty} \text{Beta}(d\beta_j^p | 1, \alpha_0) \]

\[
\text{Pois}(n | \gamma) \left[ \prod_{i=1}^{n} p_j(a, x_i) \delta_{x_i \in a(dx)} \right]
\]

Gamma Process

\[
\prod_{k=1}^{\infty} \text{Beta}(d\beta_k^\pi | 1, \alpha_0^\pi) H_0(d\theta_k)
\]

Poisson Process
Marginalization

Dirichlet Process Mixture

\[
\left( \text{CRP}(\rho(dz)|\alpha_0^\pi) \prod_{B \in \rho(z)} m(d\alpha_B) \right)
\]

(Fully Marginalized)

Gamma Process

\[
\text{NegBin} \left( n \left| \frac{1}{c+1}, \alpha_0 \right. \right) \text{CRP} (\rho(dx)|\alpha_0, N = n)
\]

Poisson Process
Marginalization

Dirichlet Process Mixture

\[
\text{CRP} (\rho(dz)|\alpha_0) \prod_{B \in \rho(z)} m(da_B)
\]

(Fully Marginalized)

Gamma Process

\[
\text{NegBin} \left( n \left| \frac{1}{c+1}, \alpha_0 \right. \right) \text{CRP} (\rho(dx)|\alpha_0, N = n)
\]

Poisson Process

Only Latent Variable
Data Analysis Results

Vancouver Posterior Predictive

- non-informative priors on hyperparameters
- 20,000 MCMC iterations
- Each sweep involved:
  - Gibbs step for each atom’s cluster membership
Data Analysis Results

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Data Analysis Results

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- Each sweep involved:
  - Gibbs step for each atom's cluster membership
Data Analysis Results

Vancouver Posterior Predictive

- non-informative priors on hyperparameters
- 20 000 MCMC iterations
- Each sweep involved:
  - Gibbs step for each atom’s cluster membership
  - Metropolis Hastings proposal for Hyperparameters
Tractable Inference

Predictive Posterior Distribution

\[
\bar{\mu}'(dy) = \frac{1}{N + \alpha_0} \sum_{i=1}^{N} \delta_{X_i}(dy) + \frac{\alpha_0}{N + \alpha_0} \frac{1}{T + \alpha_0^\pi} \left( \sum_{i=1}^{K} |B_i| m(dy|a_{B_i}) + \alpha_0^\pi m(dy) \right).
\]
Tractable Inference

Predictive Posterior Distribution

\[
\bar{\mu}'(dy) = \frac{1}{N + \alpha_0} \sum_{i=1}^{N} \delta X_i(dy) + \frac{\alpha_0}{N + \alpha_0} \frac{1}{T + \alpha_0^\pi} \left( \sum_{i=1}^{K} |B_i| m(dy|a_{B_i}) + \alpha_0^\pi m(dy) \right).
\]
Tractable Inference

Predictive Posterior Distribution

\[ \tilde{\mu}'(dy) = \frac{1}{N + \alpha_0} \sum_{i=1}^{N} \delta_{X_i}(dy) + \frac{\alpha_0}{N + \alpha_0} \frac{1}{T + \alpha_0^\pi} \left( \sum_{i=1}^{K} |B_i| m(dy|a_{B_i}) + \alpha_0^\pi m(dy) \right). \]
Tractable Inference

Predictive Posterior Expression

\[ \tilde{\mu}'(dy) = \frac{1}{N + \alpha_0} \sum_{i=1}^{N} \delta_X(x_i)(dy) + \frac{\alpha_0}{N + \alpha_0} \frac{1}{T + \alpha_0^\pi} \left( \sum_{i=1}^{K} |B_i| m(dy|a_{B_i}) + \alpha_0^\pi m(dy) \right) \]
Data Analysis Results

Vancouver Posterior Predictive
Data Analysis Results

Vancouver Posterior Predictive

\[ P(\text{new location}) = 0.102 \]

4 observed clusters
Another Example

Spatial Distribution of Graffiti in Manhattan
Another Example

Spatial Distribution of Graffiti in Manhattan

\[ P(\text{new location}) = 0.7501 \]

5 observed clusters
Model Comparisons

Competitors Considered

• Atomic Spatial Process
• Dirichlet Process Mixture Model (DPM)
• Empirical Distribution Grid Approach
• DPM mixed with Empirical Distribution
Mass Functions versus Densities

Mass

Density
Mass Functions versus Densities

Mass
Unitless (Probabilities)

Density
Units: 1/Area (Prob/Area)

Not directly comparable.
Mass Functions versus Densities

**Mass**
- Unitless (Probabilities)

**Density**
- Units: 1/Area (Prob/Area)

Instead:
Compare mass in square neighbourhoods
Model Comparisons

Non-standard comparison: predictive mass in square epsilon regions around held-out points.
Model Comparisons

Predictive Performance

<table>
<thead>
<tr>
<th></th>
<th>Vancouver</th>
<th>New York</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epsilon—neighbourhood</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Likelihood</td>
<td></td>
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</tr>
</tbody>
</table>

Model types:
- ASP
- Mixed
- Empirical
- DPM
Model Comparisons

Runtime Performance

![Box plot comparing walltime for different models in Vancouver and New York]
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