Vector-Valued Property Elicitation

Rafael Frongillo & Ian A. Kash

Harvard University, Microsoft Research

July 5, 2015

ERM $\iff$ Property Elicitation

ERM: minimizing a loss over your data

$$\hat{w} = \operatorname*{argmin}_{w \in \mathcal{W}} \sum_{y \in \text{data}} L(w, y)$$

ERM $\iff$ Property Elicitation

ERM: minimizing a loss over your data

$$\hat{w} = \operatorname*{argmin}_{w \in \mathcal{W}} \sum_{y \in \text{data}} L(w, y)$$

Property elicitation: minimizing expected loss

$$\Gamma(p) = \operatorname*{argmin}_{w \in \mathcal{W}} \mathop{\mathbb{E}}_{y \sim p} [L(w, y)]$$

*We say L **elicits** Γ*

ERM $\iff$ Property Elicitation

- $L(w, y) = (w - y)^2$, $\Gamma(p) = \text{mean}(p)$
- $L(w, y) = |w - y|$, $\Gamma(p) = \text{median}(p)$
- $L(w, y) = (1_{w \geq y} - \alpha)(w - y)$, $\Gamma(p) = \alpha\text{-quantile}(p)$
- $L(w, y) = |1_{w \geq y} - \tau|(w - y)^2$, $\Gamma(p) = \tau\text{-expectile}(p)$

Property elicitation: minimizing expected loss

$$\Gamma(p) = \operatorname*{argmin}_{w \in \mathcal{W}} \mathop{\mathbb{E}}_{y \sim p} [L(w, y)]$$

*We say L **elicits** Γ*

Central Questions

Q: Which properties are elicitable?
Q: What losses elicit particular properties?

Central Questions

Q: Which properties are elicitable?
Q: What losses elicit particular properties?

- Much known for **real-valued** case $\Gamma : \mathcal{P} \to \mathbb{R}$
  Osband'85, Lambert et al.'08, Gneiting'11, Steinwart'14...
  Γ elicitable $\iff$ Γ has convex maximal level sets

- Many results for **expected values** $\Gamma(p) = \mathbb{E}_p[X]$
  Savage'71, Osband & Reichelstein'85, Banerjee'05...
  L elicits Γ $\iff$ L = Bregman divergence

- **Vector-valued** case generally wide open!

Our Contributions

1. Unify **expected value** results
   Remove differentiability assumptions

2. Loss **separability**: $L(w, y) = \sum L_i(w_i, y)$
   $\exists$ *non-separable losses? Yes and no.*

3. Construction of **identification functions**
   Intuitively, derivatives of losses

4. **Convex maximal** level sets not sufficient
   In contrast to real-valued case

Future

- Full elicitability characterization
- Elicitation complexity

Come by the poster!