Getting lost in space
Large sample analysis of the resistance/commute distance

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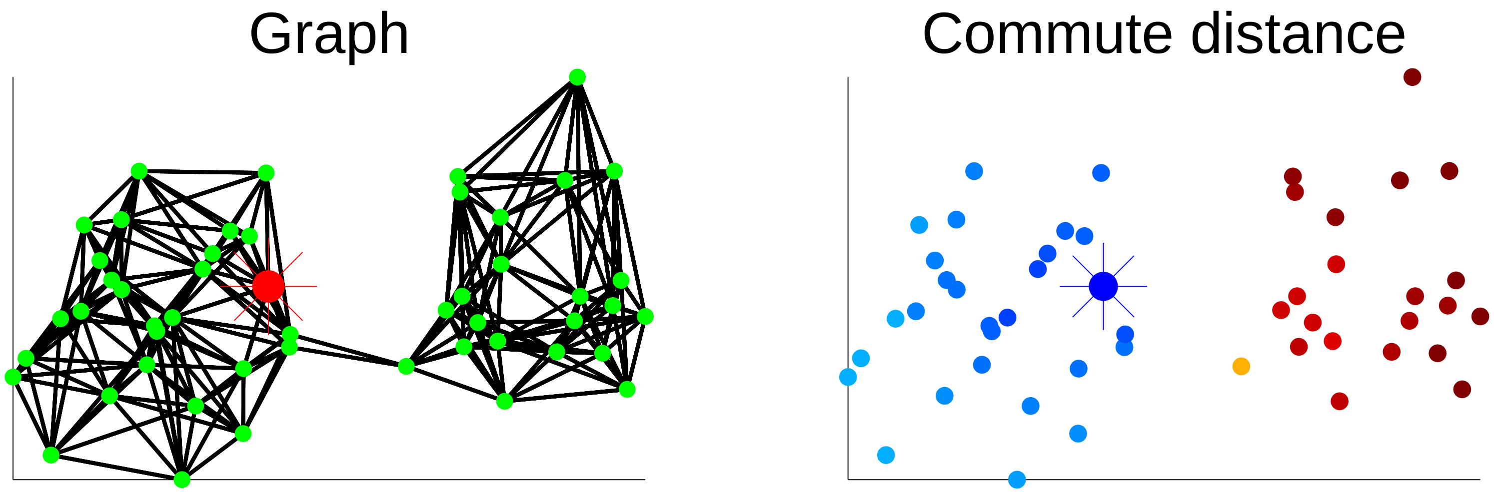
The commute distance

Idea of similarity graphs in machine learning:

- connect similar points - build global structure from local structure



Commute distance on graphs:

- expected number of steps for the random walk on the graph to go from one vertex to another vertex and back.

Usage: ranking, clustering, semi-supervised learning, network analysis

Intuition: commute distance captures cluster structure in the data !

Random geometric graphs:

- samples are drawn i.i.d. from a probability measure in $\mathbb{R}^d$
- k-nearest neighbor or ε-neighborhood graph

Sketch of Main Result:

Commute distance C_{ij} is meaningless for **large** random geometric graphs:

$$C_{ij} \approx \frac{1}{d_i} + \frac{1}{d_j} \qquad \text{for} \;\; i \neq j,$$

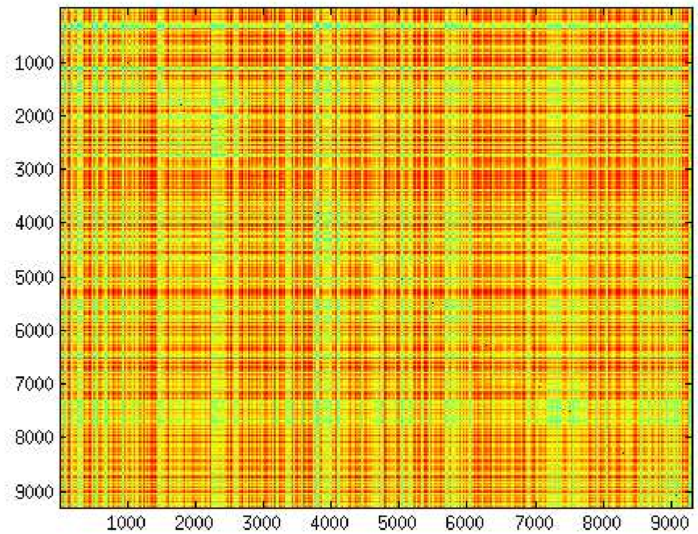
where $d_i = \sum_{s=1}^{n} w_{is}$ is the degree of vertex i.

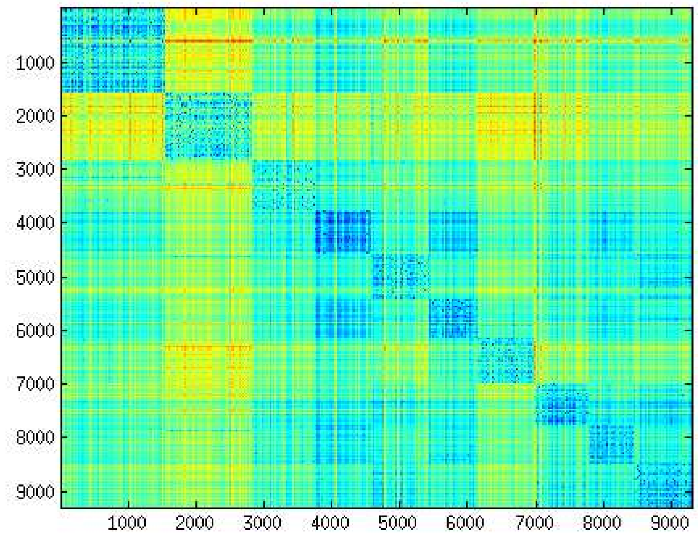
Implications:

- depends only on **local** connectivity - **no global** structure incorporated
- all points have the same nearest neighbor: the point with maximal degree

Result generalizes to other (random) graph types

USPS: 10-nearest neighbor graph





Left: Original commute distance, Right: Amplified commute distance

At the poster:

- detailed limit results + discussion of more general graph types
- amplified commute distance and associated kernel
- comparison of different distances for semi-supervised learning on USPS