Nonparametric Density Estimation for Capture-Recapture

Capture-Recapture = the science of estimating the size of a population from multiple incomplete lists

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How many fish are in this lake?

(1) Catch and weigh 1000 fish, tag each one with a unique identifier, and release.

(2) Again capture and weigh 1000 fish, and observe that (say) 50 of them have a tag.

		List L_1	
		yes	no
List L_2	yes	50	950
	no	950	$c_0 = ?$

List independence $\rightarrow$ the odds ratio is 1

$$c_0 \approx (950)(950)/50 = 18050$$

But independence fails if big fish are easier to catch than small fish!

Imputation with heterogeneity

Counts:

		List L_1: yes	List L_1: no
List L_2	yes	c_{11}	c_{01}
	no	c_{10}	$c_{00} = ?$

Probabilities:

		List L_1: yes	List L_1: no
List L_2	yes	$p(11, x)$	$p(01, x)$
	no	$p(10, x)$	$p(00, x) = ?$

Conditional Probabilities:

$$\pi(\mathbf{y}, x) := \frac{p(\mathbf{y}, x)}{1 - p(\mathbf{0}, x)}$$

		List L_1: yes	List L_1: no
List L_2	yes	$\pi(11, x)$	$\pi(01, x)$
	no	$\pi(10, x)$	$\pi(00, x) = ?$

$$\hat{c}_{\mathbf{0}} = \sum_{i=1}^{n_c} \hat{\pi}(\mathbf{0}, x_i)$$

Two Frontiers in Capture-Recapture

(1) Estimation of conditional probabilities

		List L_1	
		yes	no
List L_2	yes	$\pi(11,x)$	$\pi(01,x)$
	no	$\pi(10,x)$	

Use nonparametric conditional density estimation!

(2) Imputing the missing cell

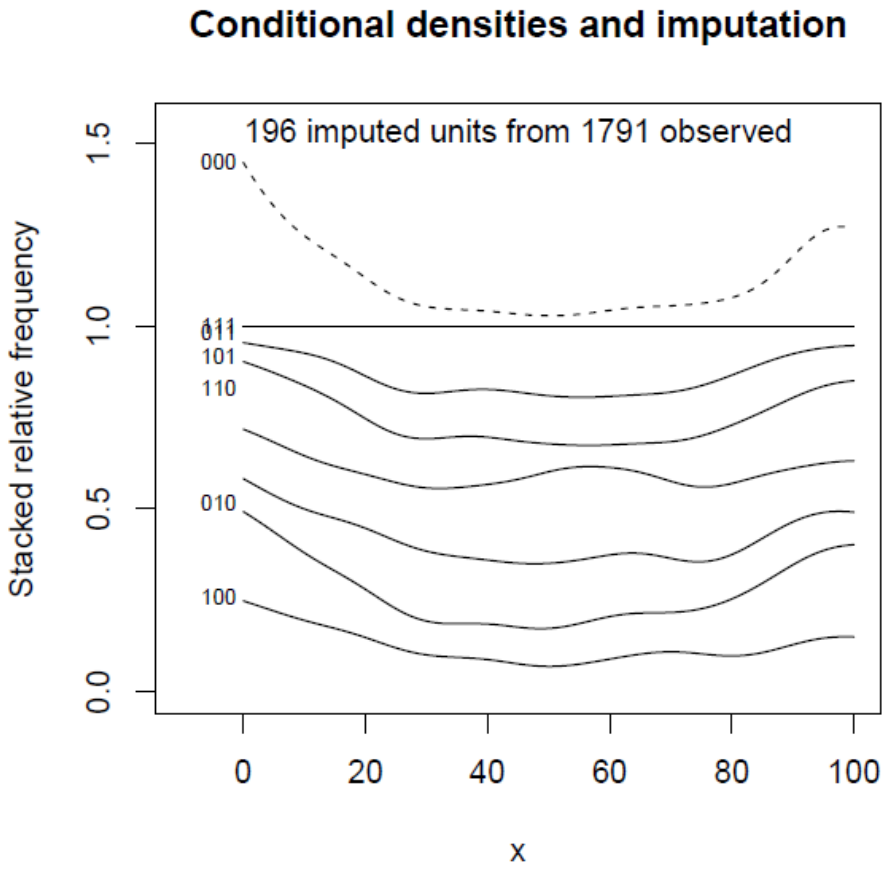
Independence model: $\pi(00,x) = \frac{\pi(01,x)\pi(10,x)}{\pi(11,x)}$

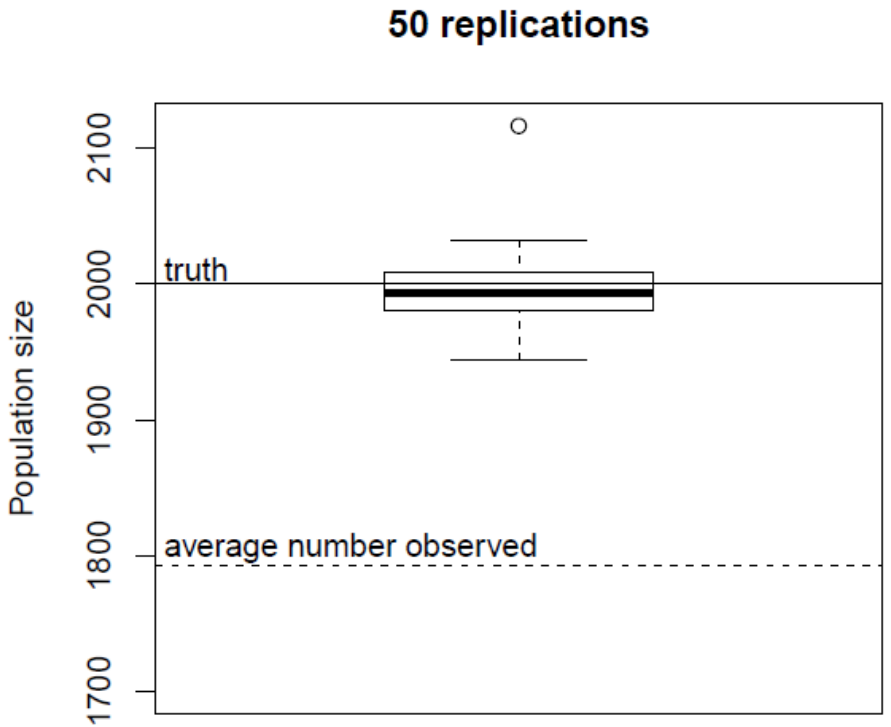
For three lists, a saturated model:

$$\log \pi(\mathbf{y},x) = u + u_1\mathbf{y}_1 + u_2\mathbf{y}_2 + u_3\mathbf{y}_3 + u_{12}\mathbf{y}_1\mathbf{y}_2 + u_{13}\mathbf{y}_1\mathbf{y}_3 + u_{23}\mathbf{y}_2\mathbf{y}_3$$

where $u = u(x)$.

Extra slide … Q&A





Challenges

- Local model selection
- Balancing model complexity over x versus complexity over y
- Dealing with heterogeneity not explained by covariates