

# Bridging Human and Machine Learning: Using discrete Markov Chain Monte Carlo with People to explore human categories

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Mind's challenge: to infer complex structure  
from limited, noisy, ambiguous input

e.g.

- Infer 3D representations from 2D retina projection
- Recognize complex sounds from pressure waves
- Learn language from hearing limited sentences  
allowed in language
- Learn categories from limited examples

Mind's challenge: to infer complex structure from limited, noisy, ambiguous input

Uses of Bayesian probability theory and machine learning methods in cognitive science

- **Benchmark**, posing ideal rational solution which is then compared with psychological behaviour to find underlying principles of how the mind works
- **Model** of psychological processes
- **Tool** for measuring mental representations

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Uses of Bayesian probability theory and machine learning methods in cognitive science

- **Benchmark**, posing ideal rational solution which is then compared with psychological behaviour to find underlying principles of how the mind works
- **Model** of psychological processes
- **Tool** for measuring mental representations structure of human categories

# Human concepts and categories

- We form categories to make sense and find structure in overwhelmingly complex world
- Concept is a mental representation that picks out a set of entities, or a category.
- Every day, we are constantly referring to our categories, updating them, and creating new ones
- Categories can span over any concept in life
  - Healthy foods, activities
  - Family and friends vs. foes
  - Preferable activities, choices
  - Efficient travel modes

# Human concepts and categories

- Background on category representation research
- Challenge: Difficult to measuring human categories in real life (high-dimensional, complex) domains
- New method of measuring people's representation of categories: Markov Chain Monte Carlo (MCMC)
  - sampling algorithm from computer science adapted to measure human representations of complex real life environments

# Background

- How we learn and represent concepts & categories is major topic of psychological research



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- One assumes category membership depends on features of an item  $x$ . Membership is not rigid.  $P(C=dog/x)$



x



x

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- One assumes category membership depends on features of an item  $x$ . Membership is not rigid.  $P(C=dog/x)$



x



x

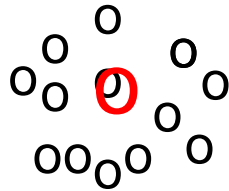
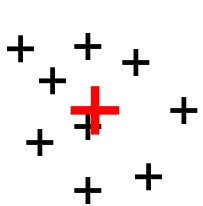
- Two popular models of category representation: *prototypes* and *exemplars*

# Background

## Models of category representation:

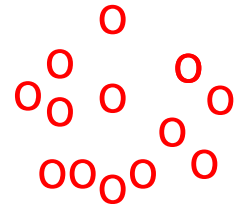
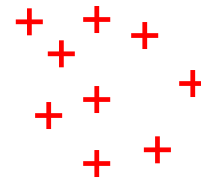
### Prototype

(Medin & Schaffer, 1978; Nosofsky, 1986)



### Exemplar

(Reed, 1972)

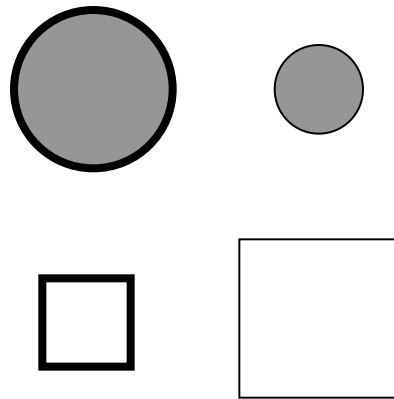


Both models can also be combined

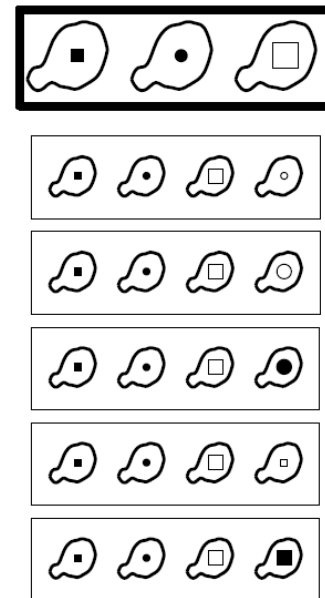
(Anderson, 1990; Griffiths et al., 2008).

# Background

## Stimuli used in categorization research



Shepard, Hovland, & Jenkins, 1963



Griffiths et al., 2008  
Feldman 2000

# Challenge of measuring human categories

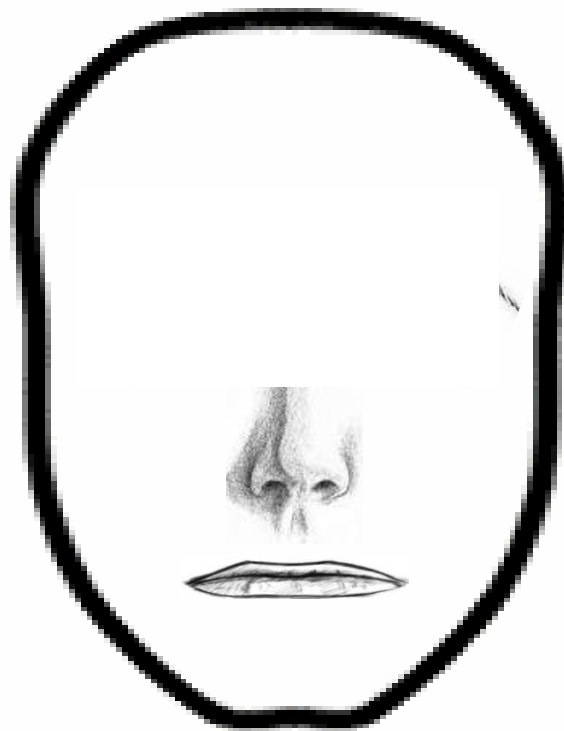
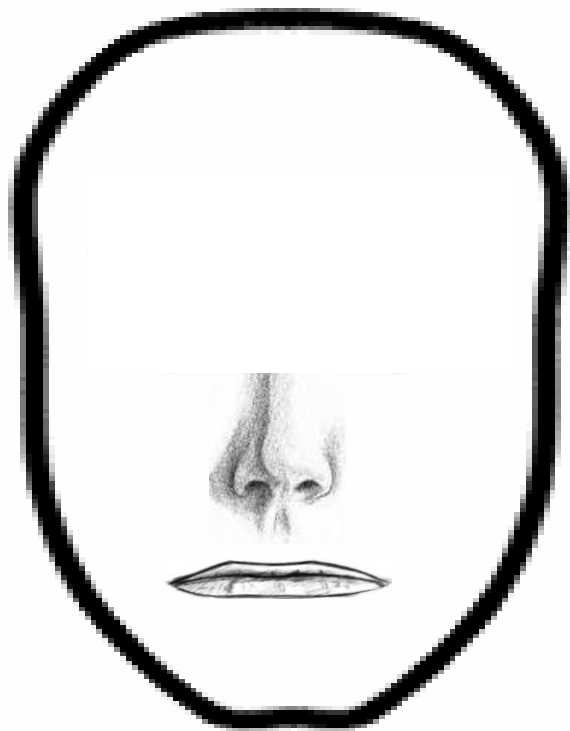
*But what about real life concepts and categories?*

- It is very difficult to measure the structure of human categories and concepts
- Real life items are complex and high dimensional



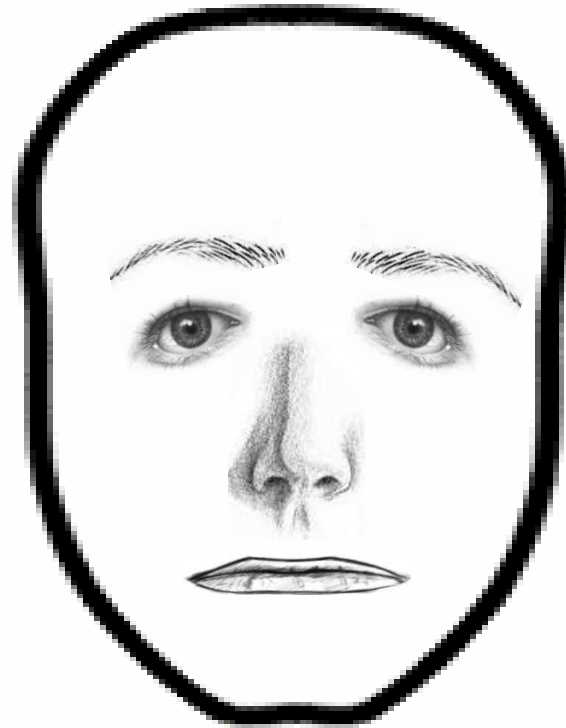
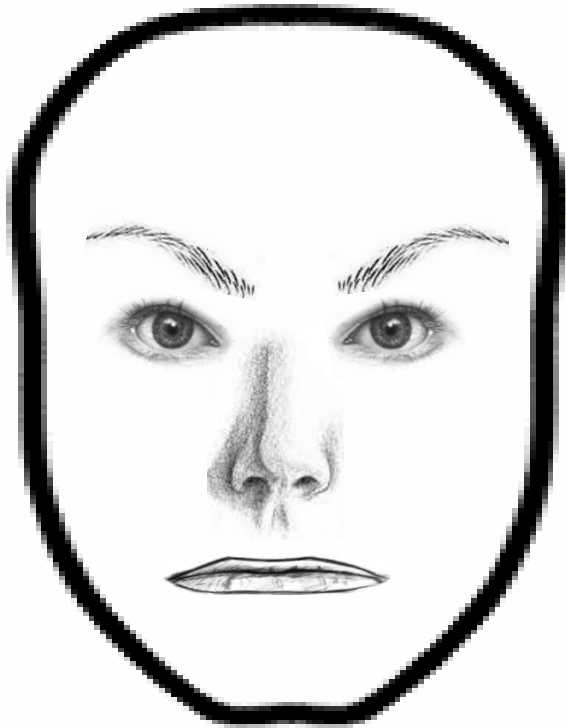
# Challenge of measuring human categories

*Features often cannot be interpreted independently*



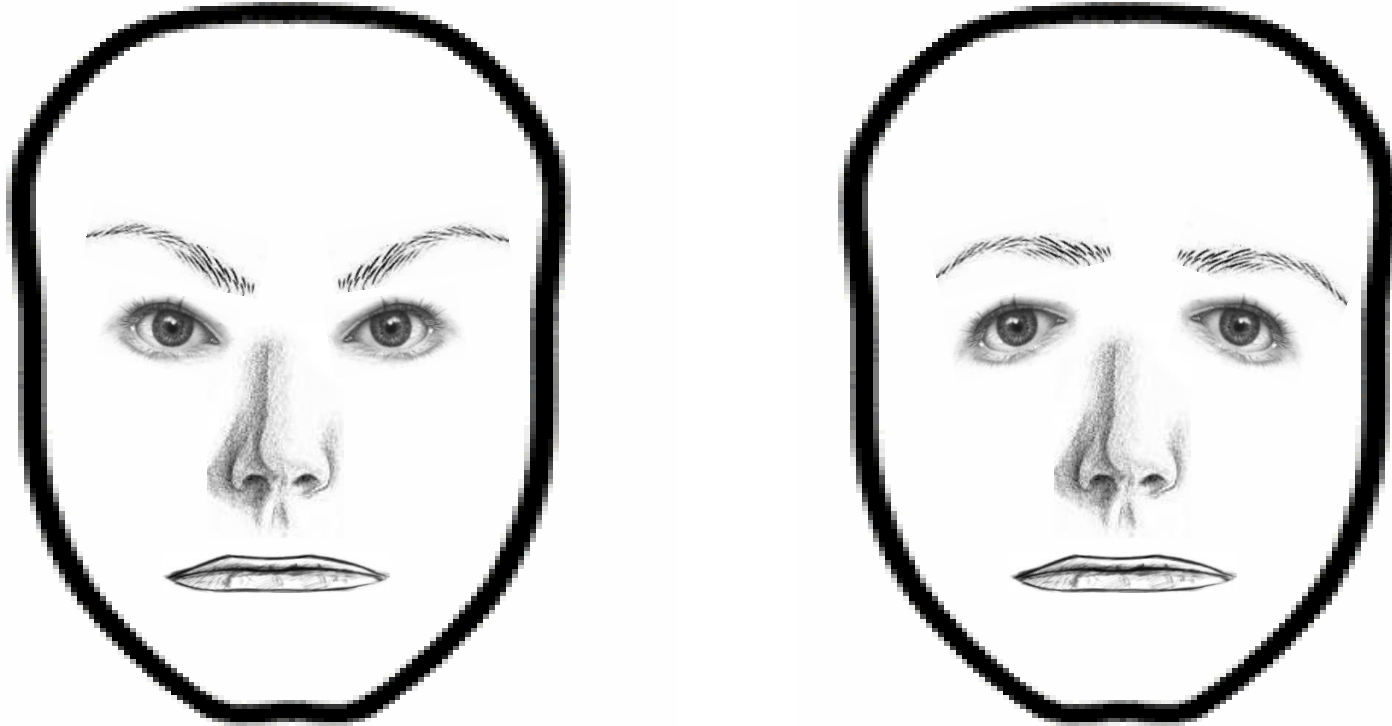
# Challenge of measuring human categories

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# Challenge of measuring human categories

*Features often cannot be interpreted independently*



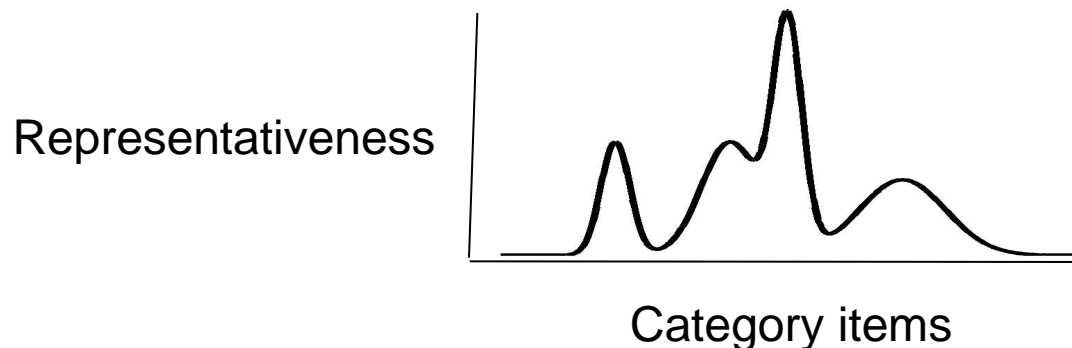
*Many interdependent features -> extremely high dimensional stimuli*



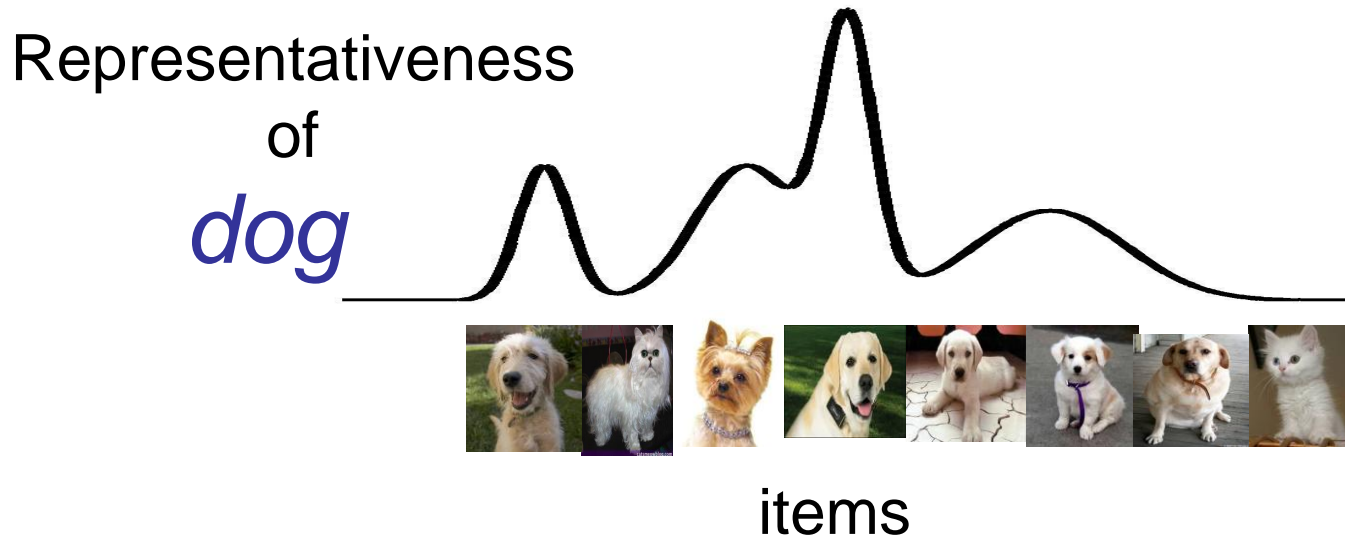
# Challenge of measuring human categories

**Problem:** Human categories often have complex structure, span high dimensional spaces. How do we measure complex, real-life categories?

**A start:** Assume all items can be assigned a weight associated with its “representativeness” of the category



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# Challenge of measuring human categories



**A start:** Assume all items can be assigned a weight associated with its “representativeness” of the category

Which is a giraffe?



Probability  $p(x|c)$

Which phone?



Preferences  $f(x)$

Which looks more happy?



Strength  $s(x)$

# Challenge of measuring human categories

## Example: Happy faces



-One possibility is to just ask:

How representative is this of a happy face?

Problems:

- Hard for people answer this question (no idea of scale)*
- Will require prohibitively large number of ratings to explore the category (1000 ratings for 1000 items)*

-Another possibility is to pick random pairs and ask:

Which is more representative of a happy face?

Advantage:

- Binary choice task is easier to answer*



Problems:

- *Will require prohibitively large number of ratings to sample all 499500 possible pairs*

**Solution:** Markov Chain Monte Carlo (MCMC) for exploring human categories

- uses easy binary choice task
- efficiently explores category space by finding and focusing on highly representative regions
- Adapted from widely and long-used algorithm from computer science (Metropolis et al. 1953)

## **MCMC method for exploring human categories:**

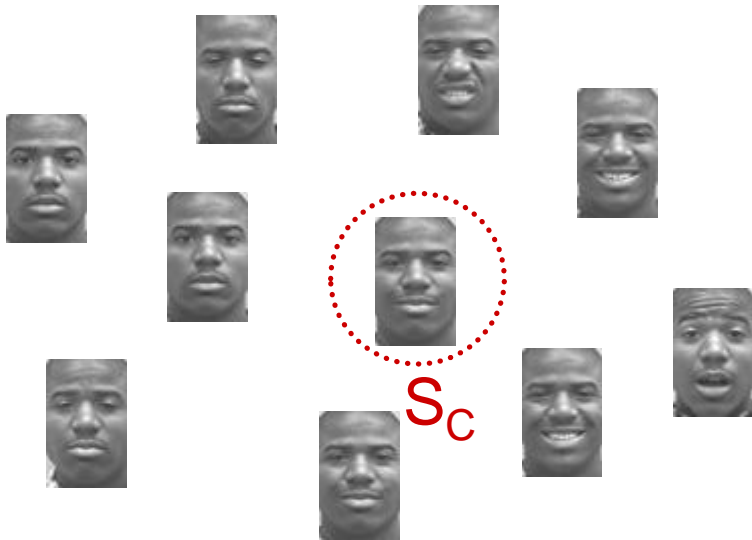
- 1) Start at current state  $S_C$
- 2) Choose at random a nearby proposal state  $S_P$
- 3) Let participant choose whether to accept proposal as new state

*-States are examples from set of stimuli you are exploring*

*-We'll need to evaluate what is "nearby", will explain later.*

# MCMC method for exploring human categories:

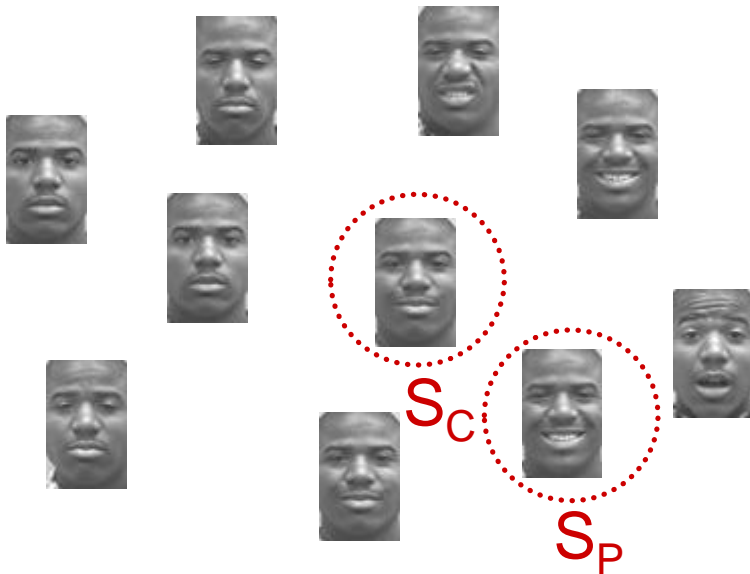
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Chain:

# MCMC method for exploring human categories:

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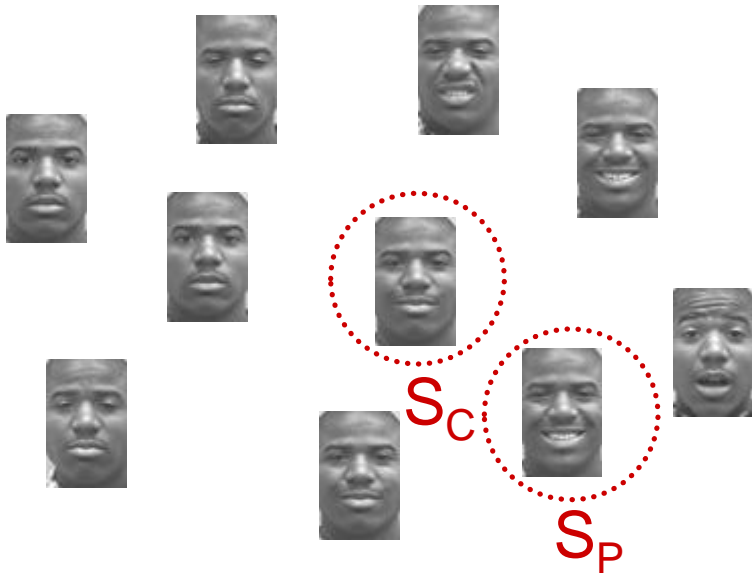


Chain:



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Experiment screen:

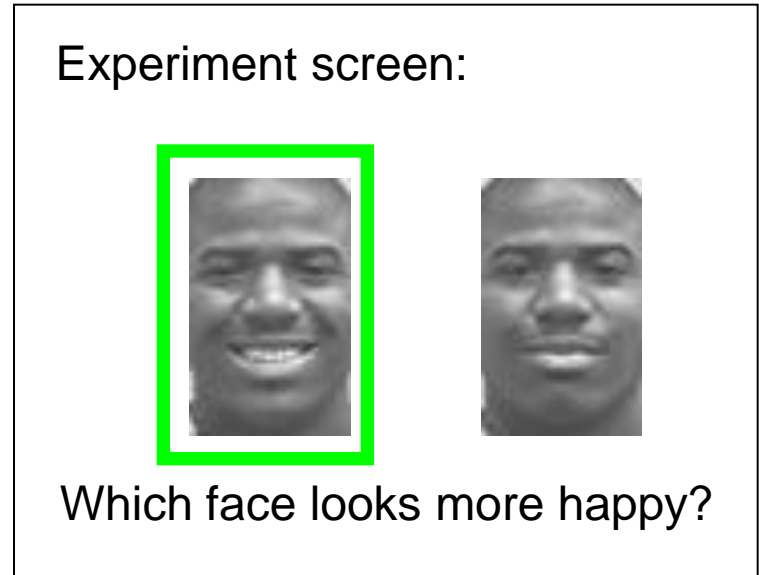
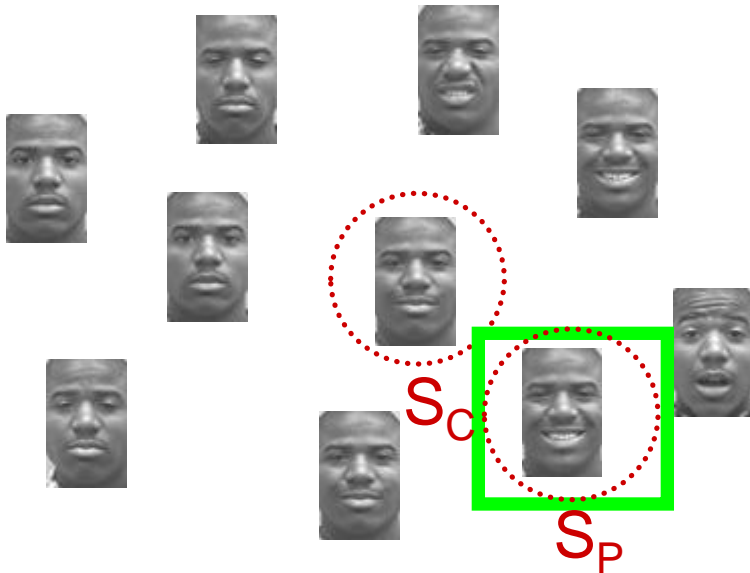


Which face looks more happy?

Chain:

# MCMC method for exploring human categories:

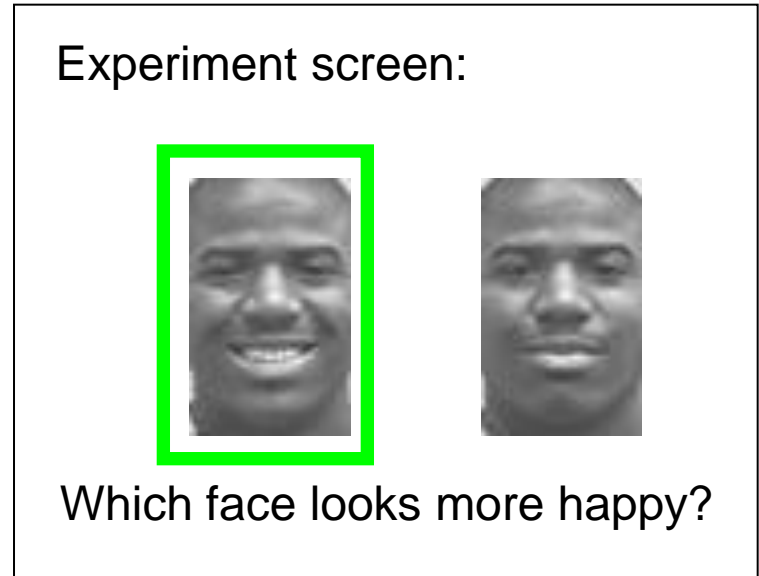
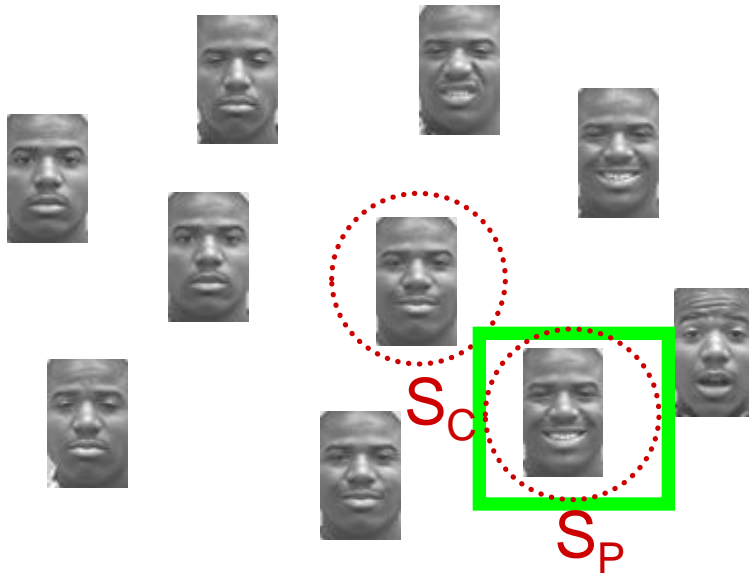
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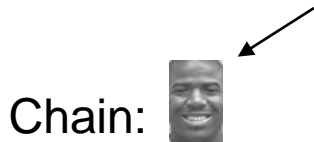
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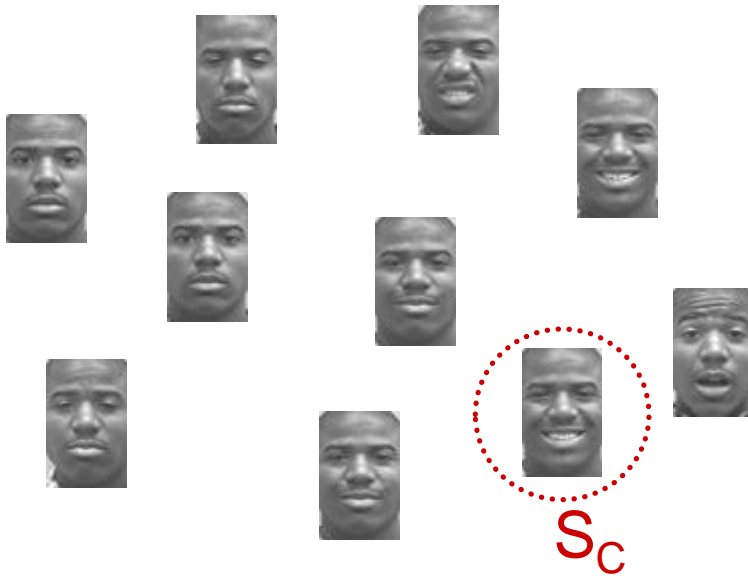


Add selected state to chain



# MCMC method for exploring human categories:

- 1) Start at current state  $S_C$
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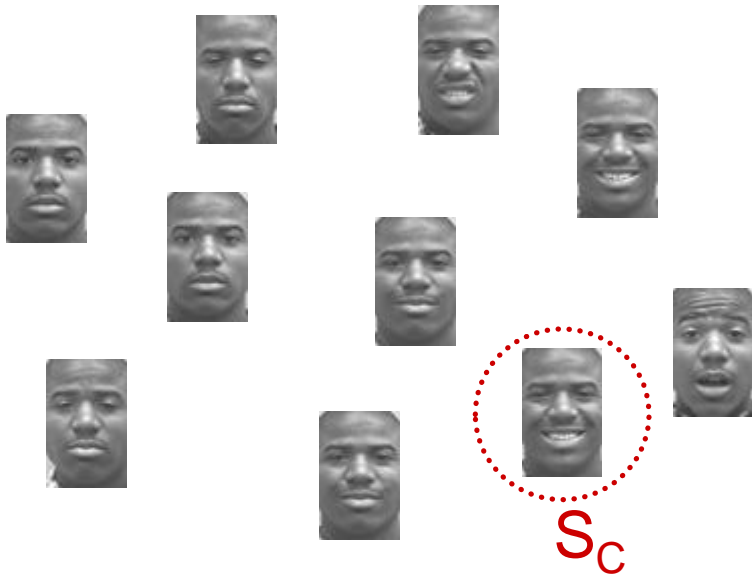
Chosen state is the new current state

*(Here the proposed state became the current state)*

Chain: 

# MCMC method for exploring human categories:

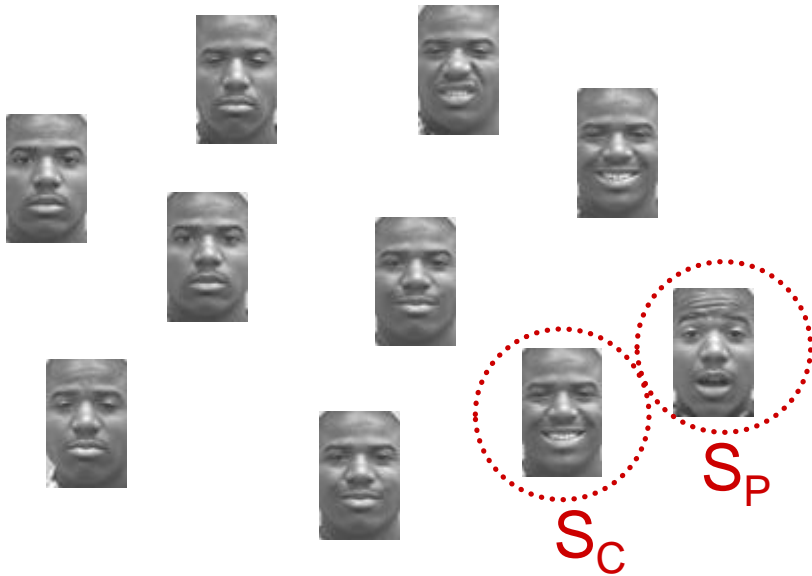
- 1) Start at current state  $S_C$
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Chain: 

# MCMC method for exploring human categories:

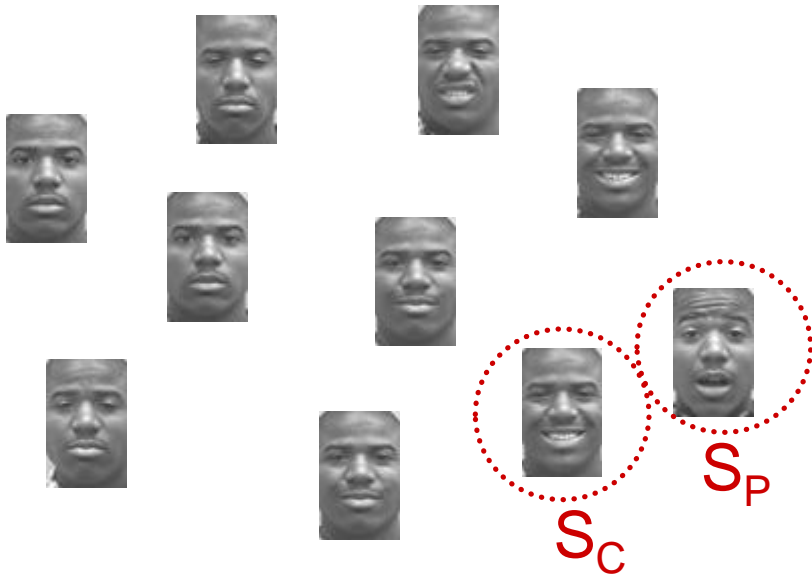
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Experiment screen:



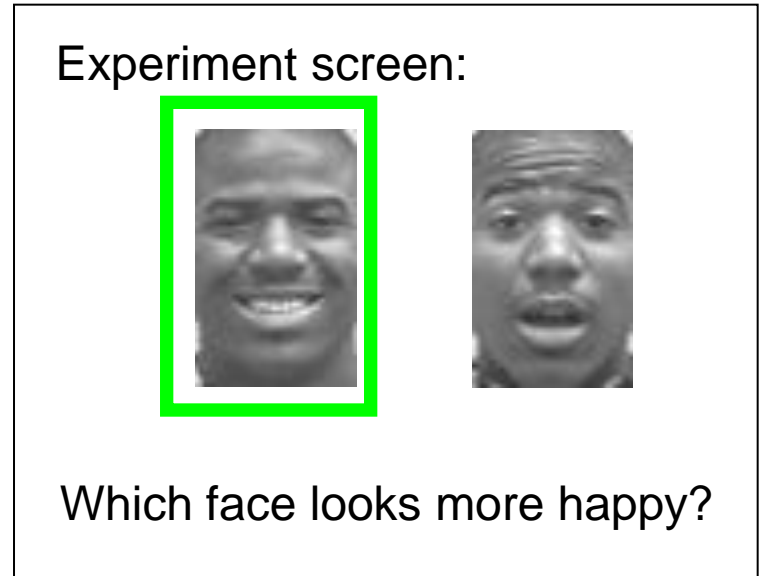
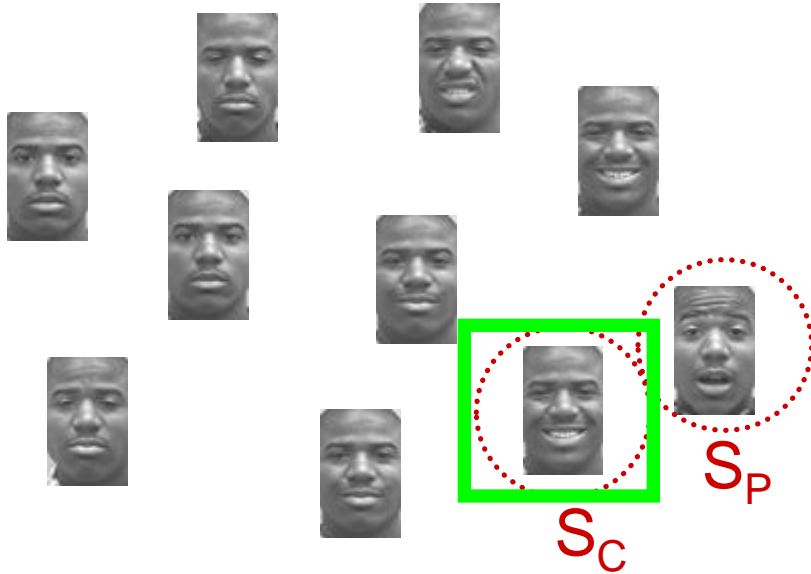
Which face looks more happy?

Chain:



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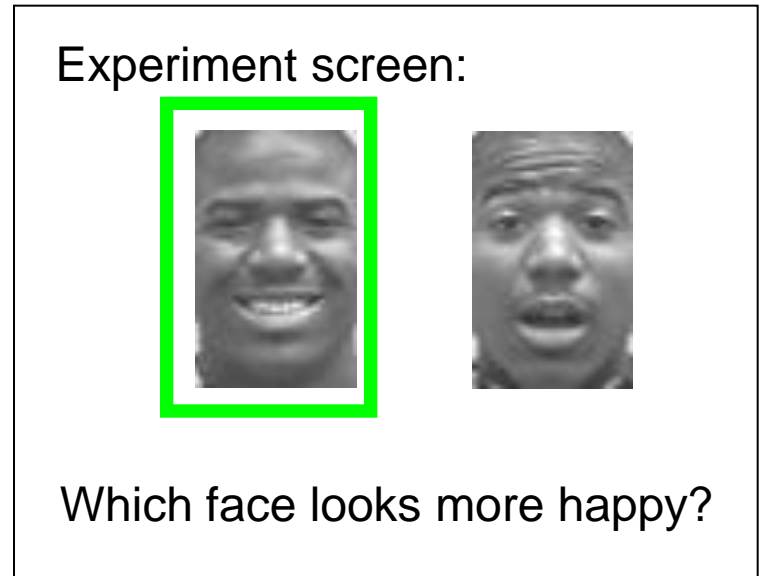
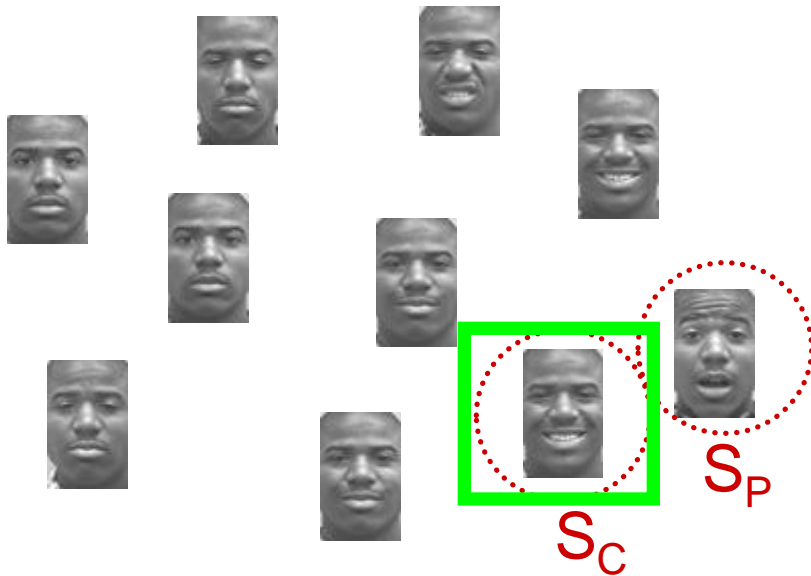


Chain: 



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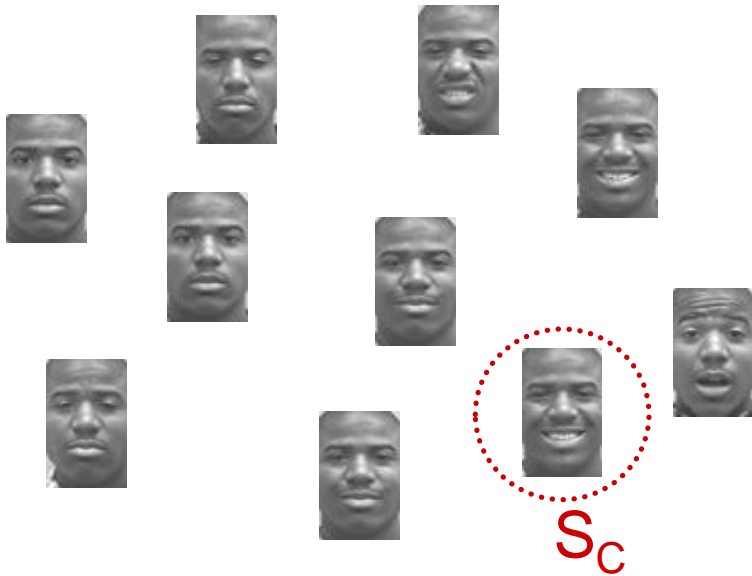


Add selected state to chain



# MCMC method for exploring human categories:

- 1) Start at current state  $S_C$
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Chosen state is the new current state

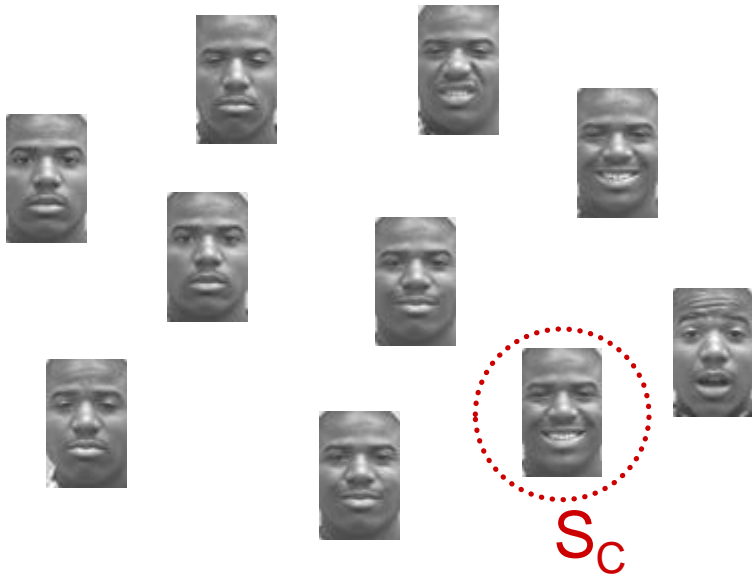
*(Here current state stayed the same)*

Add selected state to chain



# MCMC method for exploring human categories:

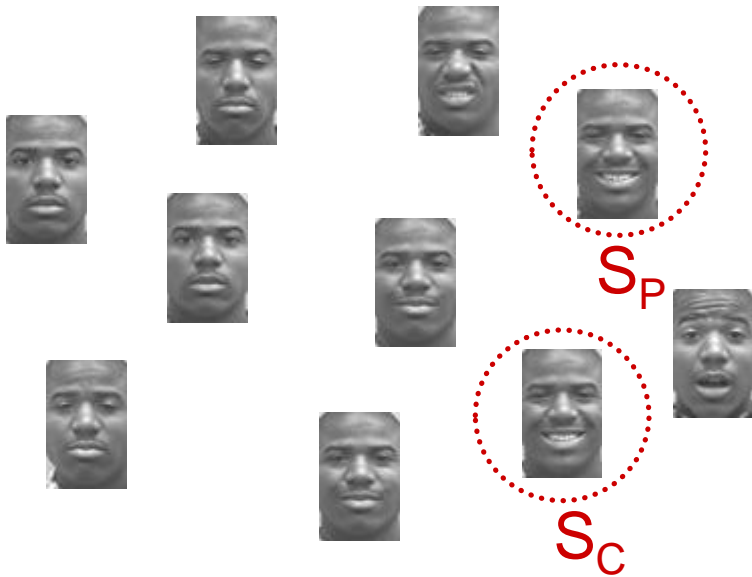
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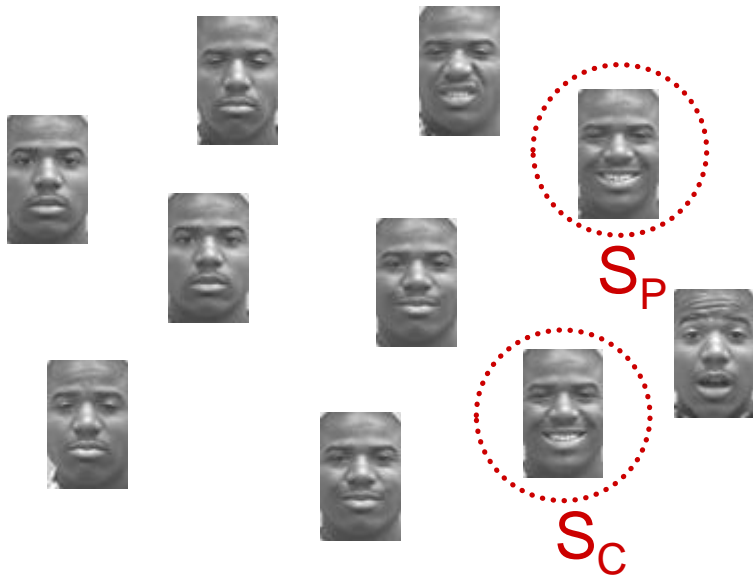
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Experiment screen:

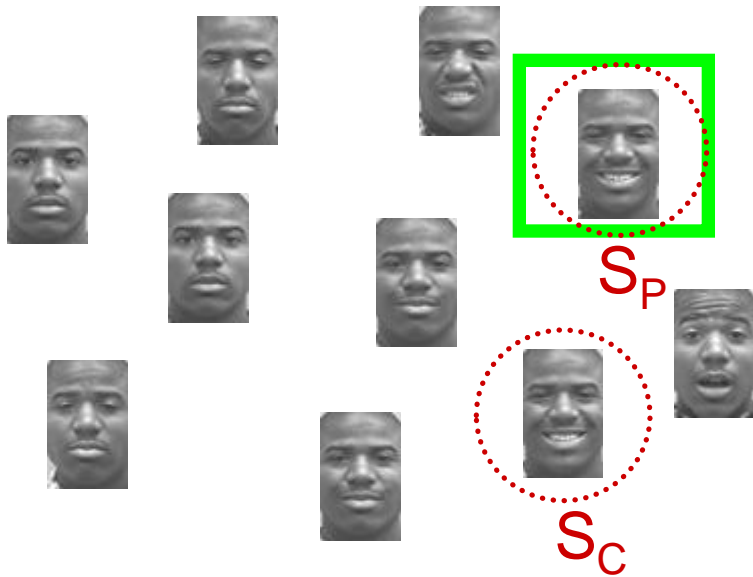


Which face looks more happy?

Chain: 

# MCMC method for exploring human categories:

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Experiment screen:



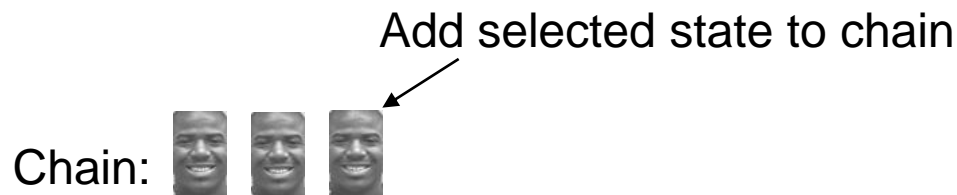
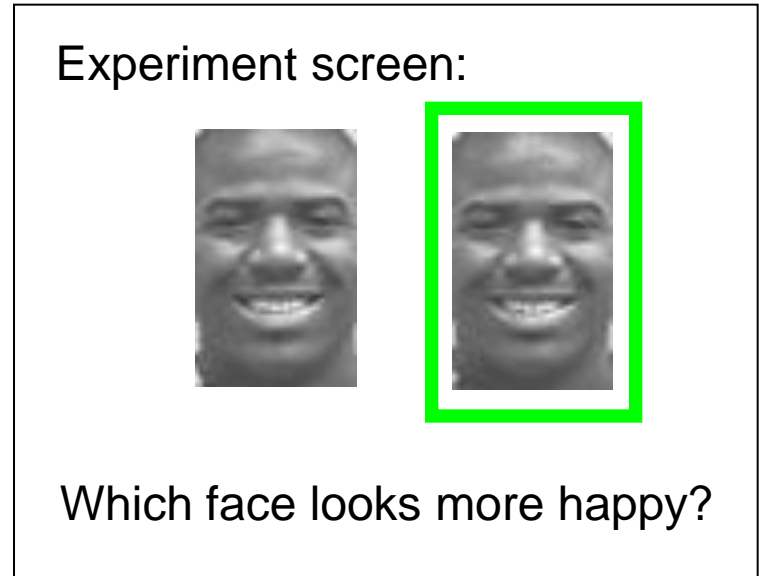
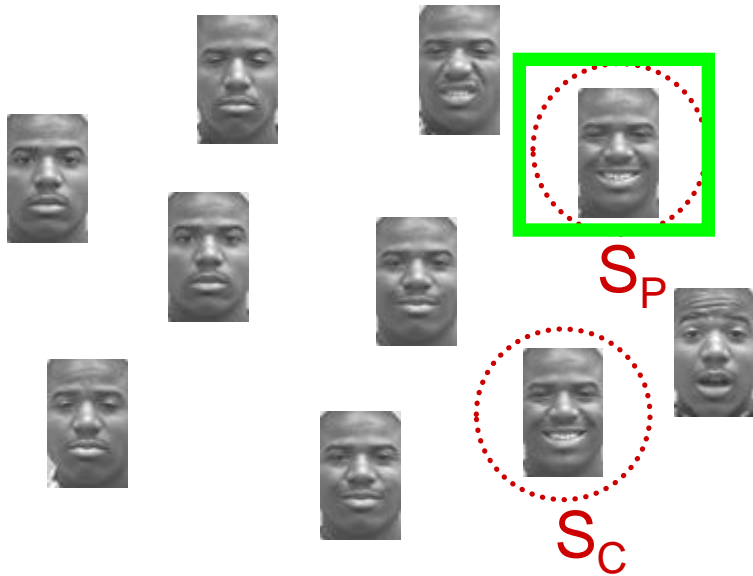
Which face looks more happy?

Chain:



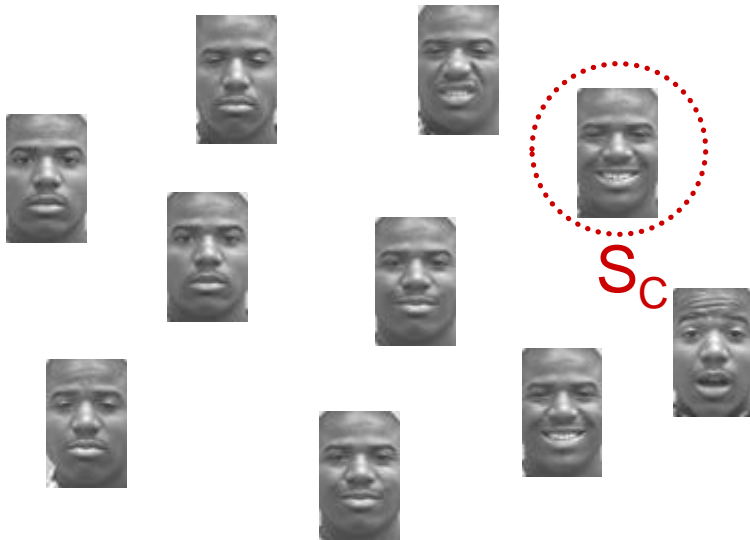
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# MCMC method for exploring human categories:

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Chosen state is the new current state

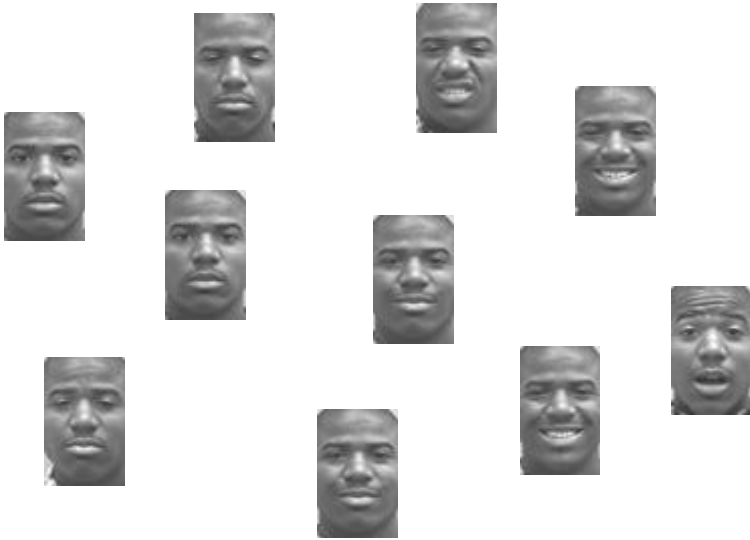
*(Here the proposed state became the current state)*

Chain: 



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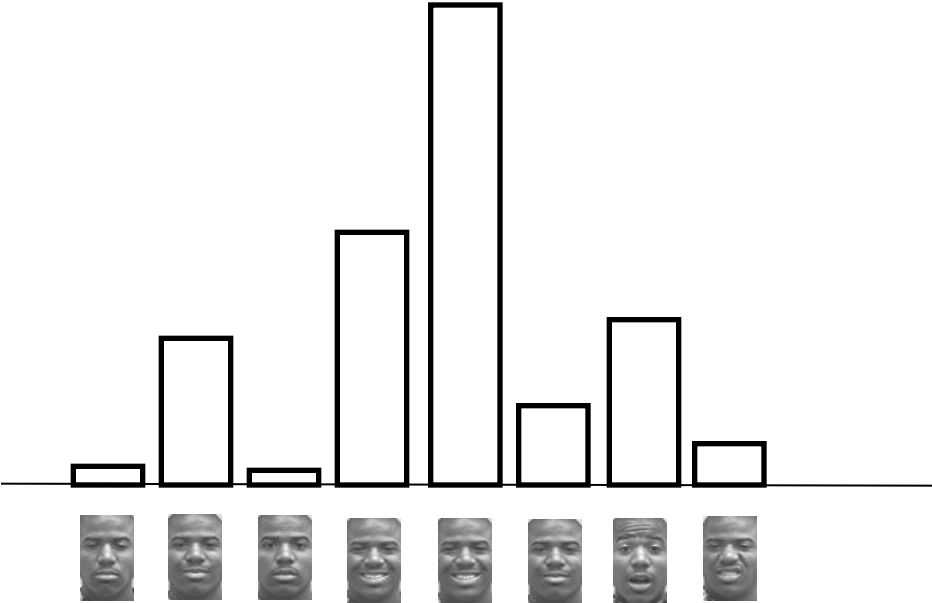
Eventually you end up with a long chain of states



# MCMC method for exploring human categories:

Chain:  ...

Build a histogram over states

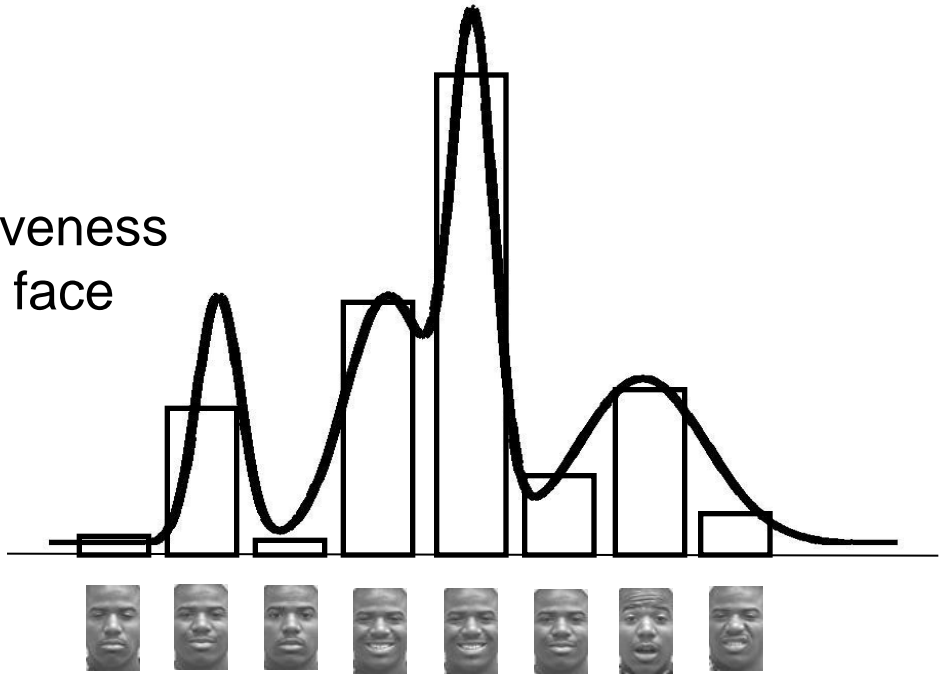


# MCMC method for exploring human categories:

Chain: 

Build a histogram over states

Representativeness  
of a happy face



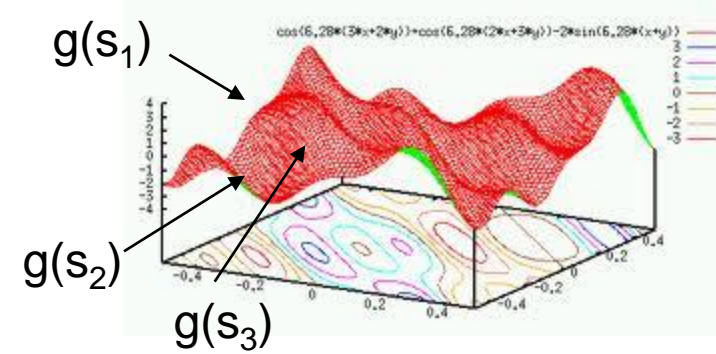
# MCMC algorithm from computer science

Goal: to sample from complex high-dimensional probability distribution

Assume: know relative weights  $g(S)$ , *the shape of distribution*

Algorithm:

- 1) Start at current state  $S_C$
- 2) Choose a nearby proposal state  $S_p$
- 3) Accept proposal as new state with probability  $A(S_p; S_C)$



MCMC acceptance probability

$$A(S_p; S_C) = \frac{g(S_p)}{g(S_C) + g(S_p)}$$

# MCMC algorithm from computer science

Goal: to sample from complex high-dimensional probability distribution

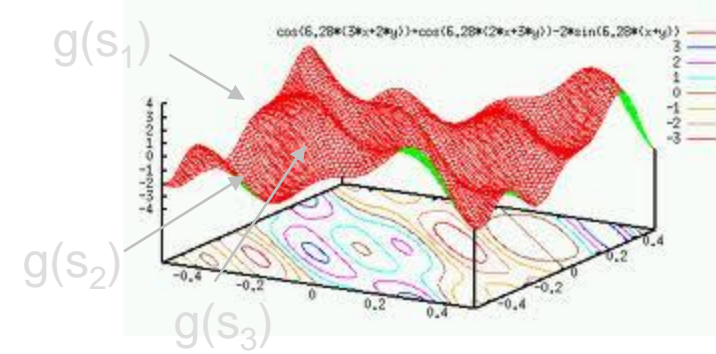
Assume: know relative weights  $g(S)$ , the shape of distribution

Computer Science Algorithm:

- 1) Start at current state  $S_C$
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MCMC acceptance probability

$$A(S_p; S_C) = \frac{g(S_p)}{g(S_C) + g(S_p)}$$



Human MCMC algorithm

- 3) Let participant choose whether to accept proposal as new state. Assume they accept proposal with  $L(S_p; S_C)$

Luce's choice rule for binary choice

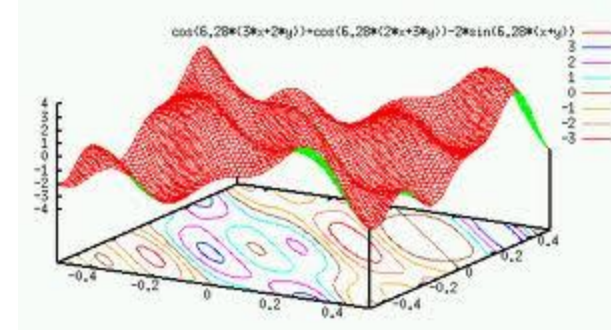
$$L(S_p; S_C) = \frac{W(S_p)}{W(S_C) + W(S_p)}$$

*Here we don't need the shape of distribution*

# MCMC algorithm from computer science

Goal: to sample from complex high-dimensional probability distribution

Assume: know relative weights  $g(S)$ , *the shape of the complex function*



Computer Science Algorithm:

- 1) Start at current state  $S_C$
- 2) Choose a nearby proposal state  $S_p$
- 3) Accept proposal as new state with probability  $A(S_p; S_C)$

Human MCMC algorithm

- 1) Start at current state  $S_C$
- 2) Choose a nearby proposal state  $S_p$
- 3) Let participant choose whether to accept proposal as new state. Assume they accept proposal with  $L(S_p; S_C)$

MCMC acceptance probability

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Luce's choice rule for binary choice

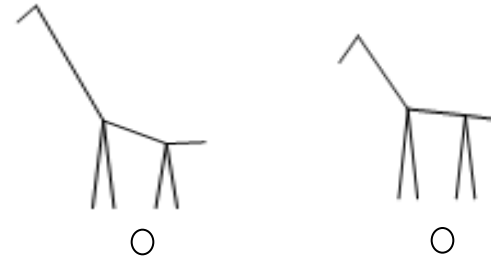
$$L(S_p; S_C) = \frac{W(S_p)}{W(S_C) + W(S_p)}$$

**Key observation: similar structure**

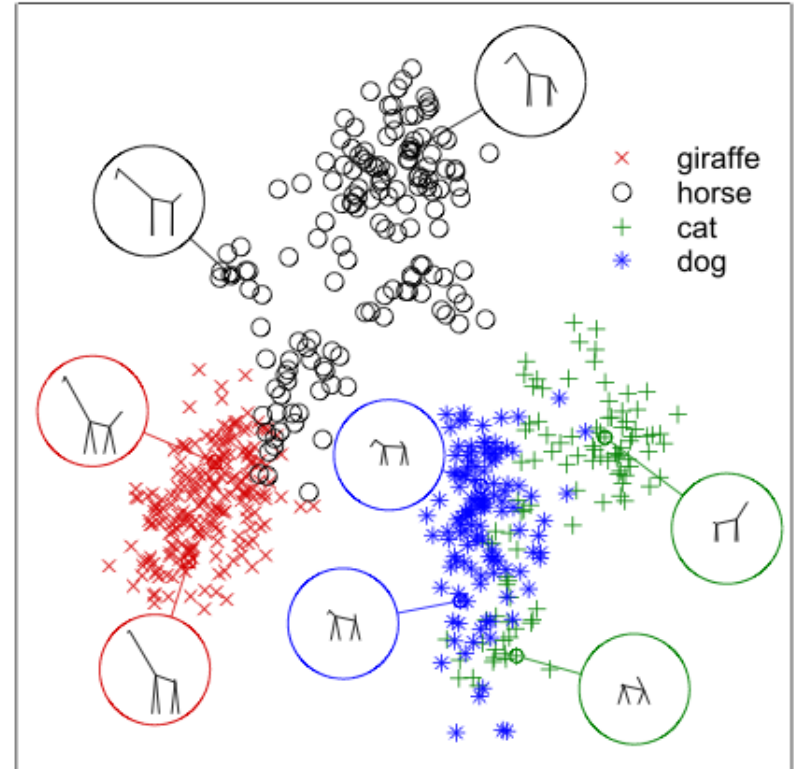
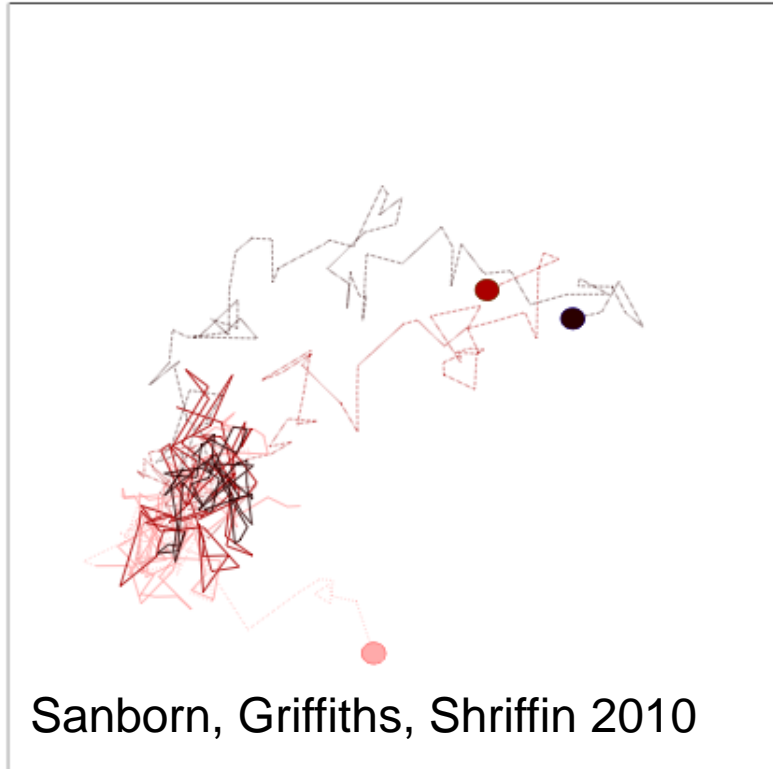
# Convergence in task with stick figure animals

*Example experiment screen:*

Which looks more like a Giraffe?



*Results:*











## How to make a proposal state?

- Previous work used easily parameterized stimuli
- We want to explore large variety of real life categories and concepts
- How do we propose a nearby state for arbitrary stimulus spaces for which definitions of “nearby” is not obvious (e.g. sets of images)?











# How to make a proposal state?

Define similarity metric and quantify similarity over all pairs

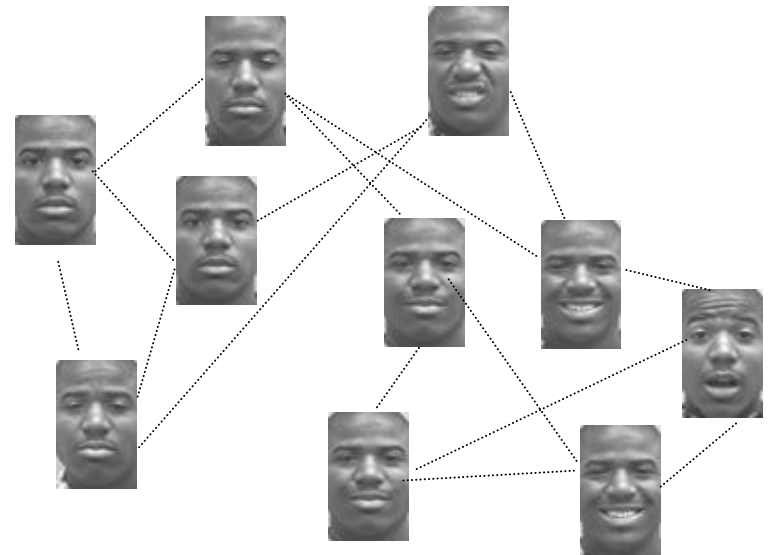
					...
	1	0.7	0.5	0.6	
	0.7	1	0.6	0.1	
	0.5	0.6	1	0.4	
	0.6	0.1	0.4	1	
⋮					

# How to make a proposal state?

Define similarity metric and quantify similarity over all pairs

					...
	1	0.7	0.5	0.6	
	0.7	1	0.6	0.1	
	0.5	0.6	1	0.4	
	0.6	0.1	0.4	1	
⋮					









Run 'B-matching' to obtain graph of N nearest neighbours (e.g. N=3)



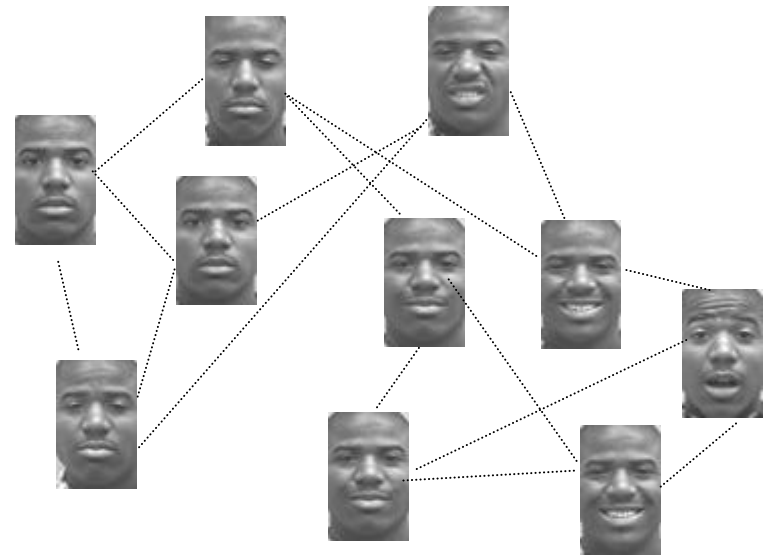
Make sure your set is fully connected

Discrete MCMC makes proposal based on similarity measure between items.

Define similarity metric and quantify similarity over all pairs

					...
	1	0.7	0.5	0.6	
	0.7	1	0.6	0.1	
	0.5	0.6	1	0.4	
	0.6	0.1	0.4	1	
⋮					









Run 'B-matching' to obtain graph of N nearest neighbours (e.g. N=3)



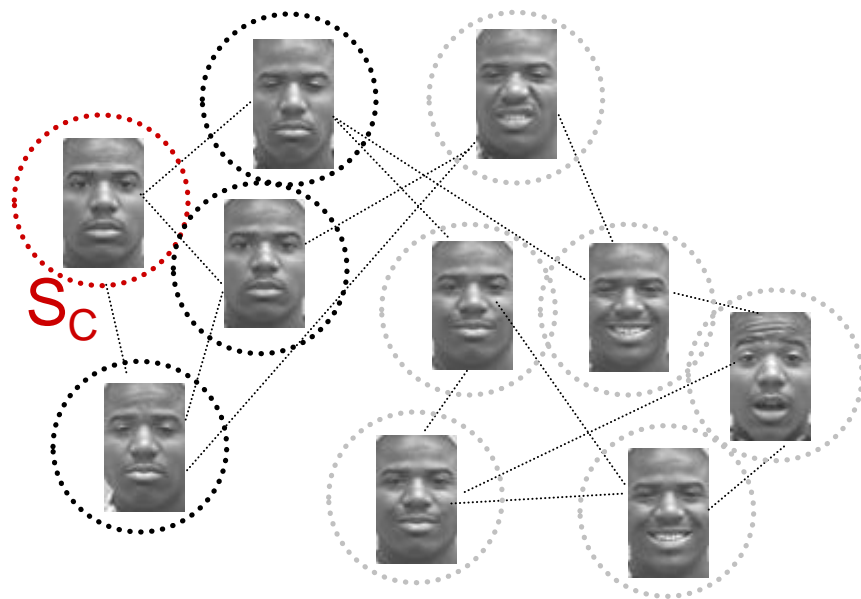
Set proposals to be uniform over neighbours, with small chance of uniform over all stimuli

Discrete MCMC makes proposal based on similarity measure between items.

Define similarity metric and quantify similarity over all pairs

					...
	1	0.7	0.5	0.6	
	0.7	1	0.6	0.1	
	0.5	0.6	1	0.4	
	0.6	0.1	0.4	1	
⋮					









Run 'B-matching' to obtain graph of N nearest neighbours (e.g. N=3)



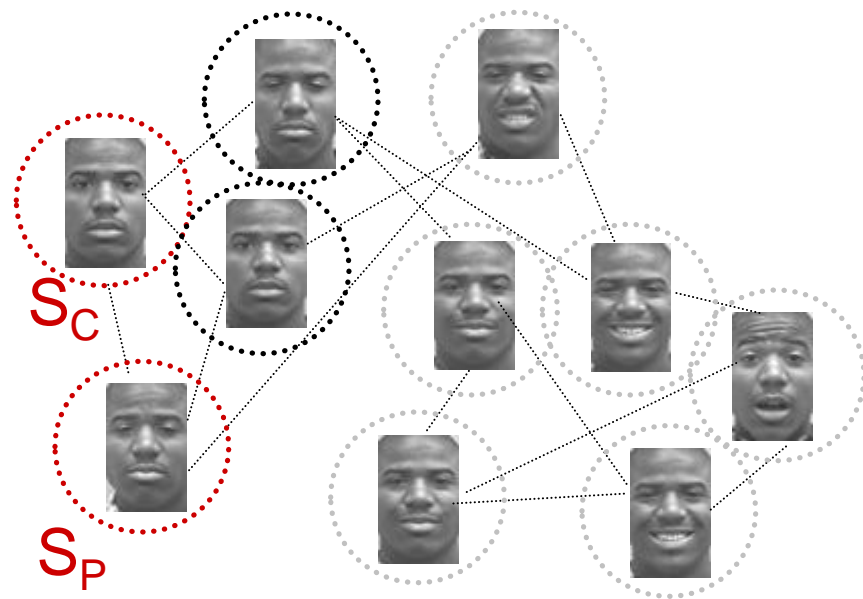
Set proposals to be uniform over neighbours, with small chance of uniform over all stimuli

Discrete MCMC makes proposal based on similarity measure between items.

Define similarity metric and quantify similarity over all pairs

					...
	1	0.7	0.5	0.6	
	0.7	1	0.6	0.1	
	0.5	0.6	1	0.4	
	0.6	0.1	0.4	1	
⋮					









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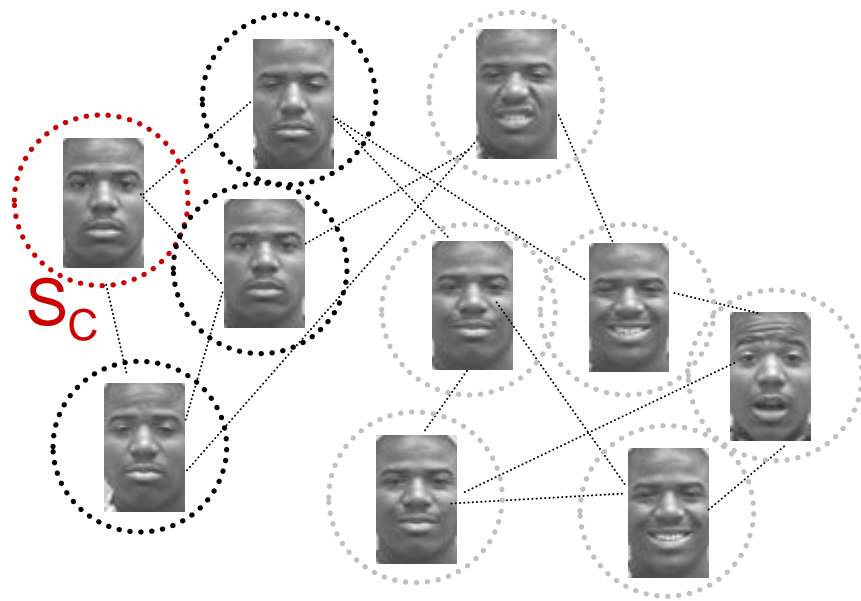
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Run 'B-matching' to obtain graph of N nearest neighbours (e.g. N=3)



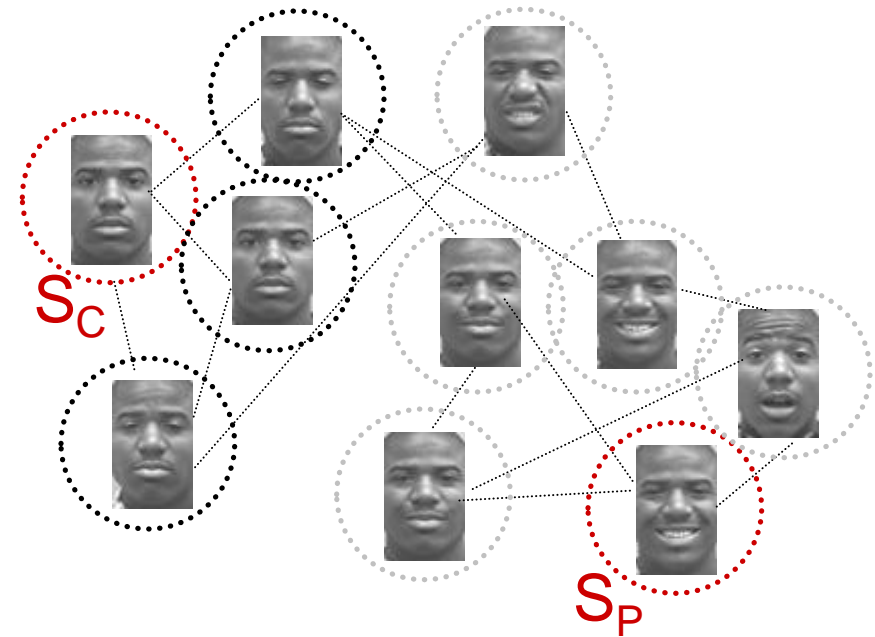
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⋮					

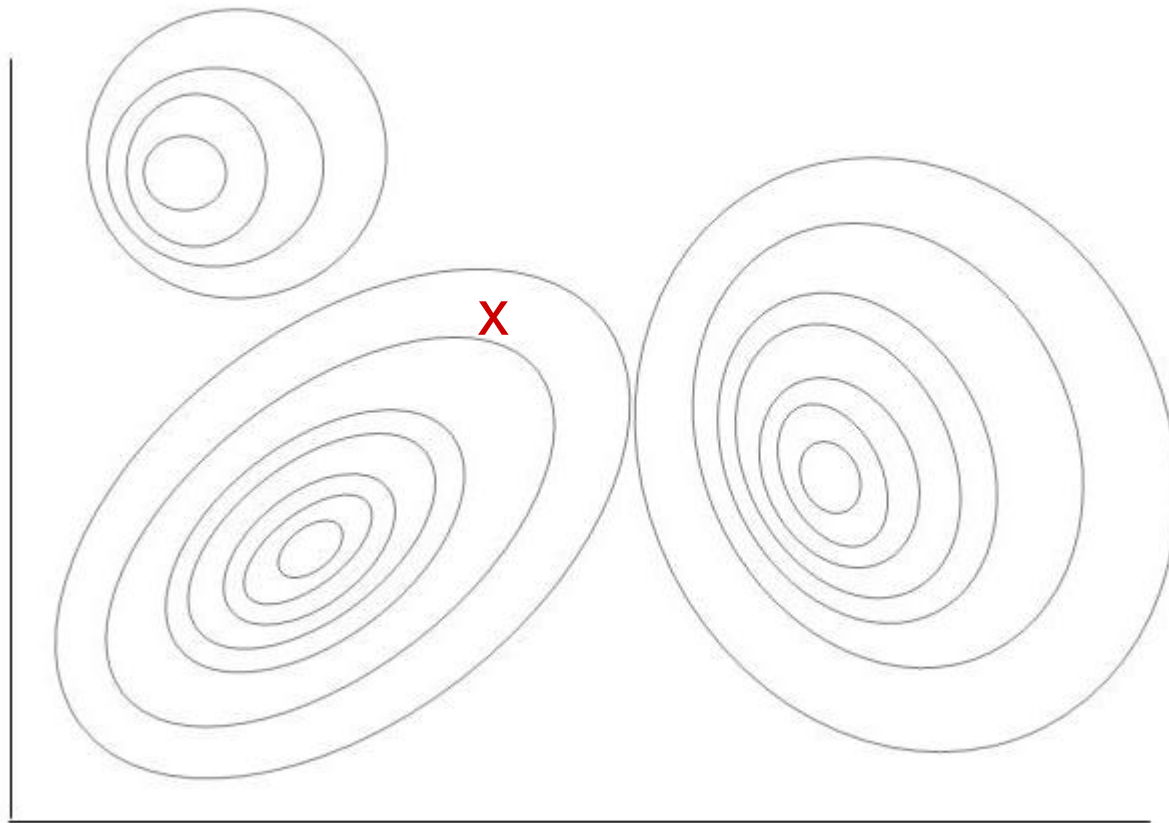
Run 'B-matching' to obtain graph of N nearest neighbours (e.g. N=3)



Set proposals to be uniform over neighbours, with small chance of uniform over all stimuli

# MCMC method for exploring human categories:

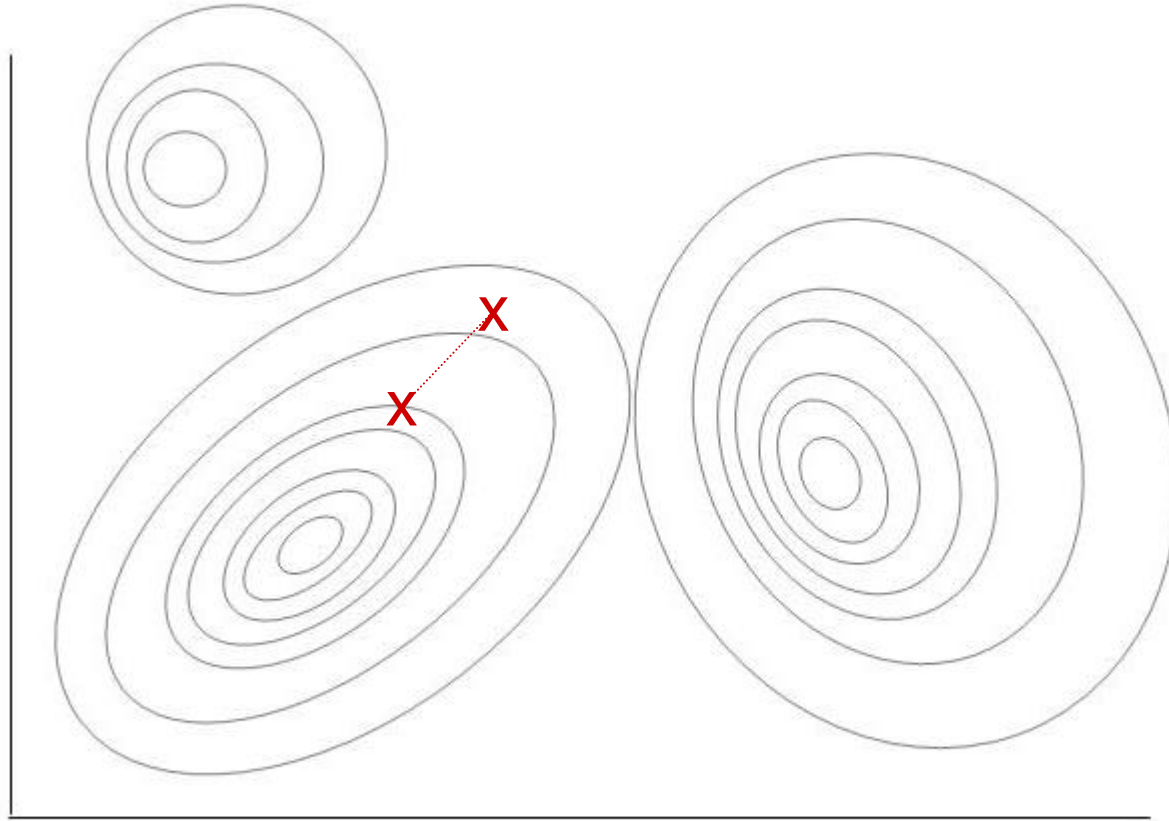
Algorithm efficiently explores regions of high representativeness





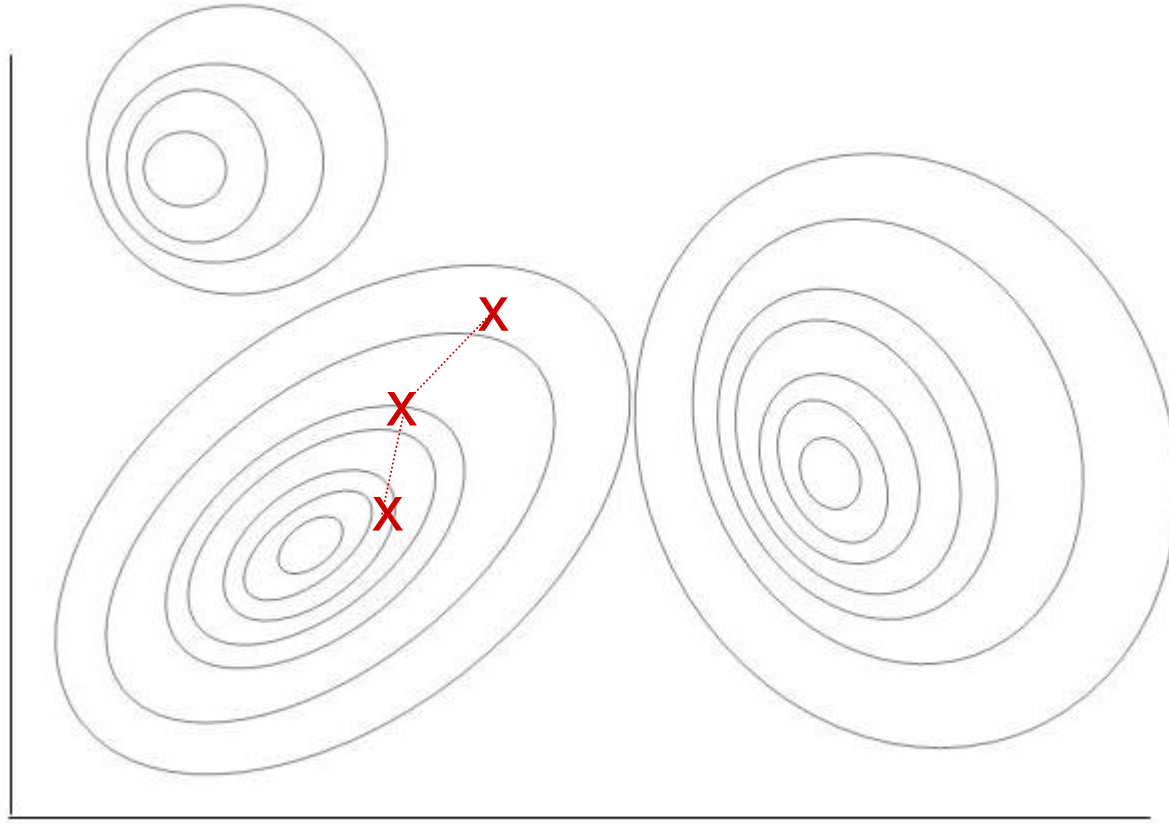
# MCMC method for exploring human categories:

Algorithm efficiently explores regions of high representativeness



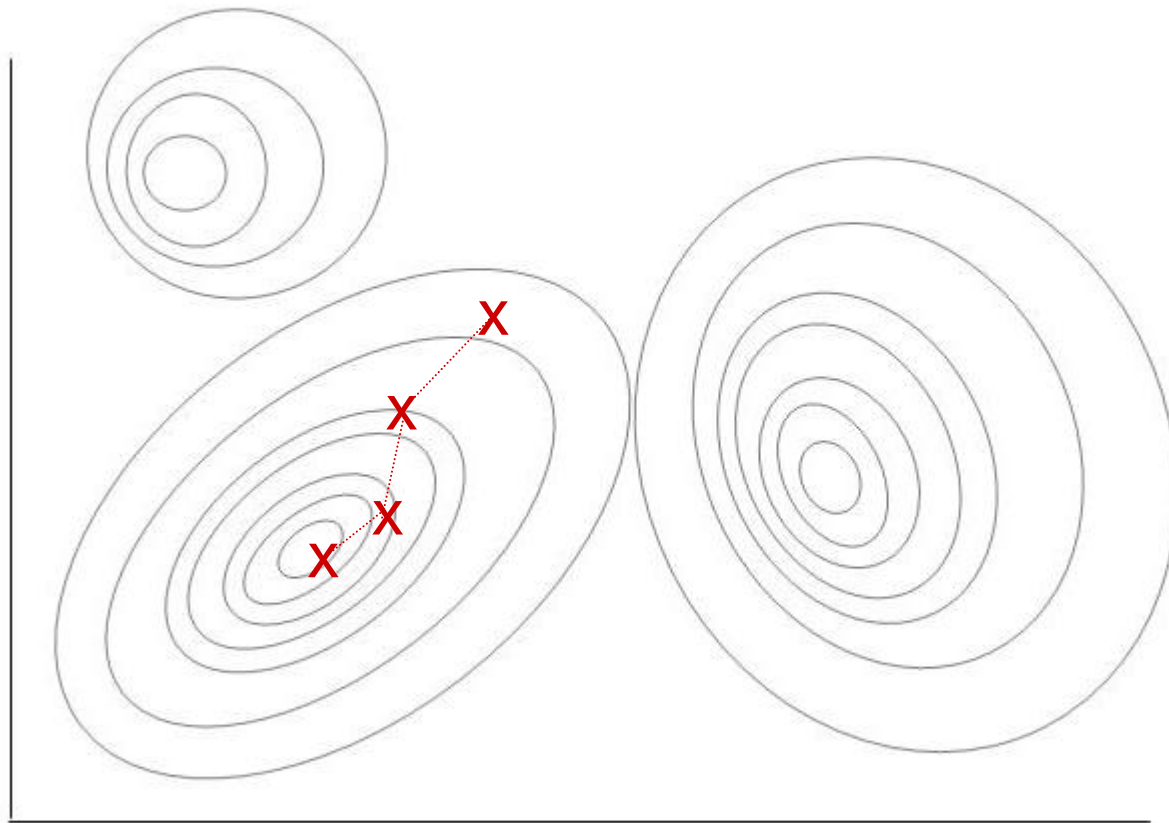
# MCMC method for exploring human categories:

Algorithm efficiently explores regions of high representativeness



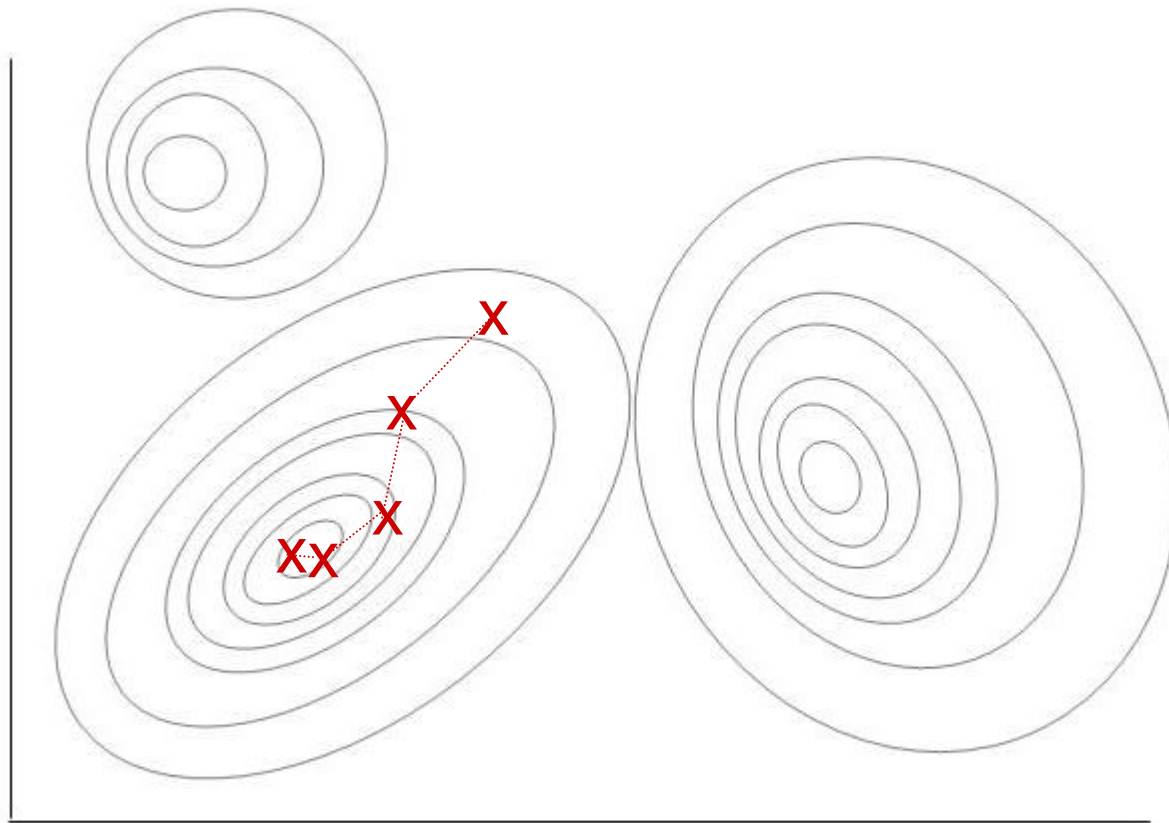
# MCMC method for exploring human categories:

Algorithm efficiently explores regions of high representativeness



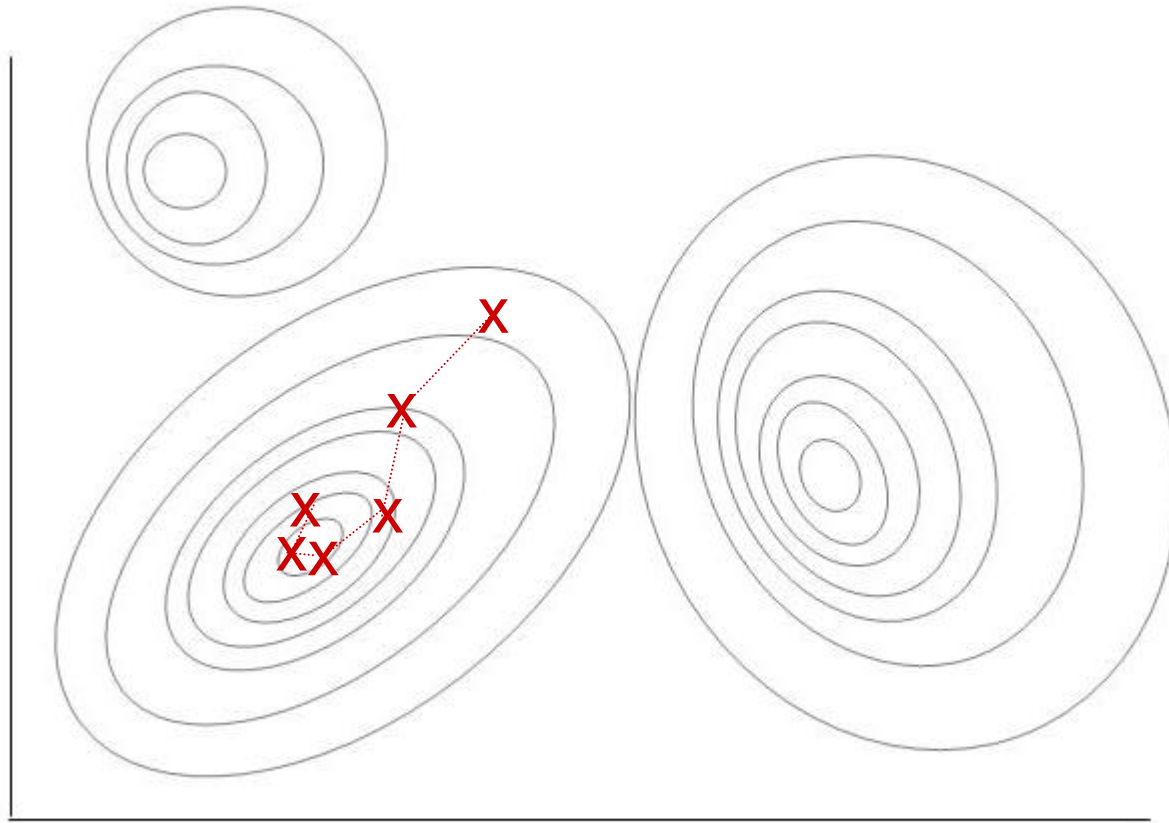
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Algorithm efficiently explores regions of high representativeness



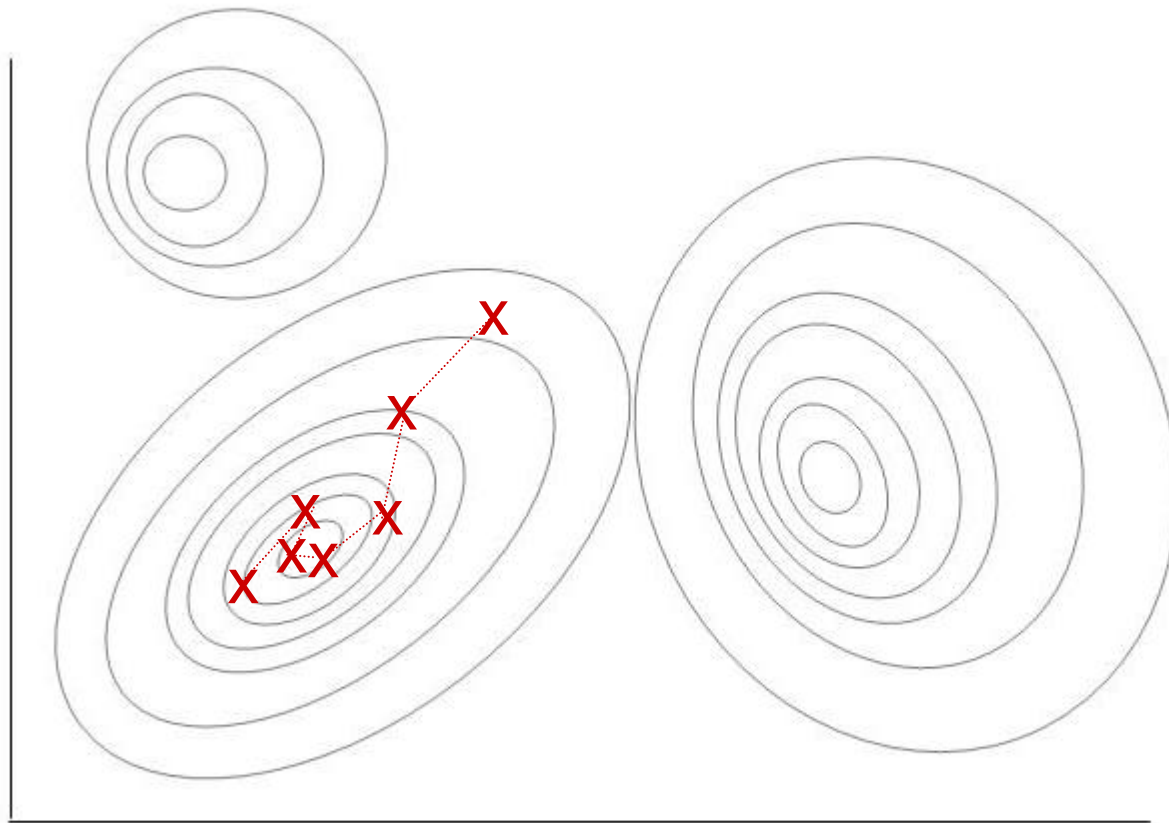
# MCMC method for exploring human categories:

Algorithm efficiently explores regions of high representativeness



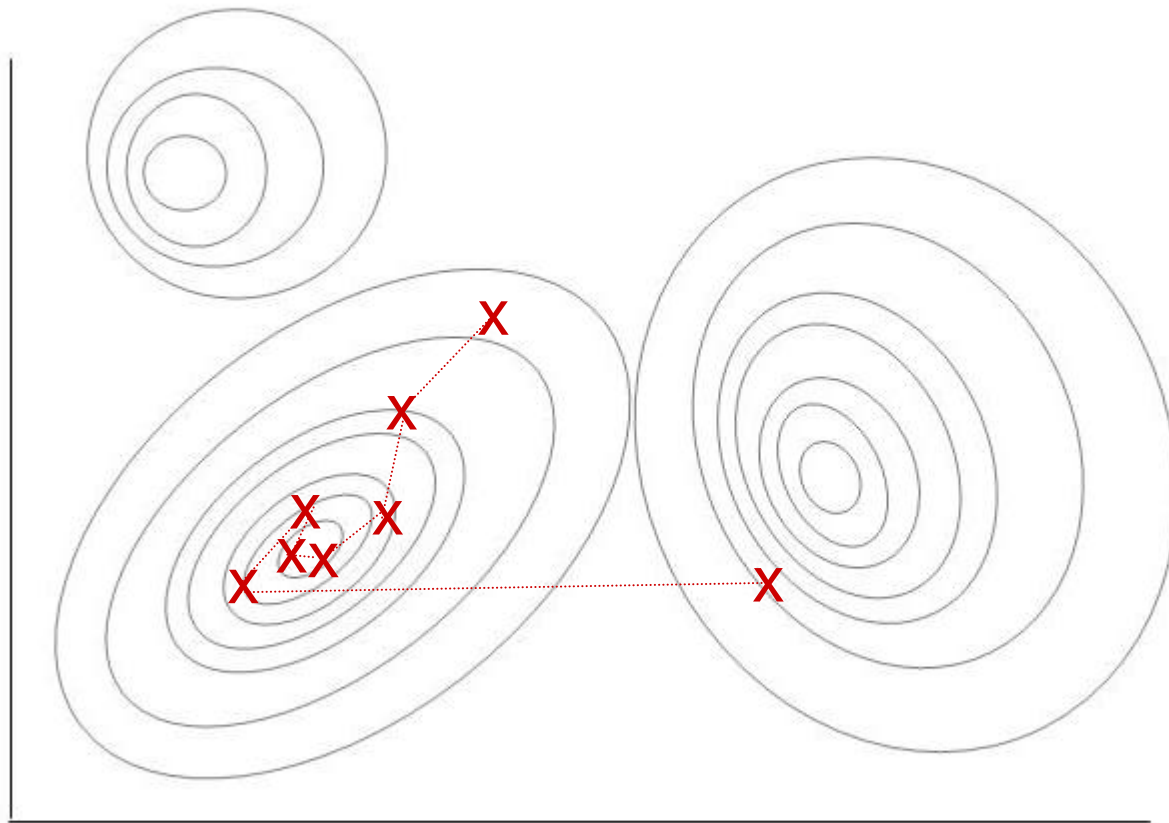
# MCMC method for exploring human categories:

Algorithm efficiently explores regions of high representativeness



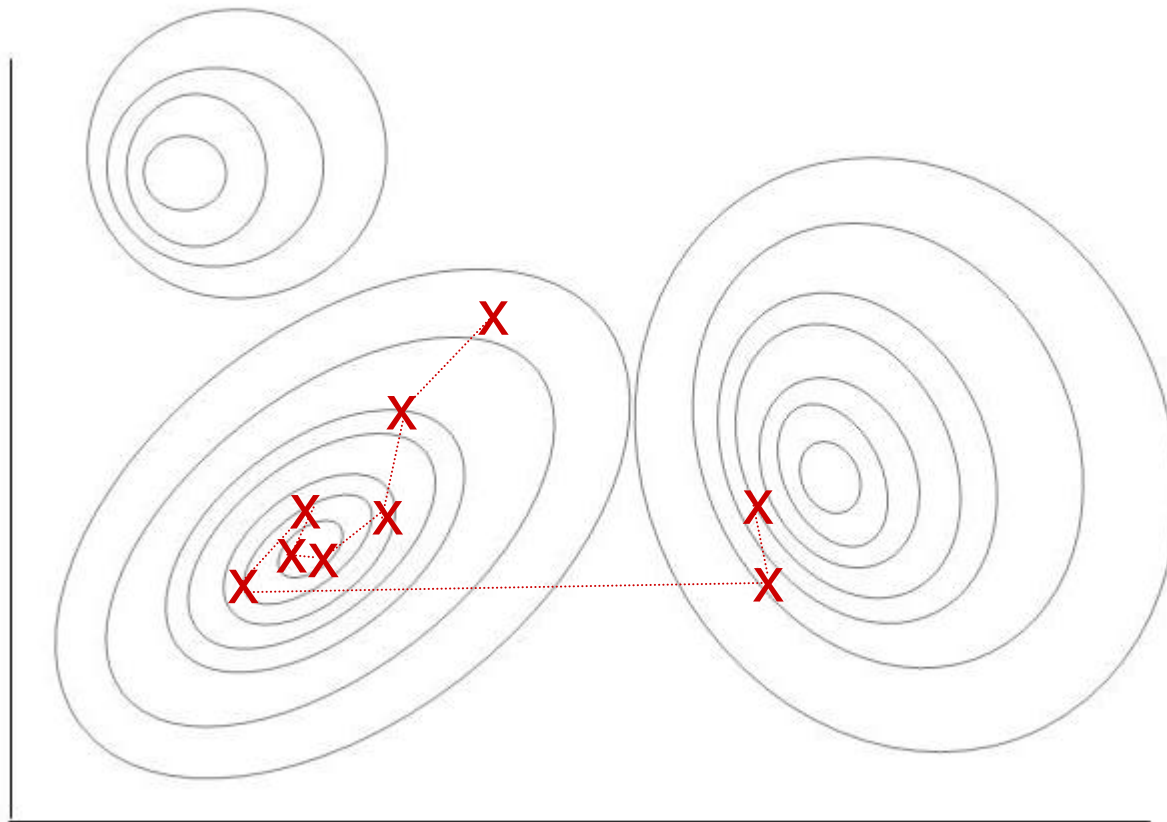
# MCMC method for exploring human categories:

Algorithm efficiently explores regions of high representativeness



# MCMC method for exploring human categories:

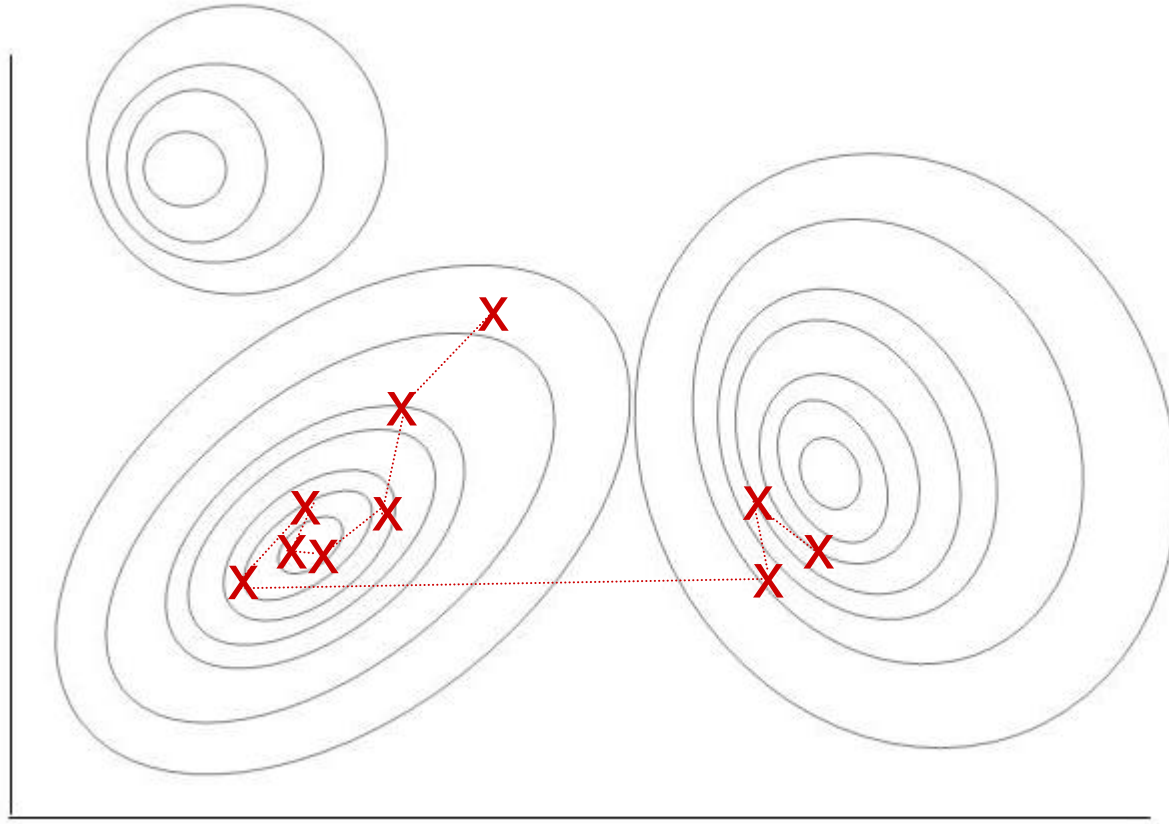
Algorithm efficiently explores regions of high representativeness





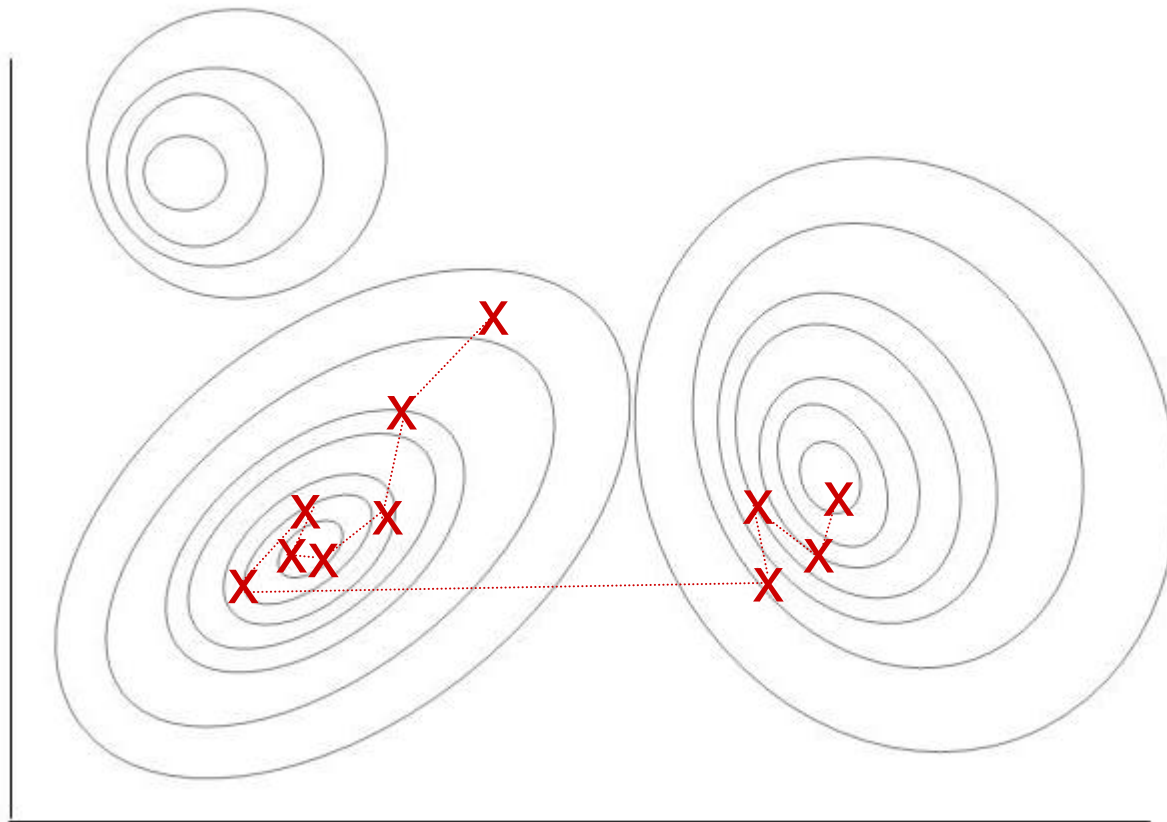
# MCMC method for exploring human categories:

Algorithm efficiently explores regions of high representativeness



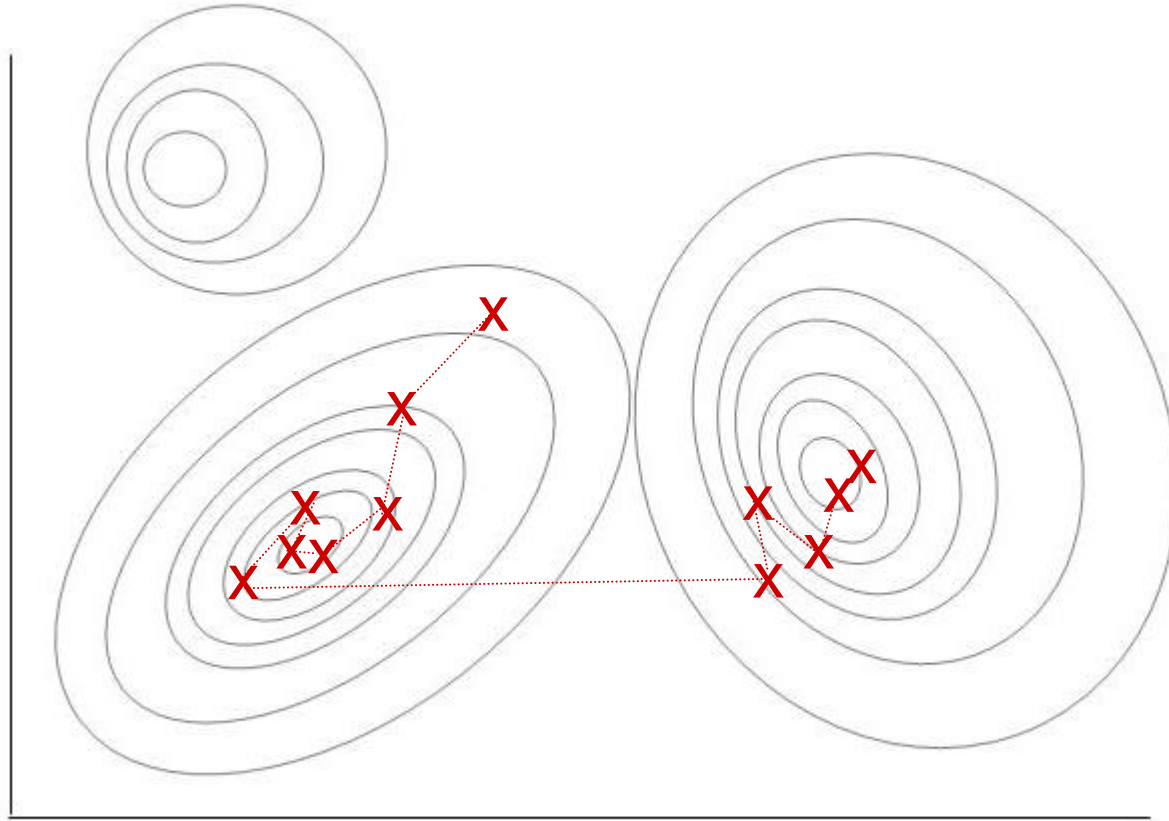
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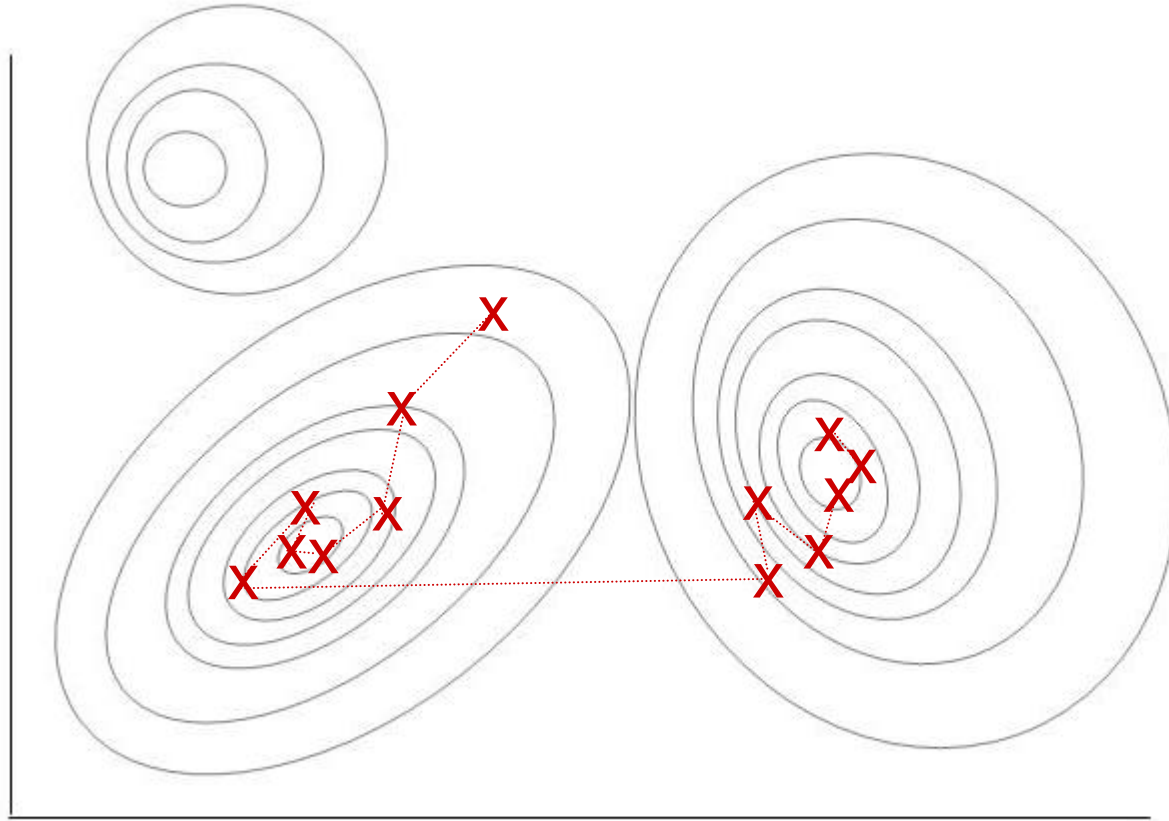
# MCMC method for exploring human categories:

Algorithm efficiently explores regions of high representativeness



# MCMC method for exploring human categories:

Algorithm efficiently explores regions of high representativeness



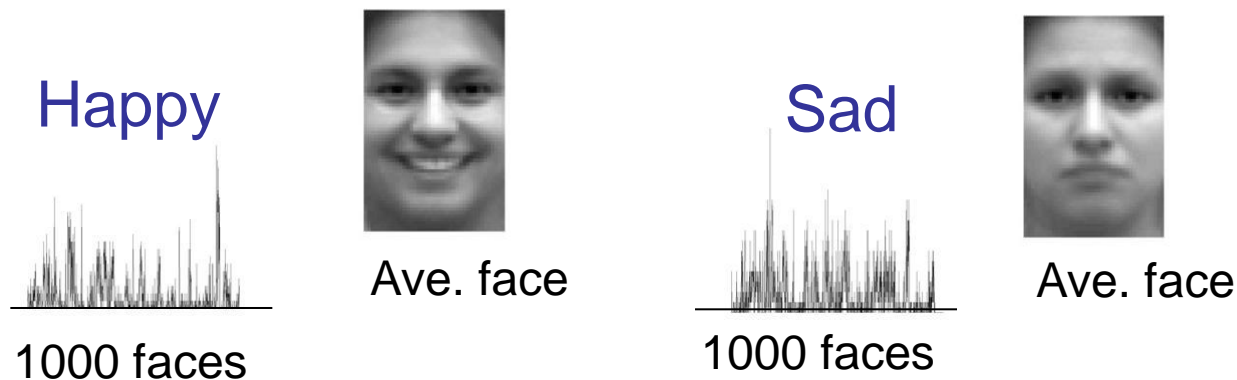
Using MCMC we can explore a wide variety of real life categories

## Emotions: Which face is more Happy/Sad?

(Hsu, A., Martin, J., Sanborn, A., Griffiths, T. *submitted*)



... set of 1000 faces



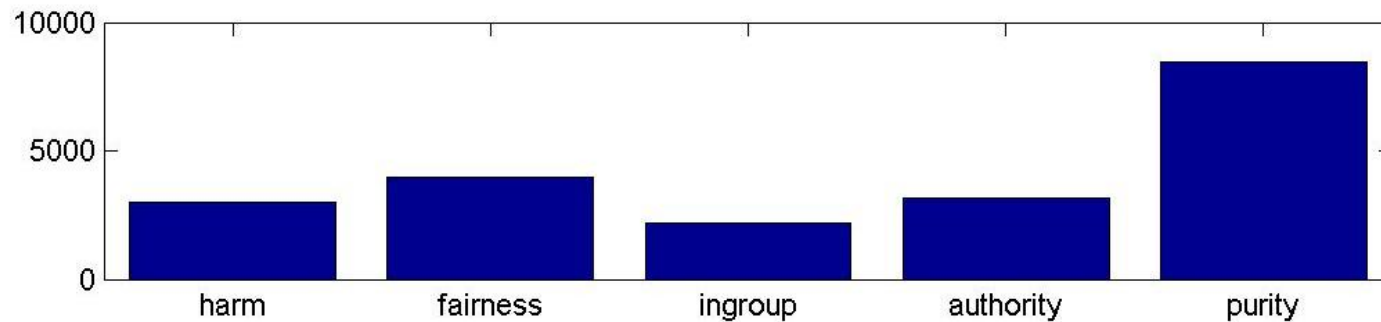


Using MCMC we can explore a wide variety of real life categories

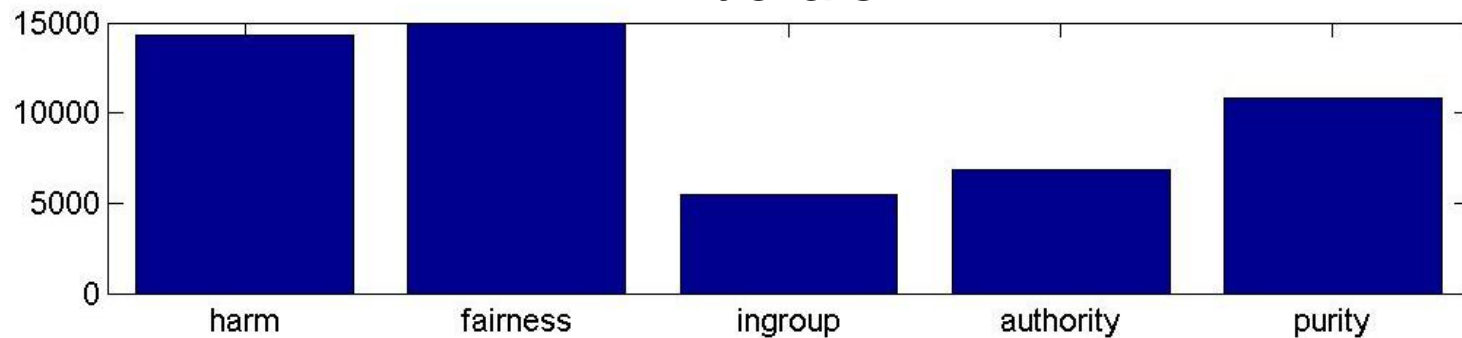
Liberal vs. Conservative Morality: Which word is more relevant to morality?

(Hsu, A., Martin, J., Sanborn, A., Griffiths, T. *In preparation*)

### Conservatives



### Liberals



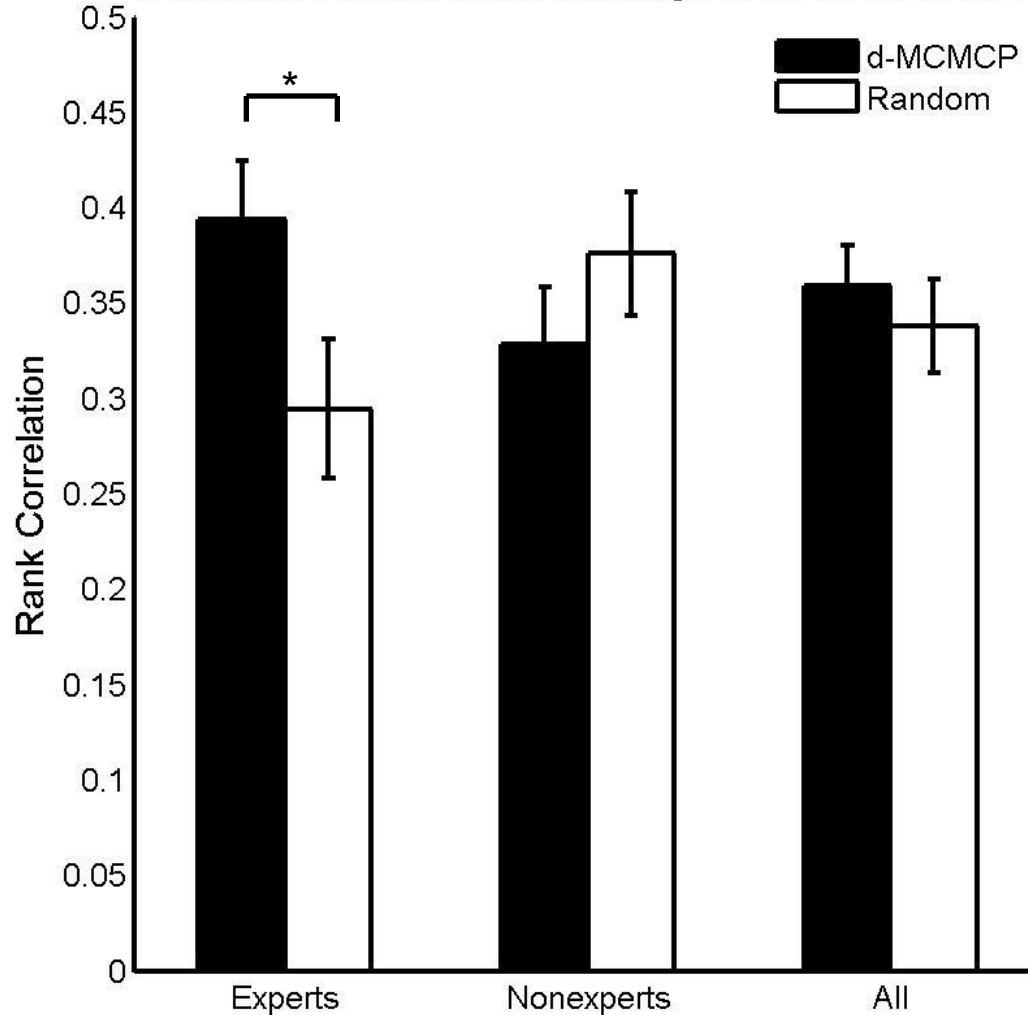
Results collapsed across 160 words pre-chosen to be relevant for morality

Using MCMC we can explore a wide variety of real life categories

## Measuring Consumer preferences: Which wine is preferred?

(Hsu, A., Coenen, A., Lewis, R., Cheung, B. *submitted*)

Predicted versus Actual Rankings of 10 Novel Wines



MCMC is able to measure preferences of experts significantly better when tested on predictions for novel wine preferences

\*  $p < 0.05$



Using MCMC we can explore a wide variety of life categories.

### **Faces**

- Emotions (Prof. Tom Griffiths; UC Berkeley)
- Trustworthiness (Costi Rezesescu; UCL)

### **Images**

- Iconic images of cities, seasons, holidays (Prof. Tom Griffiths; UC Berkeley)

### **Legal reasoning**

- Stereotypes of criminals (Dr D. Lagnado; UCL)

### **Consumer Preferences**

- Preferences for product features (Anna Coenen, Dr. R. Lewis, Dr. B. Cheung; Decision Technology)

### **Medical decision making**

- Hospital choice (Dr I. Vlaev, Dr D. King, Dr H. Lee; Imperial College)
- Surgical fracture diagnosis (Dr R. Emery, Dr I. Vlaeg; Imperial)

Thank you to:

Tom Griffiths

Jay Martin

Adam Sanborn