Dual Decomposition of Finite Horizon Markov Decision Processes

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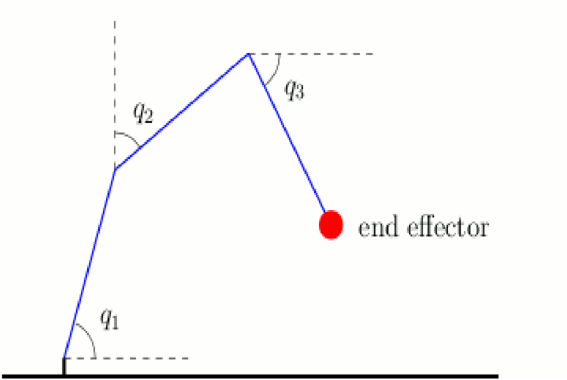
Outline

- Problem Framework
- Dual Decomposition
- Experiments
- Summary

PROBLEM FRAMEWORK

We are interested in the problem of optimal control in a dynamic environment. Examples include

- Robotics.



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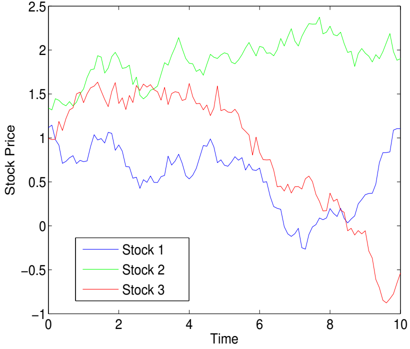

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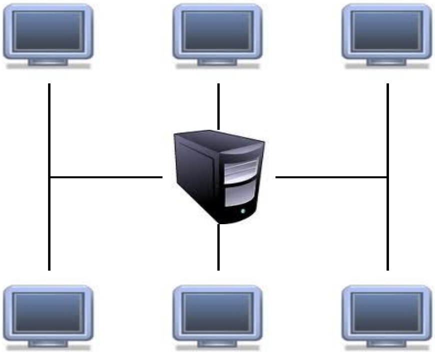
- Portfolio Optimisation.
- Network Management.



Markov Decision Processes

We consider the problem of Markov Decision Processes, which are given by

- action-state space

  action space - $a \in \mathcal{A}$ (discrete).
  state space - $s \in \mathcal{S}$ (discrete).

- initial state distribution - $p_0(s)$.
- policy

  non-stationary - $\pi_t(a|s,t) = p(a|s,t;\pi)$.
  stationary - $\pi(a|s) = p(a|s;\pi)$.

- reward - $R(a,s)$.
- transition dynamics - $p(s'|s,a)$.
- planning horizon - H (finite or infinite).

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Markov Decision Processes

Objective - Optimise π to maximise the total expected reward

$$U(\pi) = \sum_{t=1}^{H} \sum_{a_t, s_t} R(a_t, s_t) p(a_t, s_t; \pi),$$

where $p(a_t, s_t; \pi)$ is the marginal of the trajectory distribution

$$p(s_{1:H}, a_{1:H}; \pi) = p(a_H | s_H; \pi) p_0(s_1) \prod_{t=1}^{H-1} p(s_{t+1} | s_t, a_t) p(a_t | s_t; \pi).$$

Problem Framework

Interested in solving finite horizon MDP's with stationary policies, *i.e.*

- $H < \infty$,
- $\pi_t(a|s) = \pi(a|s), \qquad t = 1, ..., H.$

In particular we're interested in a **dynamic programming** 'type' solution to this problem class.

Other planning algorithms

EM - slow convergence.

Policy Gradients - susceptible to local optima.

Difficult - Bellman's *principal of optimality* no longer holds.

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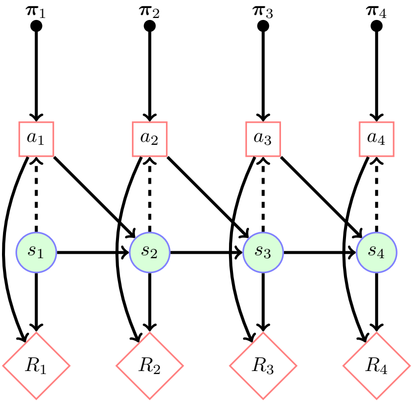
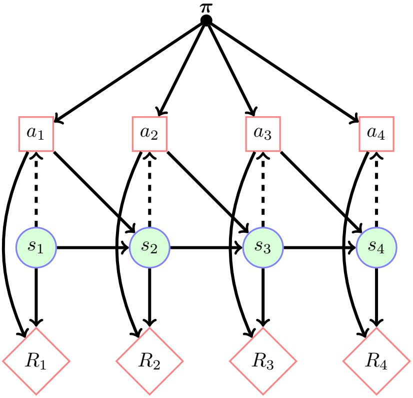
Influence Diagrams



Non-Stationary Policies

Chain Structured - Easy to Optimise



Stationary Policies

Large Policy Clique - Difficult to Optimise

DUAL DECOMPOSITION

Dual Decomposition

Use idea of **dual decomposition** to exploit the theoretical ease of optimising a finite horizon MDP with non-stationary policies.

Original maximisation problem

$$\max_{\pi} \sum_{t=1}^{H} \sum_{a_t, s_t} R(a_t, s_t) p(a_t, s_t; \pi),$$

can be rewritten as

$$\max_{\substack{\pi, \pi_{1:H} \\ \pi_t = \pi, \forall t}} \sum_{t=1}^{H} \sum_{a_t, s_t} R(a_t, s_t) p(a_t, s_t; \pi_{1:t}).$$

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Dual Decomposition

Ordinarily the constraints $\pi_t = \pi$, $t = 1, ..., H$, would be handled by adjoining

$$\sum_{t=1}^{H} \sum_{a,s} \lambda_t(a, s)(\pi_t(a|s) - \pi(a|s)),$$

to the Lagrangian.

Note - this doesn't lead to dynamic programming solution.

So we consider the equivalent constraints

$$\sum_{t=1}^{H} \sum_{a,s} \lambda_t(a, s)(\pi_t(a|s) - \pi(a|s))p(s_t = s|\pi_{1:t-1}).$$

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This leads to objective function

$$L(\pi, \pi_{1:H}, \lambda_{1:H}) = \sum_{t=1}^{H} \sum_{a_t, s_t} \left\{ \Big(R(a_t, s_t) + \lambda_t(a_t, s_t) \Big) p(a_t, s_t | \pi_{1:t}) \right.$$
$$\left. - \lambda_t(a_t, s_t) \pi(a_t | s_t) p(s_t | \pi_{1:t-1}) \right\}$$

Can perform optimisation over π.

This gives constraint set $\Lambda(\pi_{1:H})$ over Lagrange multipliers

$$\sum_{t=1}^{H} \lambda_t(a, s) p(s_t = s | \pi_{1:t-1}) = 0, \qquad \forall (a, s) \in \mathcal{S} \times \mathcal{A}.$$

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Final dual objective function

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- **slave** problems
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Slave Problem

For fixed $\lambda_{1:H}$ maximisation over $\pi_{1:H}$ takes the form

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- Objective (1) an ordinary MDP with non-stationary policies.
- Lagrange multipliers leads to non-stationary rewards.
- Solvable using dynamic programming.

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Minimisation done using a **projected sub-gradient step.**

Gradient Step - take step in direction of anti-gradient

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Projection Step - project $\lambda_{1:H}$ back down into constraint set Λ

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Algorithm Summary

Summary - dual decomposition solution iterates between **slave problem** and the **master problem** until convergence.

- **Slave Problem** - Update $\pi_{1:H}$ by solving a finite horizon MDP with
  - non-stationary policies.
  - non-stationary rewards - $\hat{R}_t = R + \lambda_t$.
- **Master Problem** - Update $\lambda_{1:H}$ using a projected sub-gradient step.

Resource Allocation

Dual decomposition algorithm adjusts non-stationary rewards (*i.e.* Lagrange multipliers) to obtain stationary policies.

Question - How are $\lambda_{1:H}$ updated?

We show the following relation

$$\lambda_t^{i+1}(s,a) \begin{cases} \leq \lambda_t^i(s,a) & \text{if } a = \underset{a}{\text{argmax}}\, \pi_t^i(a|s), \\ \geq \lambda_t^i(s,a) & \text{if otherwise.} \end{cases}$$

Additionally, the difference obeys the relation

$$|\lambda_t^{i+1}(s,a) - \lambda_t^i(s,a)| = \mathcal{O}(H - N_i(s,a)),$$

where $N_i(s,a)$

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Example - Consider an MDP with 2 actions.

If in a given a state, s, the previous slave problem found

- action a_1 was optimal for a **large** number of time points,
- while action a_2 was optimal for only a **few** time points,

then

- for time-points where a_1 was **optimal**

  $\lambda_t(a_1, s)$ - would **decrease only slightly**
  $\lambda_t(a_2, s)$ - would **increase only slightly**

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Resource Allocation - An Example

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EXPERIMENTS

Comparisons

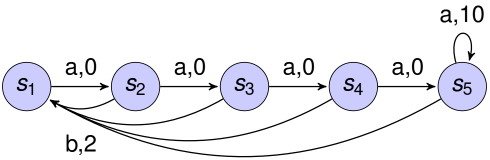
We compare our Dual Decomposition Dynamic Programming (DD DP) algorithm against;

- Expectation Maximisation (EM)
- Policy Gradients (PG)
    - Fixed Step Size
    - Line Search
- Expectation Maximisation - Policy Gradients (EM-PG)

Chain Problem

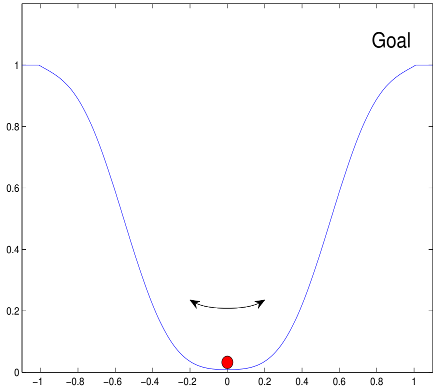
Objective - For $H = 25$ it is optimal to manoeuvre the agent to the right-most end of the chain.

- $|\mathcal{S}| = 5$.
- $|\mathcal{A}| = 2$.
- H = 25.



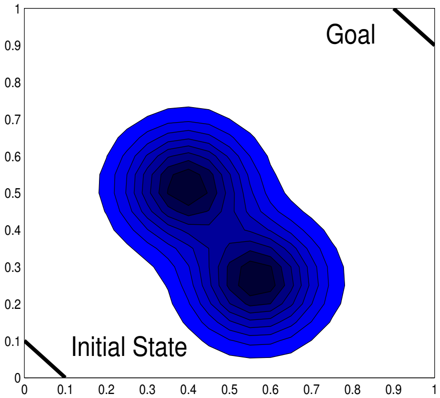
Objective - Manoeuvre the agent to the goal region at the right-most peak of the valley.

- $|\mathcal{S}| = 231$.
- $|\mathcal{A}| = 3$.
- H = 25.



Objective - Manoeuvre the agent to the goal region whilst avoiding the puddles, which cause a negative reward.

- $|\mathcal{S}| = 441$.
- $|\mathcal{A}| = 4$.
- H = 50.



Results

		Algorithms				
		DD DP	EM	F-PG	LS-PG	EM-PG
Chain Problem	$U(\pi^*)$	86	85	75	65	86
	Iterations	3	100	100	3	100
Mountain Car	$U(\pi^*)$	19	19	16	14	19
	Iterations	7	100	100	3	100
Puddle World	$U(\pi^*)$	42	39	N/A	0	N/A
	Iterations	30	1000	N/A	10	N/A

SUMMARY
&
FUTURE WORK

Summary & Future Work

Summary
We have presented that dual decomposition algorithm for finite horizon MDP's with stationary policies.

Future work

- Extend to continuous state-action domains.
- Extend to more complex domains, such as *partially observable Markov decision processes*.