Verification of OWL-Time Ontology

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Time Ontologies

There are over forty first-order theories of time

All of these time ontologies are represented using either:

- Points
- Intervals
- Both points and intervals

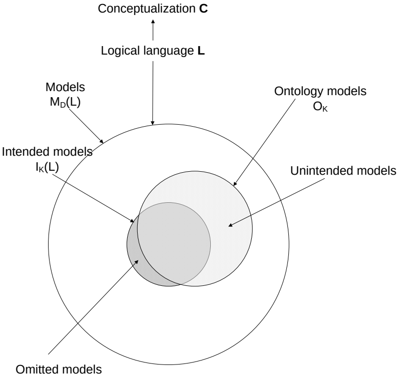
Hobbs and Pan proposed a first-order axiomatization of OWL-Time (`http://www.w3.org/TR/owl-time/`) as an ontology of time for the Semantic Web that also includes both timepoints (referred to as instants) and intervals.

Objectives

- Provide a characterization of the models of $T_{owltime}$ up to isomorphism using the notion of theory reducibility.
- Specify a modularization of $T_{owltime}$.
- Identify incorrect or missing axioms in the current axiomatization of $T_{owltime}$.
- Use the reduction to compare $T_{owltime}$ to other time ontologies for points and intervals, and address the question as to whether $T_{owltime}$ forms an adequate core theory for time ontologies, or whether it is too weak or too strong to play such a role.

Ontology Verification: Intuitions

The relationship between the intended models for an ontology and the actual models of its axioms, reproduced from Guarino (2009)



Ontology Verification: Model Theory

The necessary direction of a representation theorem (i.e. if a structure is intended, then it is a model of the ontology's axiomatization) can be stated as

$$\mathcal{M} \in \mathfrak{M}^{intended} \Rightarrow \mathcal{M} \in Mod(T_{onto})$$

The sufficient direction of a representation theorem (any model of the ontology's axiomatization is also an intended structure) can be stated as

$$\mathcal{M} \in Mod(T_{onto}) \Rightarrow \mathcal{M} \in \mathfrak{M}^{intended}$$

Ontology Verification: Reasoning

Theorem

A theory T_1 is definably equivalent with a theory T_2 iff the class of models $Mod(T_1)$ can be represented by $Mod(T_2)$.

The necessary direction of the representation theorem is equivalent to the following reasoning task:

$$T_{onto} \cup \Delta \models T_1 \cup \ldots \cup T_n \qquad \textbf{(Rep-1)}$$

The sufficient direction of the representation theorem is equivalent to the following reasoning task:

$$T_1 \cup \ldots \cup T_n \cup \Pi \models T_{onto} \qquad \textbf{(Rep-2)}$$

... But It's Too Hard!

- Relationships between first-order ontologies within a repository can be used to support verification
- The fundamental insight is that we can use the relationships between ontologies to assist us in the characterization of the models of the ontologies
- The objective is the construction of the models of one ontology from the models of another ontology by exploiting the relationships between these ontologies and their modules in the repository

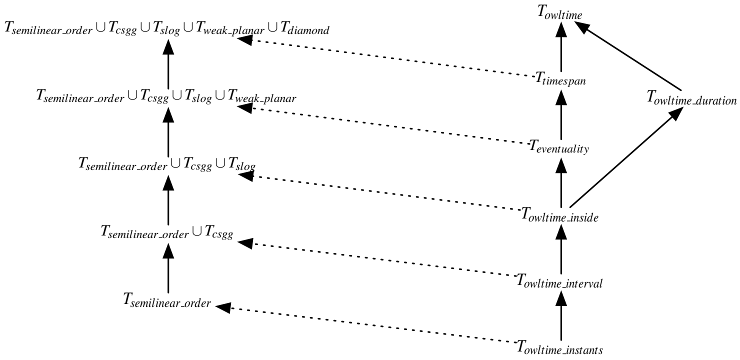
COLORE

- The COLORE (Common Logic Ontology Repository) project is building an open repository of first-order ontologies that serve as a testbed for ontology evaluation and integration techniques, and that can support the design, evaluation, and application of ontologies in first-order logic
- All ontologies are specified using Common Logic (ISO 24707)
  `http://stl.mie.utoronto.ca/colore`
  `http://code.google.com/p/colore/`

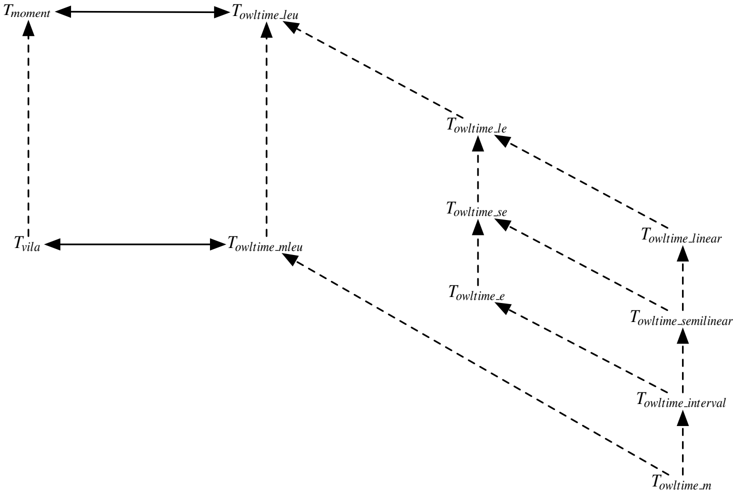
Approach

1. Find a set of theories in the repository that are definably equivalent to the ontology.
2. Specify the intended models of the ontology.
3. Prove the representation theorem for the ontology by using the representation theorems for the theories from the repository.

Modularization of $T_{owltime}$



Theories Related to $T_{owltime\_interval}$



Loop Graphical Incidence Structures

Definition

A loop graphical incidence structure is a bipartite incidence structure

$$\mathbb{S} = \langle X, Y, \mathbf{in}^{\mathbf{S}} \rangle$$

such that all elements of Y are incident with either one or two elements of X, and for each pair of points $\mathbf{p}, \mathbf{q} \in X$ there exists a unique element in Y that is incident with both $\mathbf{p}$ and $\mathbf{q}$, and for each point $\mathbf{r} \in X$ there exists a unique element in Y that incident only with $\mathbf{r}$.

Theorem

Let $G = (V, E)$ be a complete graph with loops.
A bipartite incidence structure is a loop graphical incidence structure iff it is isomorphic to $\mathbb{I} = (V, E, \in)$.

Translation Definitions

Let Δ_1 be the following set of translation definitions:

$$(\forall x)\ point(x) \equiv Instant(x)$$

$$(\forall x)\ line(x) \equiv Interval(x)$$

$$(\forall x, y)\ in^G(x, y) \equiv (begins(x, y) \vee ends(x, y))$$

$$(\forall x, y)\ before(x, y) \equiv lt(x, y)$$

Let Π_1 be the following set of translation definitions:

$$(\forall x)\ Instant(x) \equiv point(x)$$

$$(\forall x)\ Interval(x) \equiv line(x)$$

$$(\forall x, y)\ begins(x, y) \equiv ((in^G(x, y) \wedge ((\forall z)\ in^G(z, y) \supset lt(x, z))$$

$$(\forall x, y)\ ends(x, y) \equiv ((in^G(x, y) \wedge ((\forall z)\ in^G(z, y) \supset lt(z, x))$$

$$(\forall x, y)\ before(x, y) \equiv lt(x, y)$$

Verification of $T_{owltime\_leu}$

Theorem

$T_{owltime\_leu}$ is definably equivalent to

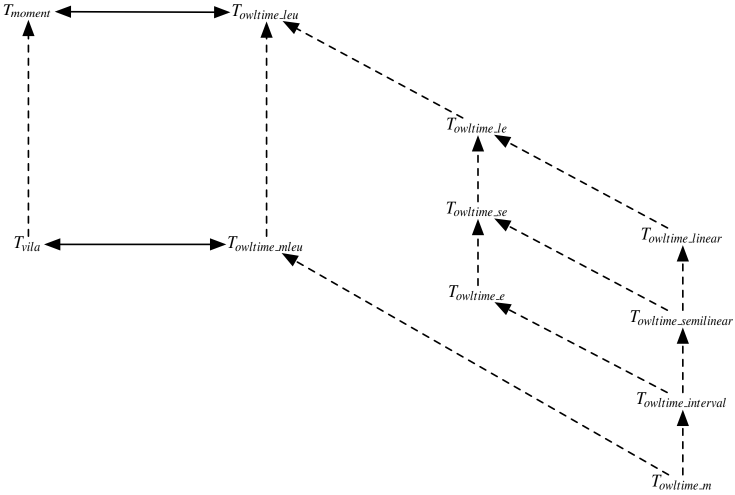
$$T_{linear\_ordering} \cup T_{loop\_graphical}$$

Proof:

$$T_{owltime\_leu} \cup \Delta \models T_{linear\_ordering} \cup T_{loop\_graphical}$$

$$T_{linear\_ordering} \cup T_{loop\_graphical} \cup \Pi \models T_{owltime\_leu}$$

Theories Related to $T_{owltime\_interval}$



Impact

$T_{owltime\_interval}$ is not interpretable by all of the existing ontologies for timepoints and intervals; in other words, there are ontologies that are not definably equivalent to any consistent extension of $T_{owltime\_interval}$.

In particular, it is not interpretable by any time ontology that prevents the existence of moments, such as $T_{endpoints}$.

Critique of Axioms for *inside*

- Mace can be used to construct a model of $T_{owltime\_interval}$ that satisfies the sentence

  $$(\exists t_1, t_2, i) inside(t_1, i) \wedge inside(t_2, i) \wedge \neg before(t_1, t_2) \wedge \neg before(t_2, t_1) \vee (t_1 \neq t_2))$$

  that is, a model in which the Instants in an Interval are not linearly ordered, even though the axioms do entail the condition that the beginning and end instants themselves are linearly ordered.
- We add the following axiom to $T_{owltime}$ to guarantee that all Instants in an Interval are linearly ordered:

$$(\forall t_1, t_2, i)\ inside(t_1, i) \wedge inside(t_2, i)$$

$$\supset (before(t_1, t_2) \vee before(t_2, t_1) \vee (t_1 = t_2)) \qquad (1)$$

Critique of Axioms for *inside*

The axiomatization in $T_{owltime\_inside}$ does not quite capture the following intuition of Hobbs and Pan:

> The concept of inside is not intended to include the beginnings and ends of intervals.

Mace can be used to construct models of $T_{owltime\_inside}$ that falsify each of the following sentences:

$$(\forall i, t_1)\ ProperInterval(i) \wedge begins(t_1, i) \supset \neg inside(t_1, i)$$

$$(\forall i, t_1)\ ProperInterval(i) \wedge ends(t_1, i) \supset \neg inside(t_1, i)$$

$$(\forall i, t_1)\ Interval(i) \wedge begins(t_1, i) \supset \neg inside(t_1, i)$$

$$(\forall i, t_1)\ Interval(i) \wedge ends(t_1, i) \supset \neg inside(t_1, i)$$

In other words, $T_{owltime\_inside}$ is not strong enough to eliminate models in which only the beginnings or ends of intervals are included as instants inside the interval.

Extension of $T_{owltime}$

If we are to entail these sentences (which should follow from the original intuition), we need to extend $T_{owltime}$ with the following two sentences:

$$(\forall i, t_1, t_2)\ inside(t_1, i) \land begins(t_2, i) \supset before(t_2, t_1) \tag{2}$$

$$(\forall i, t_1, t_2)\ inside(t_1, i) \land ends(t_2, i) \supset before(t_1, t_2) \tag{3}$$

Verification of $T_{owltime\_inside}$

Definition

A semilinear ordered geometry is a structure $\mathbb{L} = \langle X, Y, \mathbf{B}, \mathbf{in^L} \rangle$ such that

1. $\mathbb{B} = \langle X, \mathbf{B} \rangle$ is a semilinear betweenness relation;
2. $\mathbb{I} = \langle X, Y, \mathbf{in^L} \rangle$ is a weak bipartite incidence structure;
3. any triple of points that are incident with the same line in Y are ordered by the betweenness relation $\mathbf{B}$.

Eventualities

Hobbs and Pan introduce the class of eventualities to *"cover events, states, processes, propositions, states of affairs, and anything else that can be located with respect to time."*

Models of $T_{eventuality}$ and $T_{timespan}$

Definition

A weak planar geometry is a tripartite incidence structure

$$\mathbb{E} = \langle X, Y, Z, \mathbf{in}^{\mathbf{E}} \rangle$$

in which $N(\mathbf{q})$ is a linear ordered geometry for each $\mathbf{q} \in Z$.

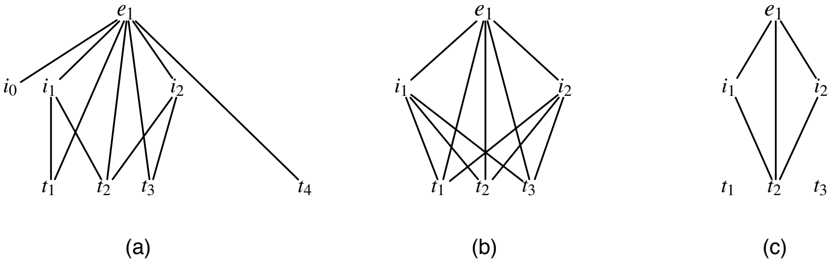


Figure: Examples of weak planar geometries and diamond geometries.

Translation Definitions for $T_{eventuality}$ and $T_{timespan}$

Let Δ_3 be the set of translation definitions in Δ_2 together with:

$$(\forall x, y)\ in^E(x, y) \equiv (inside(x, y) \lor during(x, y) \lor atTime(x, y))$$

Let Π_3 be the set of translation definitions in Π_2 together with:

$$(\forall x)\ eventuality(x) \equiv plane(x)$$

$$(\forall x, y)\ during(x, y) \equiv in^E(x, y) \land plane(x) \land line(y)$$

$$(\forall x, y)\ atTime(x, y) \equiv in^E(x, y) \land plane(x) \land point(y)$$

Representation Theorem for $T_{owltime}$

$\mathcal{M} \in \mathfrak{M}^{owltime}$ iff

$$\mathcal{M} \cong \mathcal{P} \cup \mathbb{G} \cup \mathbb{L} \cup \mathbb{E} \cup \mathbb{D}$$

where

1. $\mathcal{P} = \langle P, < \rangle$ is a linear ordering;
2. $\mathbb{S} = \langle P, I, \mathbf{in}^{\mathbf{G}} \rangle$ is a closed semilinear graphical geometry;
3. $\mathbb{L} = \langle P, I, \mathbf{B}, \mathbf{in}^{\mathbf{L}} \rangle$ is a semilinear ordered geometry;
4. $\mathbb{E} = \langle P, I, E, \mathbf{in}^{\mathbf{E}} \rangle$ is a weak planar geometry;
5. $\mathbb{D} = \langle P, I, E, \mathbf{in}^{\mathbf{D}} \rangle$ is a diamond geometry.

Conclusions

- Using the COLORE ontology repository, we have proven representation theorems for the modules of OWL-Time.
- We have identified missing axioms that are needed to capture the intended semantics.
- We have characterized the metatheoretic relationships between OWL-Time and other time ontologies.
- Next Steps
    - Representation theorems for the duration axioms.
    - How are eventualities related to existing process ontologies?