ENSSAT
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Learning Commonalities in RDF

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Introduction

Least general generalization (lgg)

- Machine Learning in the early 70's by Gordon Plotkin
- Knowledge representation domain in the early 90's
- Recently in semantic web

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- Linked Data Cloud : links between datasets

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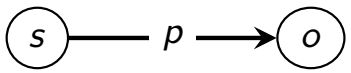
Goal

To study the problem in the setting of the *entire* RDF standard

Outline

RDF graphs

- Specification of RDF graphs with triples :
  $(\mathrm{s}, \mathrm{p}, \mathrm{o}) \in (\mathcal{U} \cup \mathcal{B}) \times \mathcal{U} \times (\mathcal{U} \cup \mathcal{L} \cup \mathcal{B})$



- Built-in property URIs to state RDF statements

RDF statement	Triple
Class assertion	$(\mathrm{s}, \mathrm{rdf{:}type}, \mathrm{o})$
Property assertion	$(\mathrm{s}, \mathrm{p}, \mathrm{o})$ with $\mathrm{p} \neq \mathrm{rdf{:}type}$

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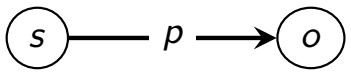

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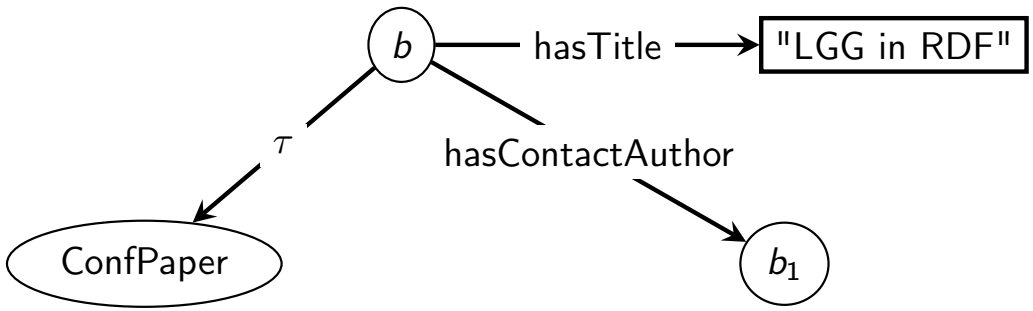

Adding ontological knowledge to RDF graphs

- Built-in property URIs to state RDF Schema statements, i.e., ontological constraints.

RDFS statement	Triple
Subclass	$(\mathsf{s}, \preceq_{\mathrm{sc}}, \mathsf{o})$
Subproperty	$(\mathsf{s}, \preceq_{\mathrm{sp}}, \mathsf{o})$
Domain typing	$(\mathsf{s}, \hookleftarrow_{d}, \mathsf{o})$
Range typing	$(\mathsf{s}, \hookrightarrow_{r}, \mathsf{o})$

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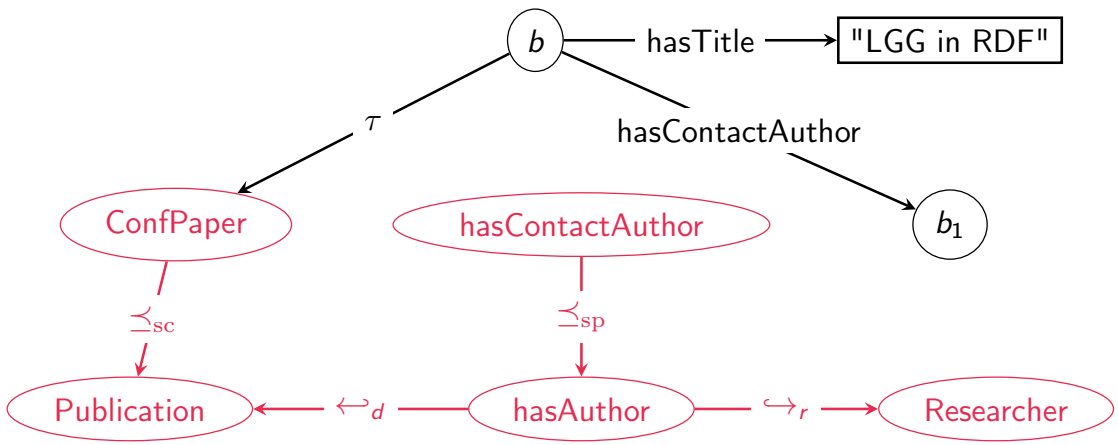


Deriving the implicit triples

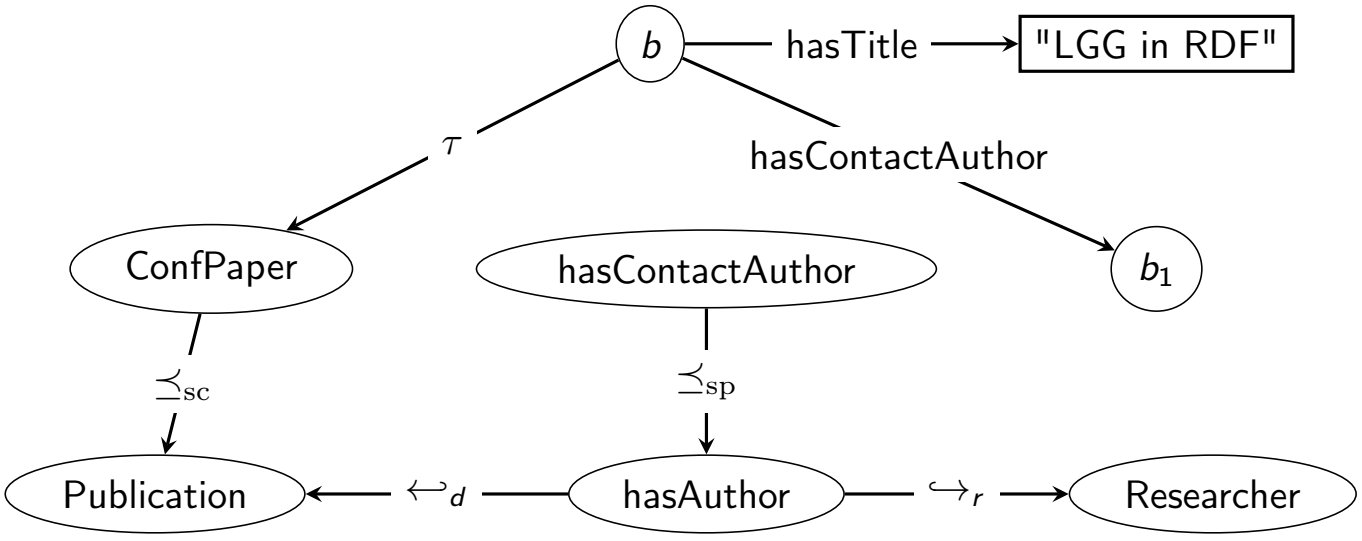
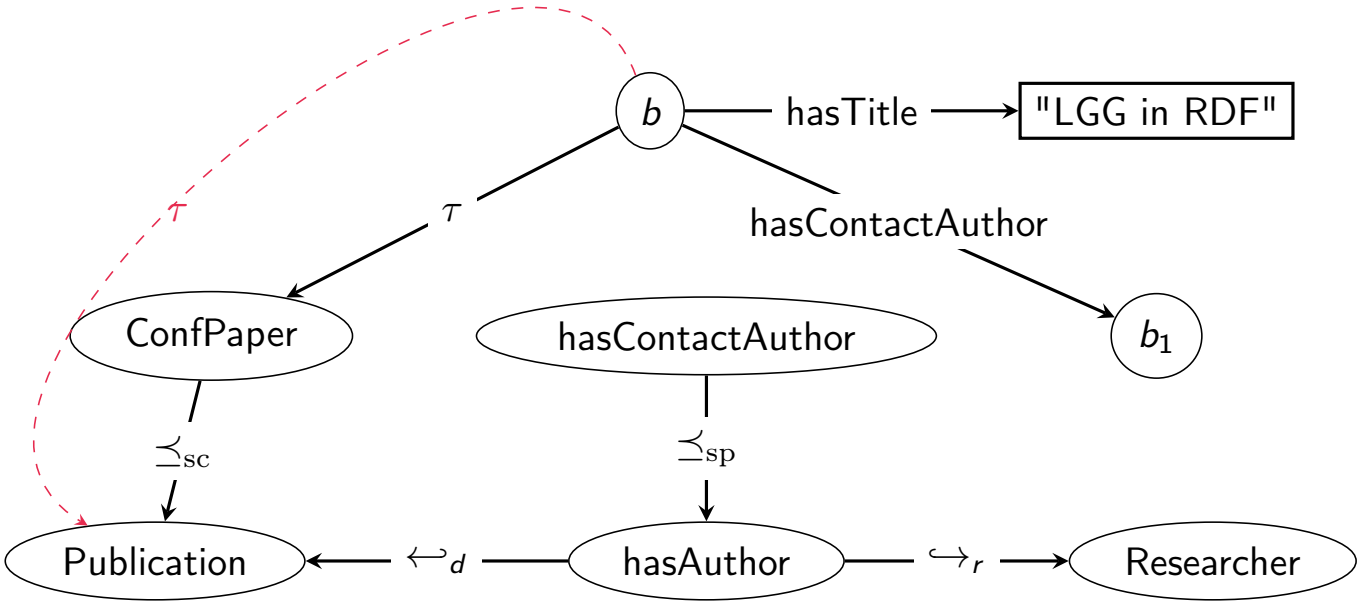


Figure: RDF graph $\mathcal{G}$

Deriving the implicit triples



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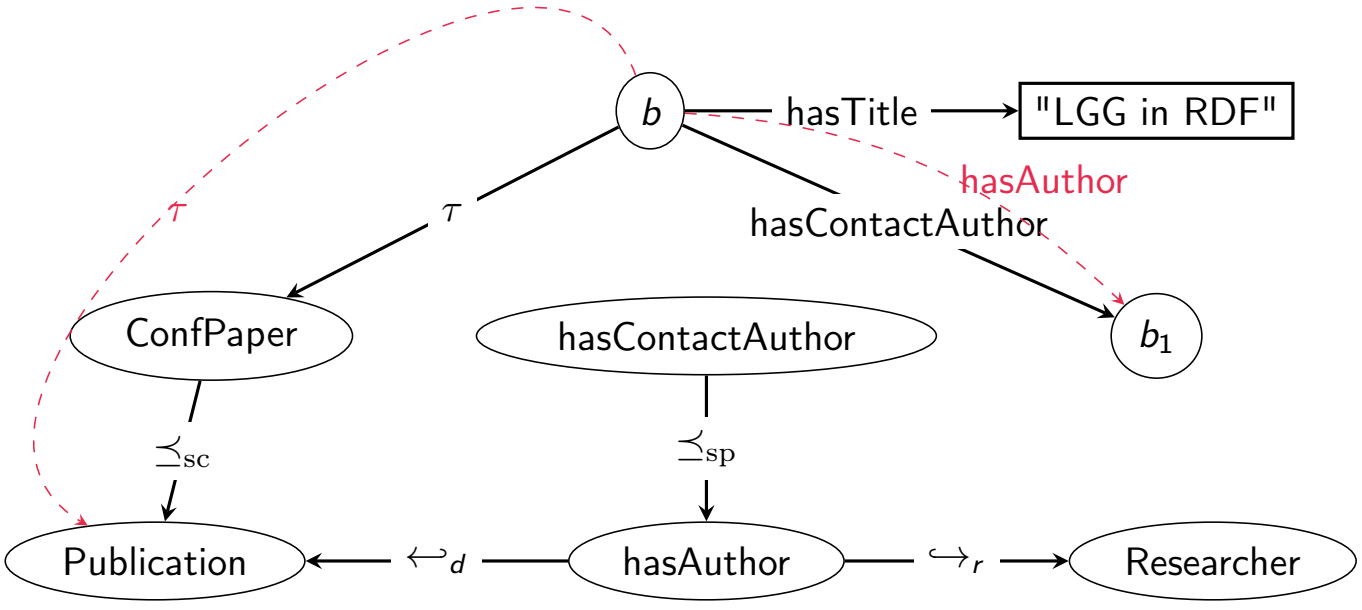

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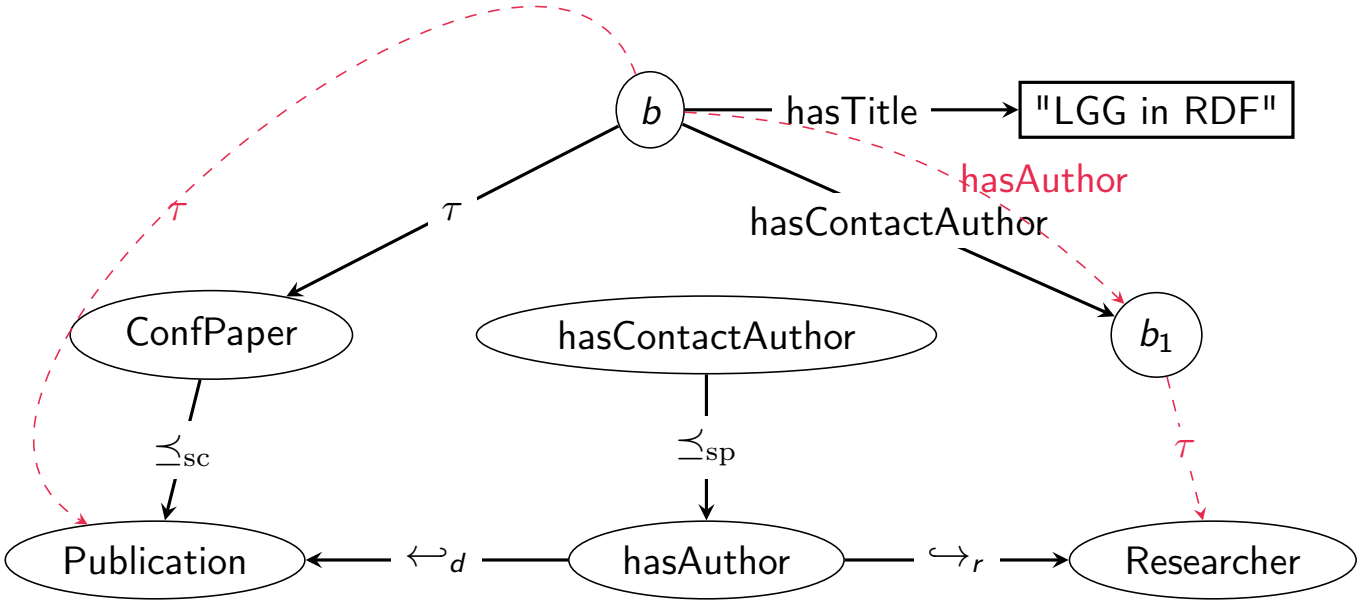

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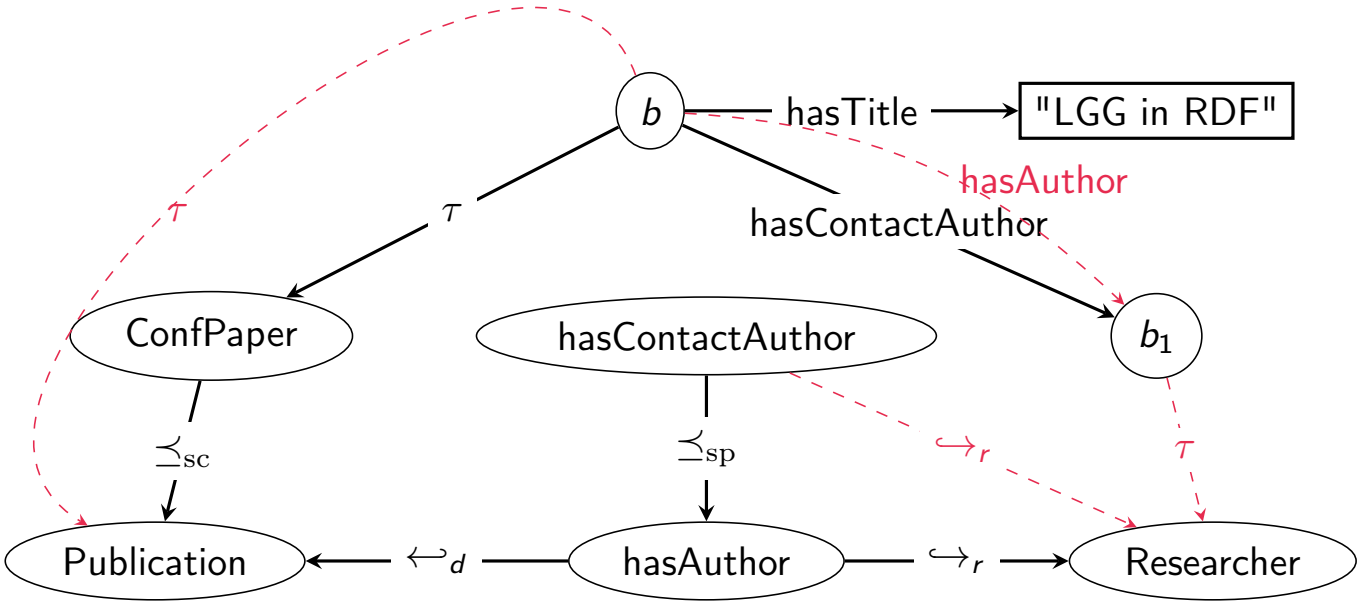

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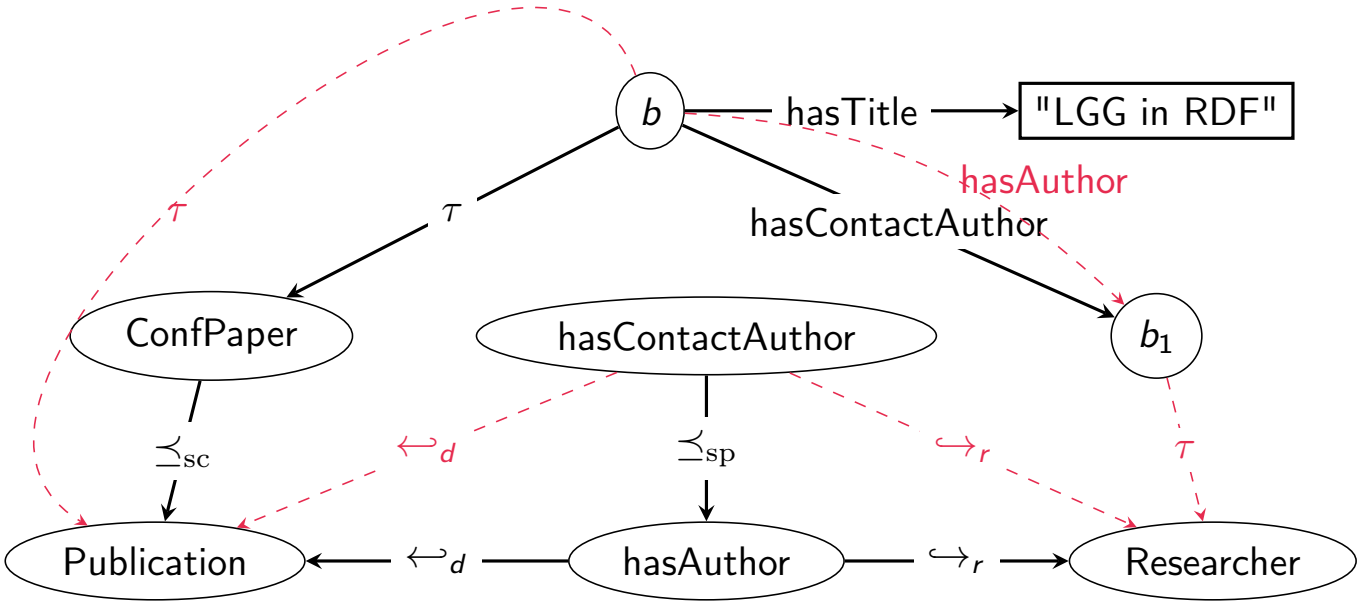

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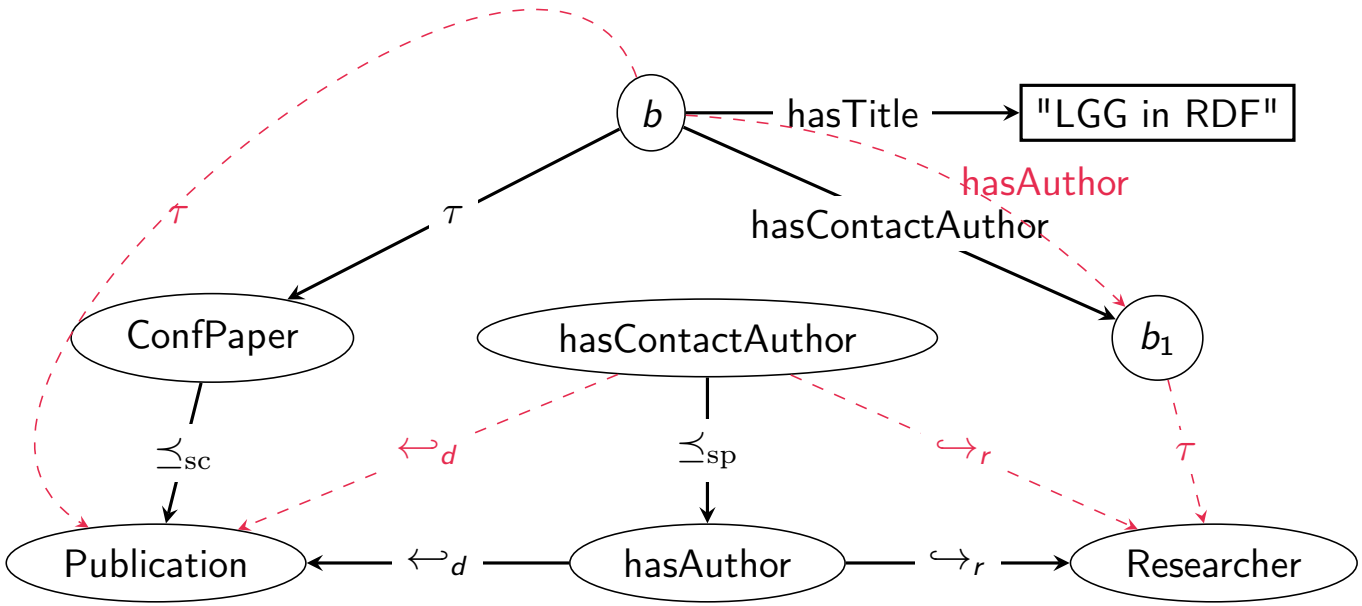

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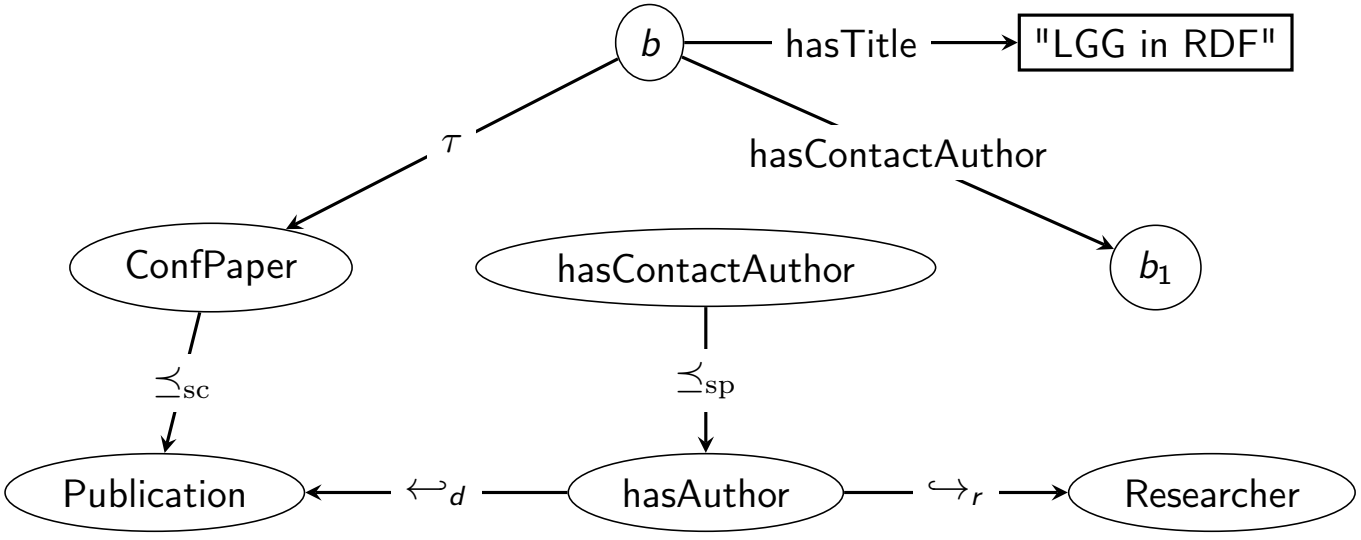
How to derive implicit triples of an RDF graph ?

Sample set of entailment rules

Rule [7]	Entailment rule
rdfs2	$(\mathtt{p}, \hookleftarrow_d, \mathtt{o}), (\mathtt{s}_1, \mathtt{p}, \mathtt{o}_1) \rightarrow (\mathtt{s}_1, \tau, \mathtt{o})$
rdfs3	$(\mathtt{p}, \hookrightarrow_r, \mathtt{o}), (\mathtt{s}_1, \mathtt{p}, \mathtt{o}_1) \rightarrow (\mathtt{o}_1, \tau, \mathtt{o})$
rdfs5	$(\mathtt{p}_1, \preceq_{\mathrm{sp}}, \mathtt{p}_2), (\mathtt{p}_2, \preceq_{\mathrm{sp}}, \mathtt{p}_3) \rightarrow (\mathtt{p}_1, \preceq_{\mathrm{sp}}, \mathtt{p}_3)$
rdfs7	$(\mathtt{p}_1, \preceq_{\mathrm{sp}}, \mathtt{p}_2), (\mathtt{s}, \mathtt{p}_1, \mathtt{o}) \rightarrow (\mathtt{s}, \mathtt{p}_2, \mathtt{o})$
rdfs9	$(\mathtt{s}, \preceq_{\mathrm{sc}}, \mathtt{o}), (\mathtt{s}_1, \tau, \mathtt{s}) \rightarrow (\mathtt{s}_1, \tau, \mathtt{o})$
rdfs11	$(\mathtt{s}, \preceq_{\mathrm{sc}}, \mathtt{o}), (\mathtt{o}, \preceq_{\mathrm{sc}}, \mathtt{o}_1) \rightarrow (\mathtt{s}, \preceq_{\mathrm{sc}}, \mathtt{o}_1)$
ext1	$(\mathtt{p}, \hookleftarrow_d, \mathtt{o}), (\mathtt{o}, \preceq_{\mathrm{sc}}, \mathtt{o}_1) \rightarrow (\mathtt{p}, \hookleftarrow_d, \mathtt{o}_1)$
ext2	$(\mathtt{p}, \hookrightarrow_r, \mathtt{o}), (\mathtt{o}, \preceq_{\mathrm{sc}}, \mathtt{o}_1) \rightarrow (\mathtt{p}, \hookrightarrow_r, \mathtt{o}_1)$
ext3	$(\mathtt{p}, \preceq_{\mathrm{sp}}, \mathtt{p}_1), (\mathtt{p}_1, \hookleftarrow_d, \mathtt{o}) \rightarrow (\mathtt{p}, \hookleftarrow_d, \mathtt{o})$
ext4	$(\mathtt{p}, \preceq_{\mathrm{sp}}, \mathtt{p}_1), (\mathtt{p}_1, \hookrightarrow_r, \mathtt{o}) \rightarrow (\mathtt{p}, \hookrightarrow_r, \mathtt{o})$

Semantics of RDF graphs



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Semantics of RDF graphs

$$rdfs9 : (\mathtt{s}, \preceq_{\mathrm{sc}}, \mathtt{o}), (\mathtt{s}_1, \tau, \mathtt{s}) \rightarrow (\mathtt{s}_1, \tau, \mathtt{o})$$

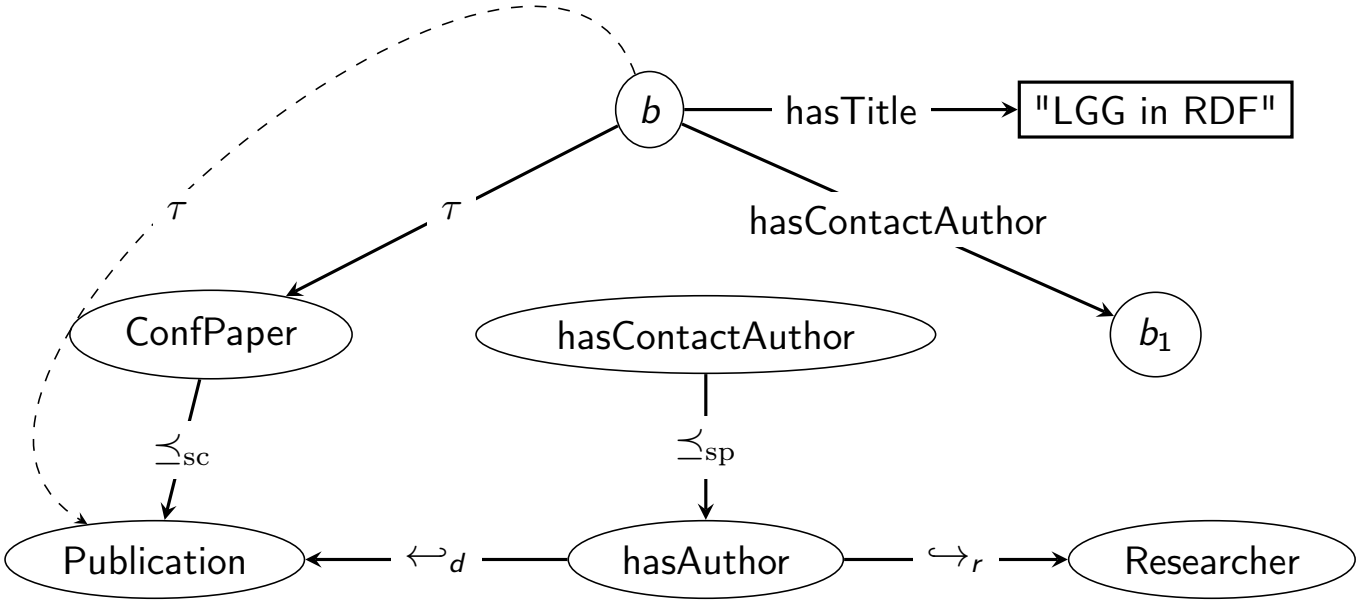

Figure: RDF graph $\mathcal{G}$

Semantics of RDF graphs

$$rdfs7 : (\mathrm{p}_1, \preceq_{\mathrm{sp}}, \mathrm{p}_2), (\mathrm{s}, \mathrm{p}_1, \mathrm{o}) \rightarrow (\mathrm{s}, \mathrm{p}_2, \mathrm{o})$$

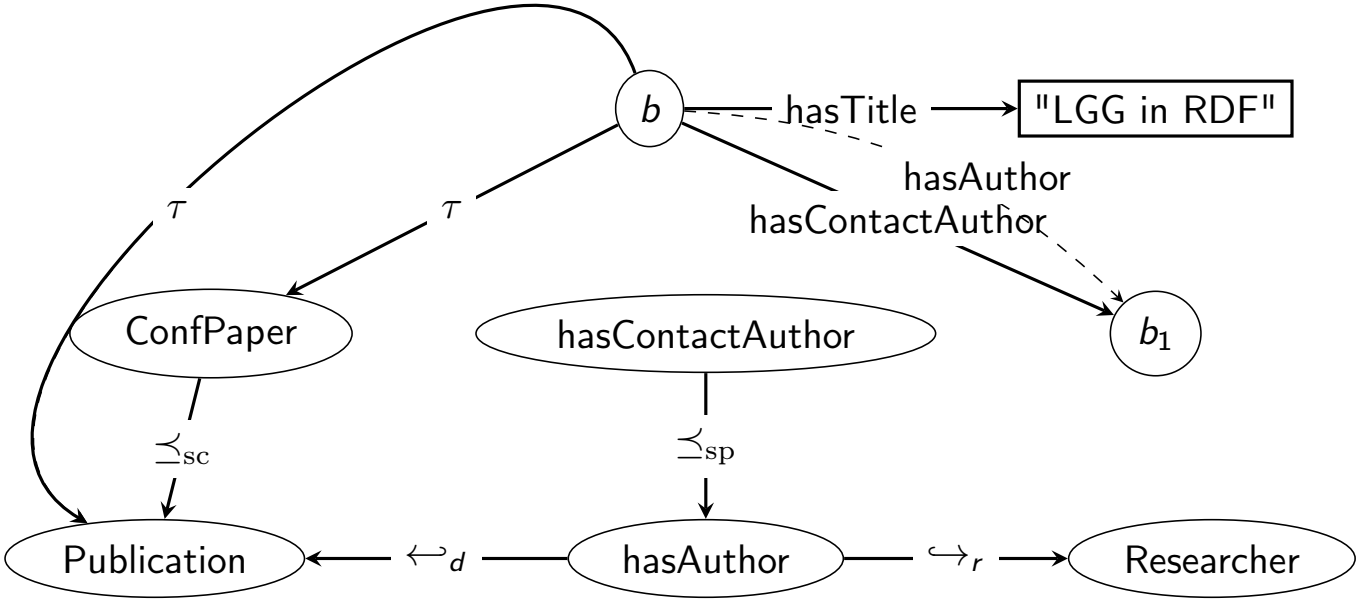

Figure: RDF graph $\mathcal{G}$

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$$rdfs3 : (\mathrm{p}, \hookrightarrow_r, \mathrm{o}), (\mathrm{s}_1, \mathrm{p}, \mathrm{o}_1) \rightarrow (\mathrm{o}_1, \tau, \mathrm{o})$$

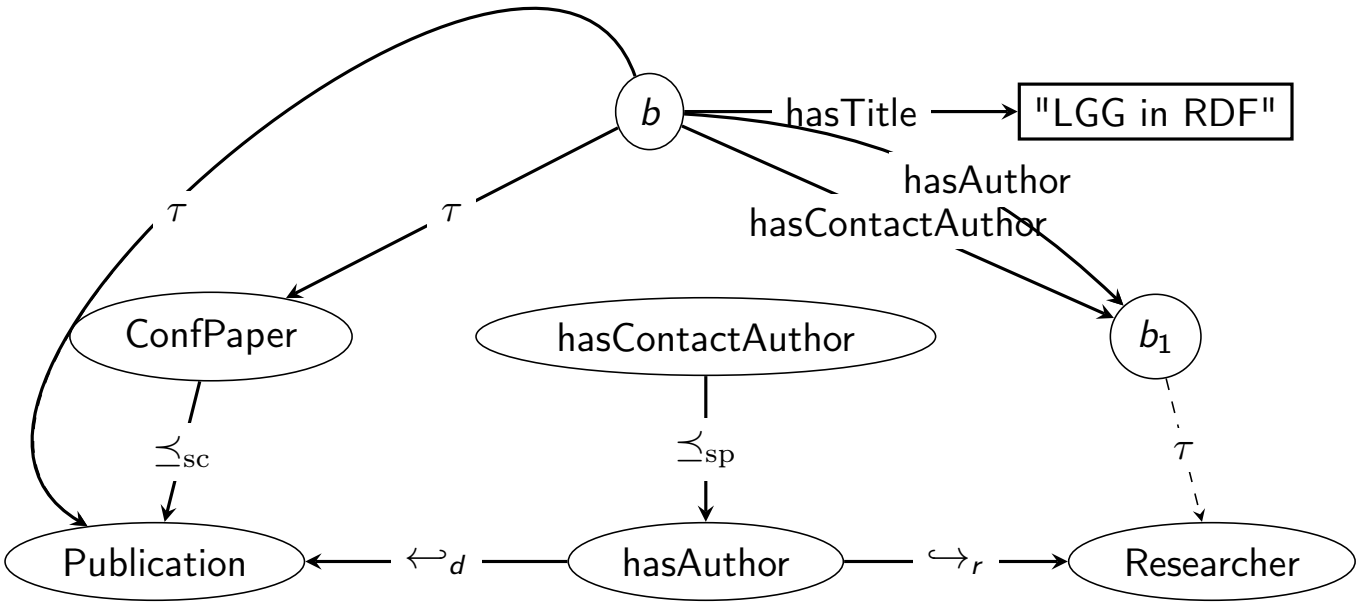


Figure: RDF graph $\mathcal{G}$

Semantics of RDF graphs

$$ext4 : (p, \preceq_{sp}, p_1), (p_1, \hookrightarrow_r, o) \rightarrow (p, \hookrightarrow_r, o)$$

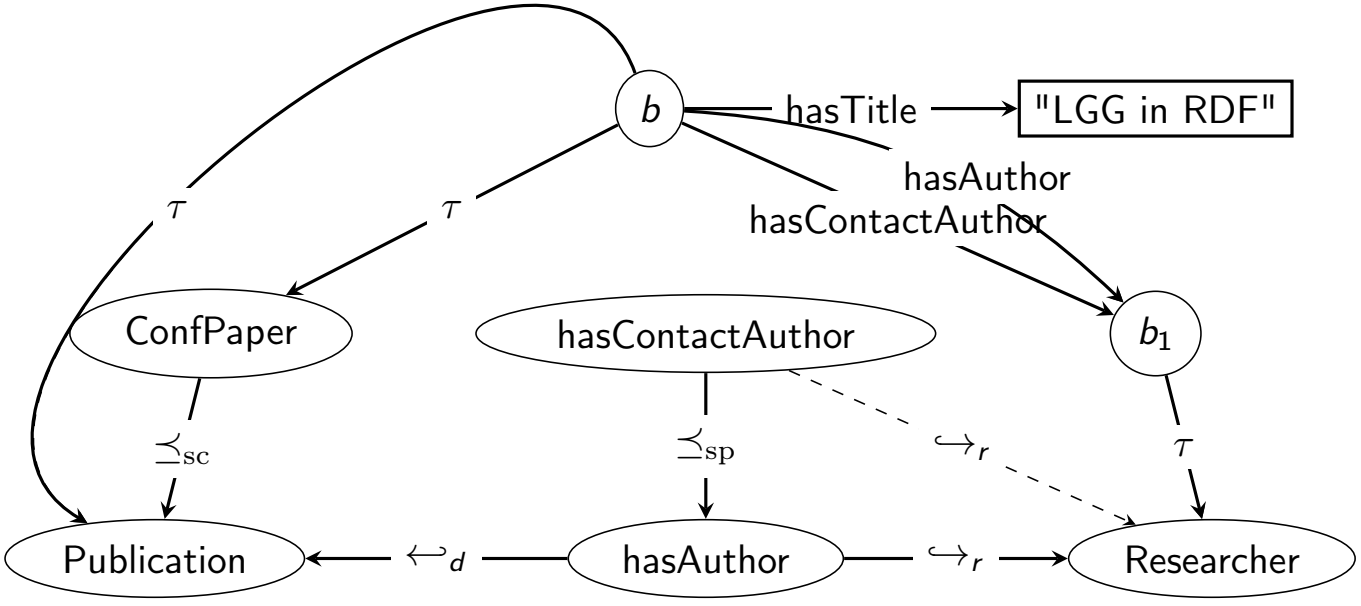
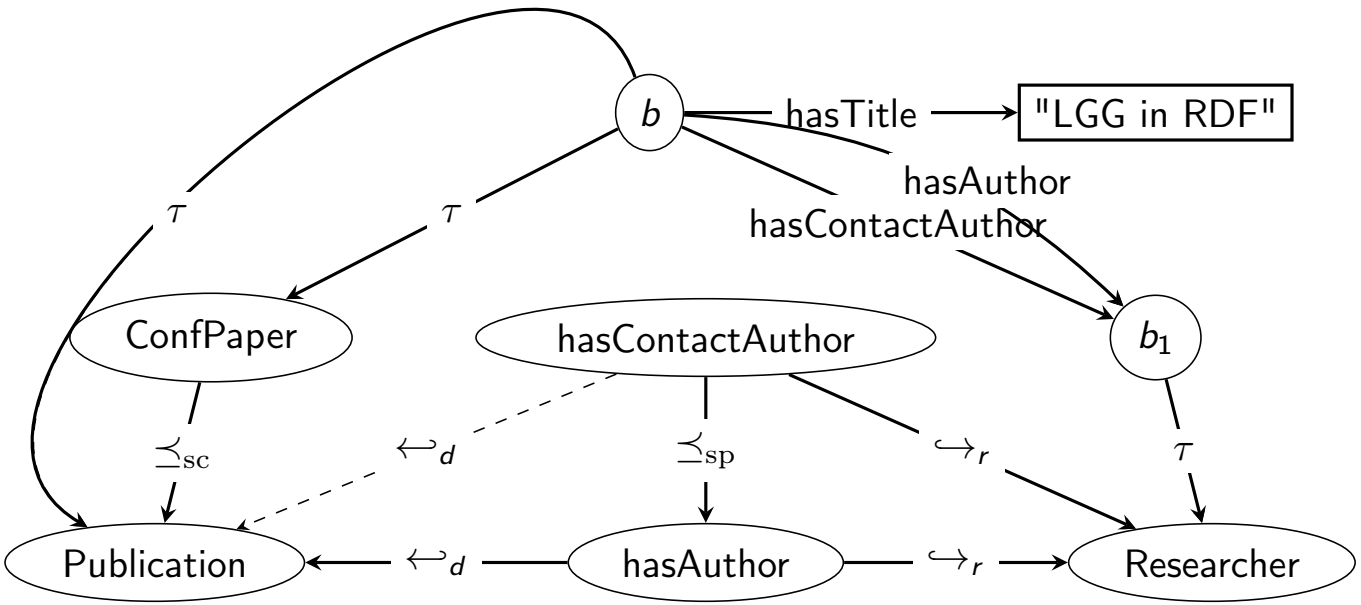


Figure: RDF graph $\mathcal{G}$

Semantics of RDF graphs

$$ext3 : (p, \preceq_{sp}, p_1), (p_1, \hookleftarrow_d, o) \rightarrow (p, \hookleftarrow_d, o)$$



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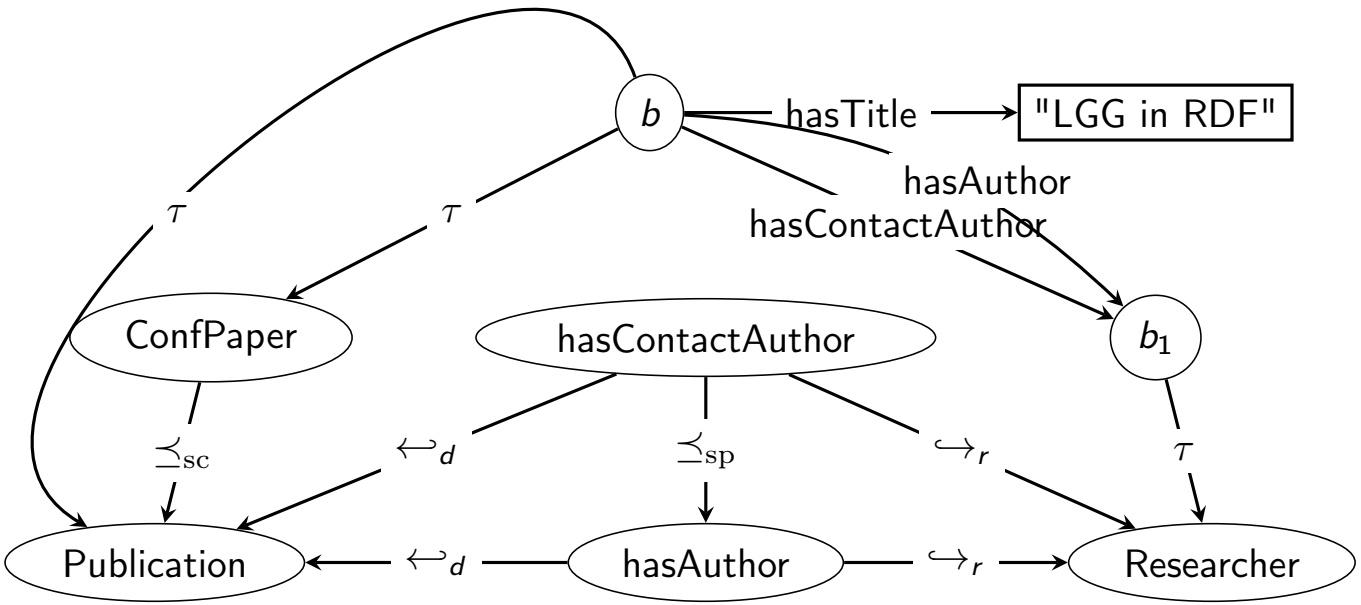


Figure: Saturated RDF graph $\mathcal{G}^{\infty}$

Entailment between RDF graphs

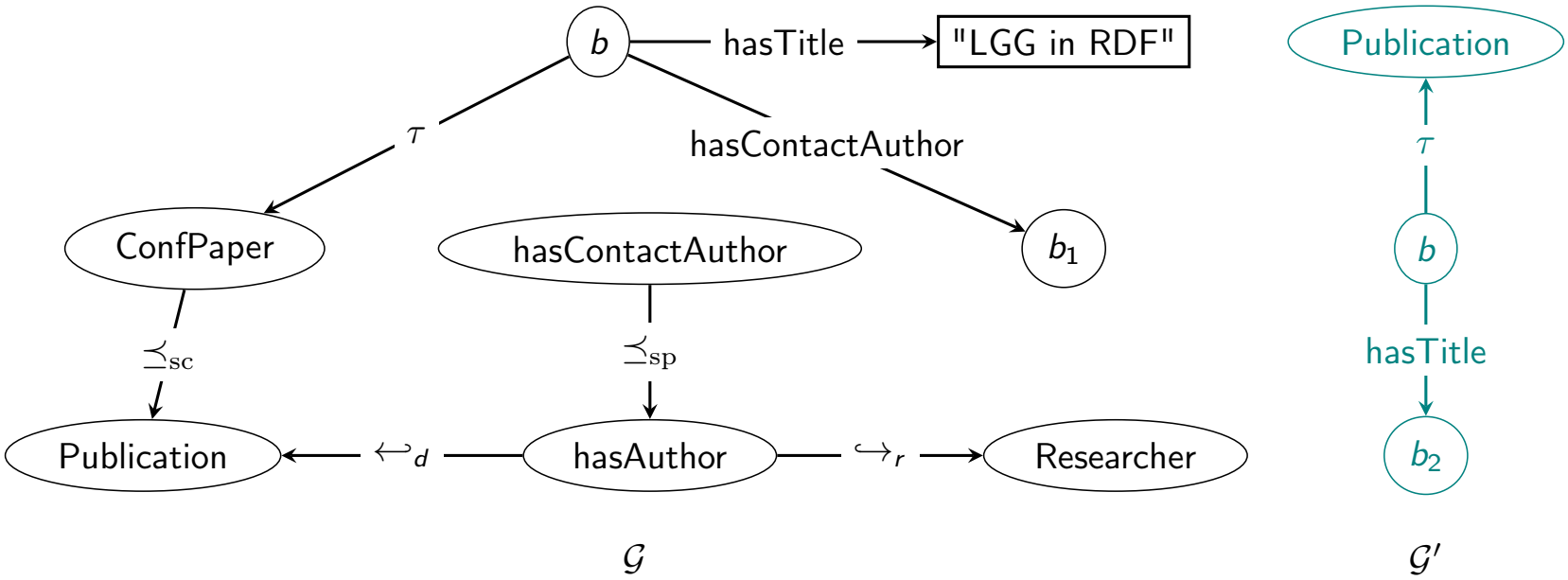
Let $\mathcal{G}$ and $\mathcal{G}'$ be two graphs RDF and $\mathcal{R}$ a set of RDF entailment rules. There exists relationship to compare $\mathcal{G}$ and $\mathcal{G}'$ called *entailment between graphs*
$\mathcal{G}$ is more specific than $\mathcal{G}'$:

- $\mathcal{G} \models_{\mathcal{R}} \mathcal{G}' \iff \mathcal{G}^{\infty} \models \mathcal{G}'$

There must exist an embedding of $\mathcal{G}'$ in $\mathcal{G}^{\infty}$.

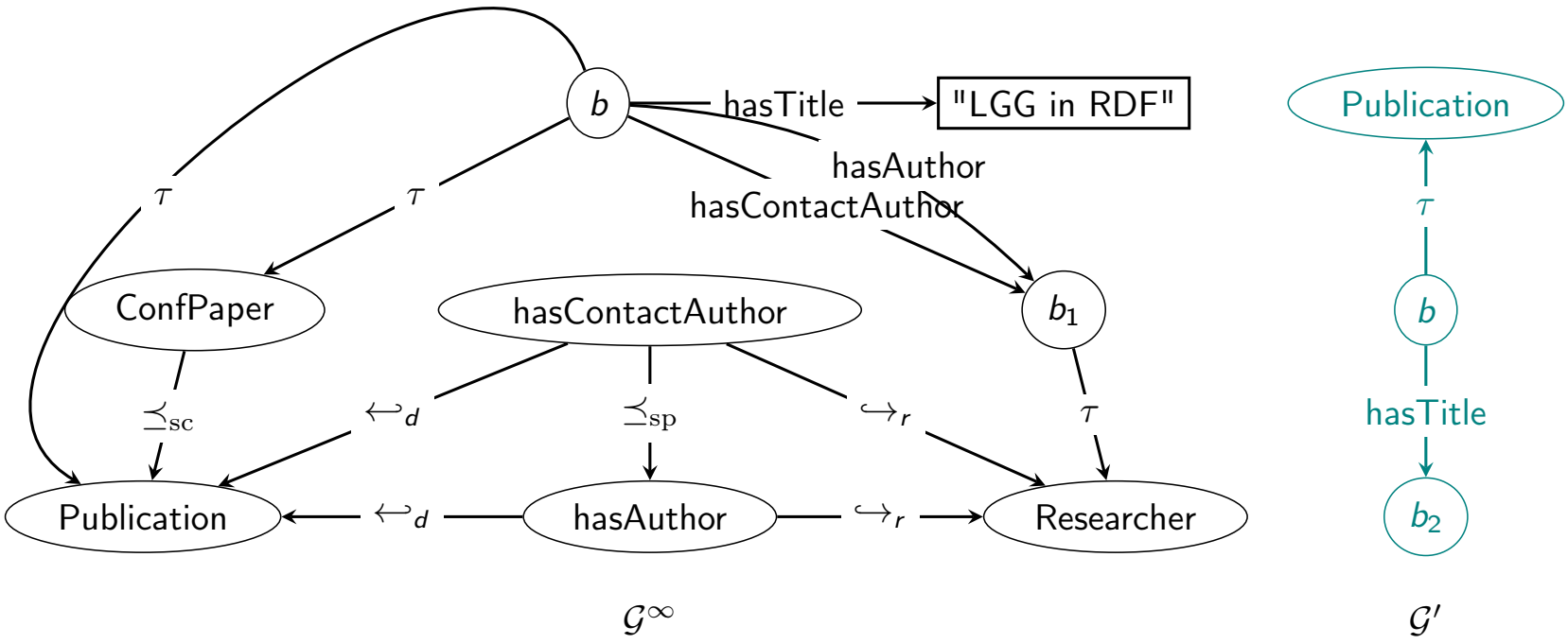
Entailment between RDF graphs

$$\mathcal{G} \overset{?}{\models}_{\mathcal{R}} \mathcal{G}'$$



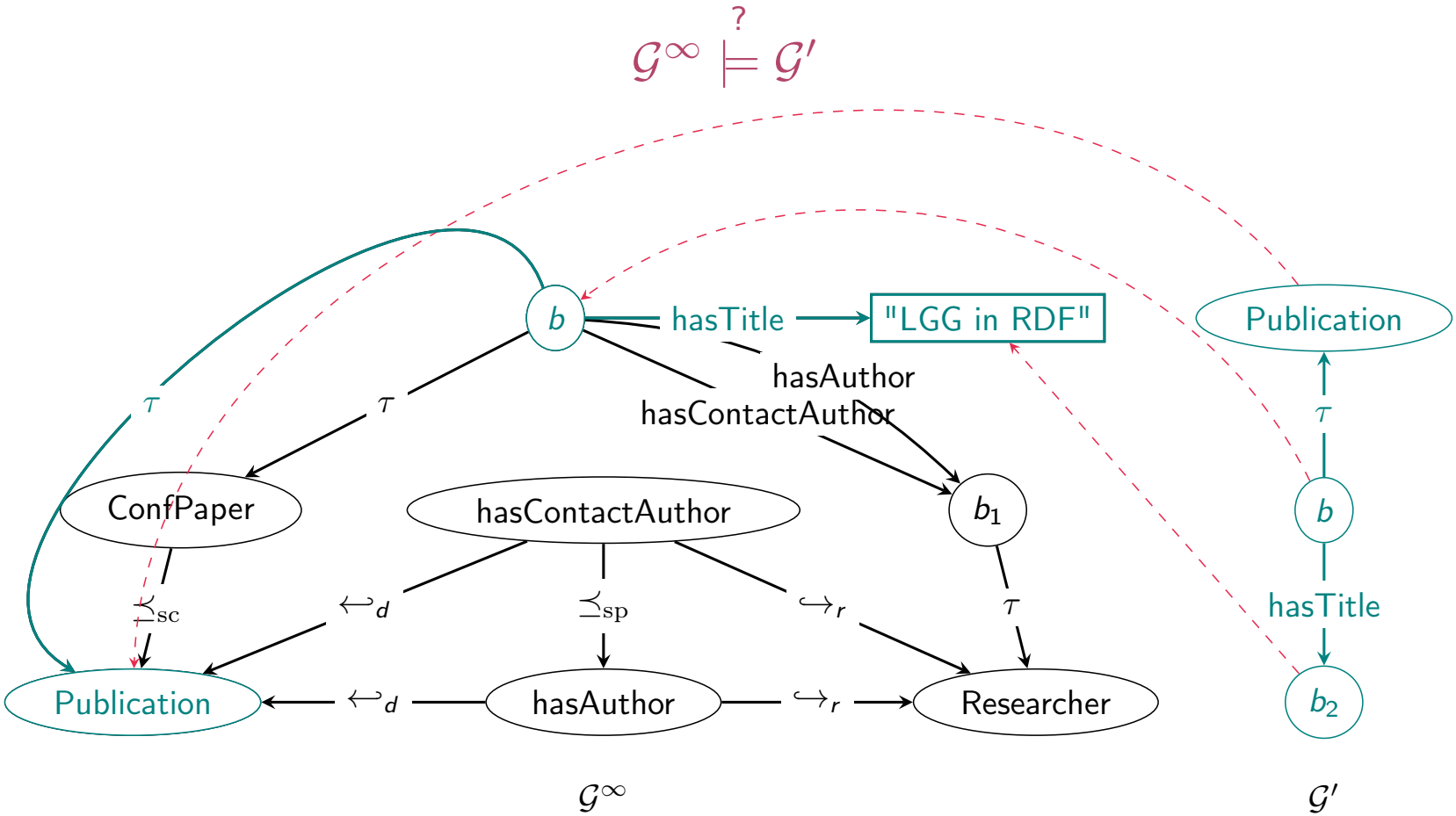
Entailment between RDF graphs

$$\mathcal{G}^{\infty} \overset{?}{\models} \mathcal{G}'$$



$\mathcal{G}^{\infty}$ $\mathcal{G}'$

Entailment between RDF graphs



RDF graph $\mathcal{G}$ is more specific than RDF graph $\mathcal{G}'$

Outline

Towards defining lgg in RDF

A *least general generalization* (lgg) of n descriptions $d_1, \ldots, d_n$ is a most specific description d generalizing every $d_{1 \leq i \leq n}$ for some generalization/specialization relation between descriptions (G.Plotkin).

lgg in RDF

- descriptions are RDF graphs
- relation generalization/specialization is entailment between RDF graphs

Defining the lgg of RDF graphs

Definition (lgg of RDF graphs)

Let $\mathcal{G}_1, \ldots, \mathcal{G}_n$ be RDF graphs and $\mathcal{R}$ a set of RDF entailment rules.

- A *generalization* of $\mathcal{G}_1, \ldots, \mathcal{G}_n$ is an RDF graph $\mathcal{G}_g$ such that $\mathcal{G}_i \models_{\mathcal{R}} \mathcal{G}_g$ holds for $1 \leq i \leq n$.
- A *least general generalization* (lgg) of $\mathcal{G}_1, \ldots, \mathcal{G}_n$ is a generalization $\mathcal{G}_{\text{lgg}}$ of $\mathcal{G}_1, \ldots, \mathcal{G}_n$ such that for any other generalization $\mathcal{G}_g$ of $\mathcal{G}_1, \ldots, \mathcal{G}_n$, $\mathcal{G}_{\text{lgg}} \models_{\mathcal{R}} \mathcal{G}_g$ holds.

Result : lgg of n RDF graphs vs lgg of two RDF graphs

$$
\begin{array}{ll}
\ell_3(\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3) & \equiv_{\mathcal{R}} \ \ell_2(\ell_2(\mathcal{G}_1, \mathcal{G}_2), \mathcal{G}_3) \\
\ldots & \ldots \\
\ell_n(\mathcal{G}_1, \ldots, \mathcal{G}_n) & \equiv_{\mathcal{R}} \ \ell_2(\ell_{n-1}(\mathcal{G}_1, \ldots, \mathcal{G}_{n-1}), \mathcal{G}_n) \\
& \equiv_{\mathcal{R}} \ \ell_2(\ell_2(\cdots \ell_2(\ell_2(\mathcal{G}_1, \mathcal{G}_2), \mathcal{G}_3) \cdots, \mathcal{G}_{n-1}), \mathcal{G}_n)
\end{array}
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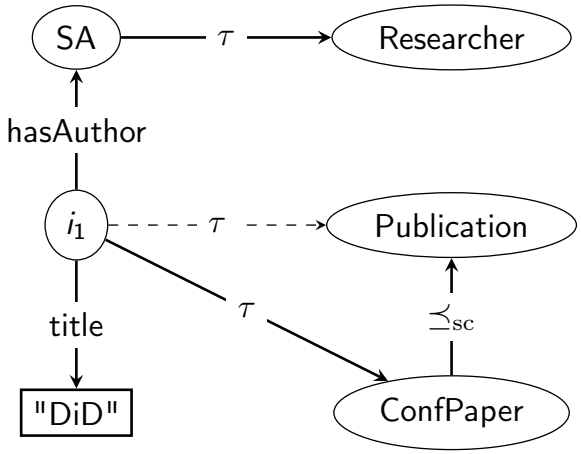
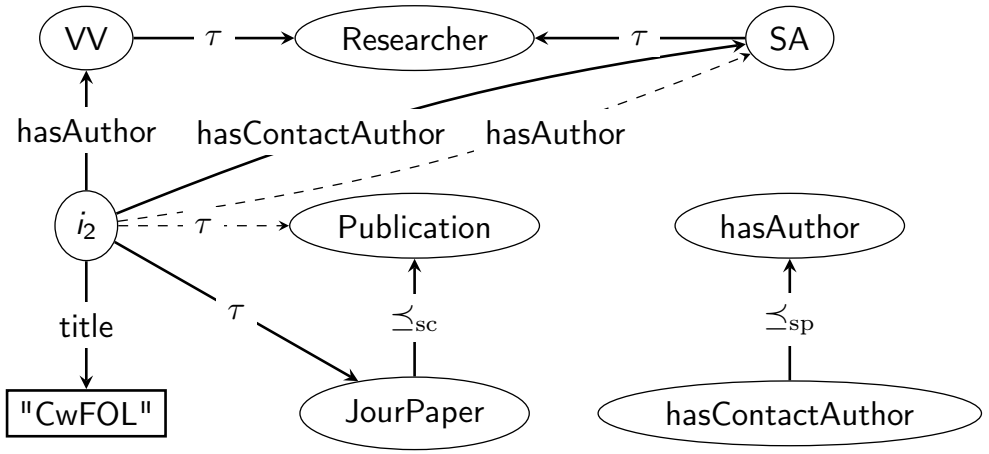
\end{array}
$$

We focus on computing lgg of two RDF graphs

Defining the lgg of RDF graphs

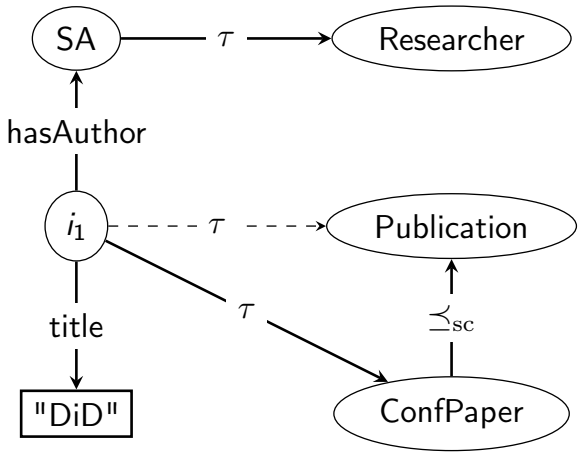


$\mathcal{G}_1$



$\mathcal{G}_2$

Defining the lgg of RDF graphs



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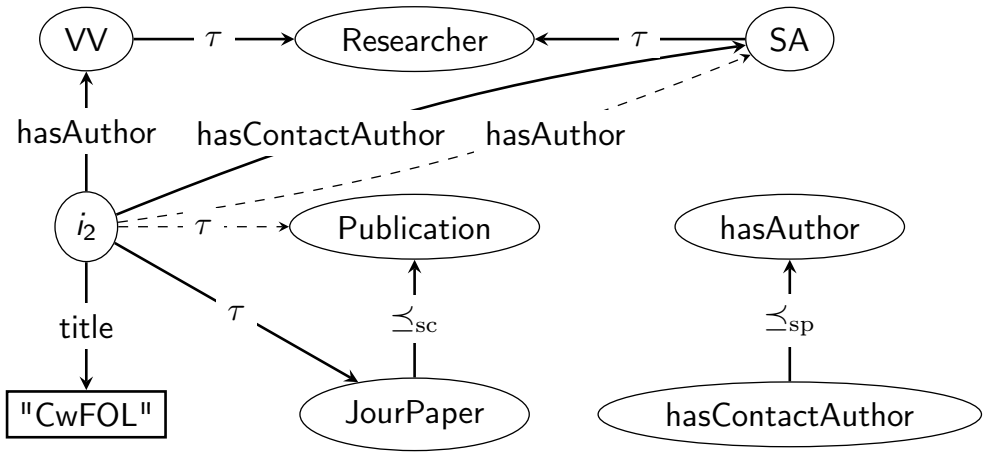
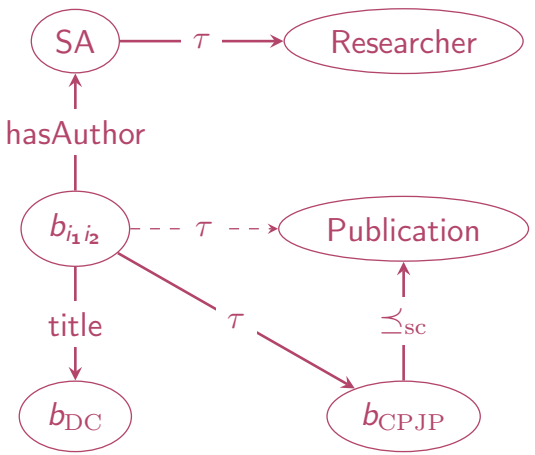
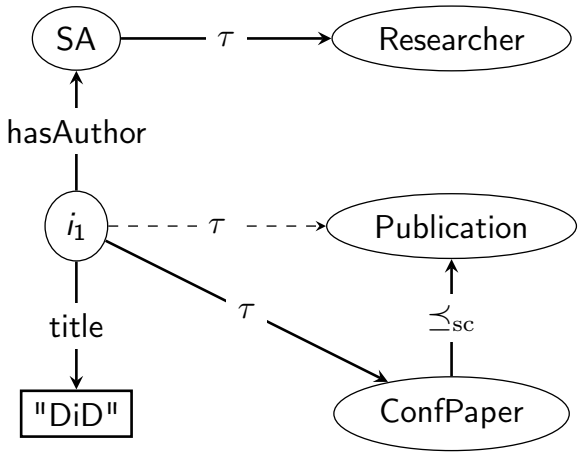


$\mathcal{G}_2$



$\mathcal{G}_{\mathtt{lgg}}$

Defining the lgg of RDF graphs



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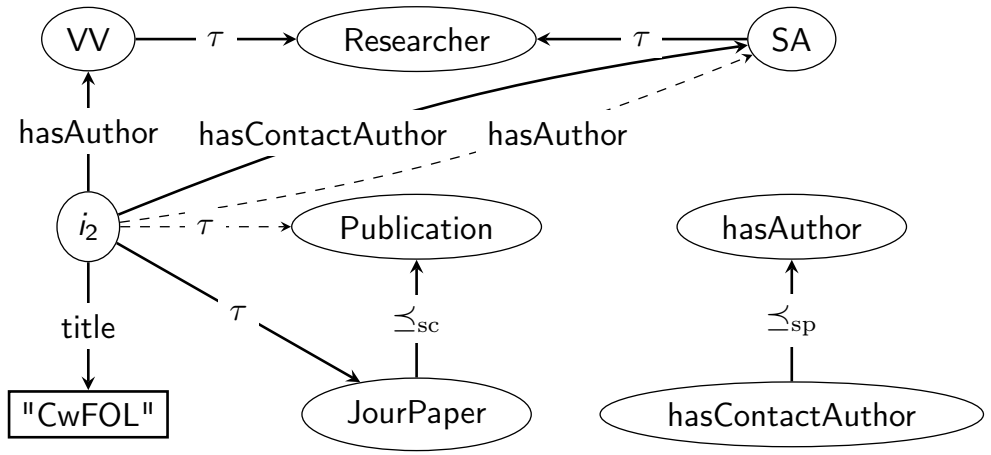
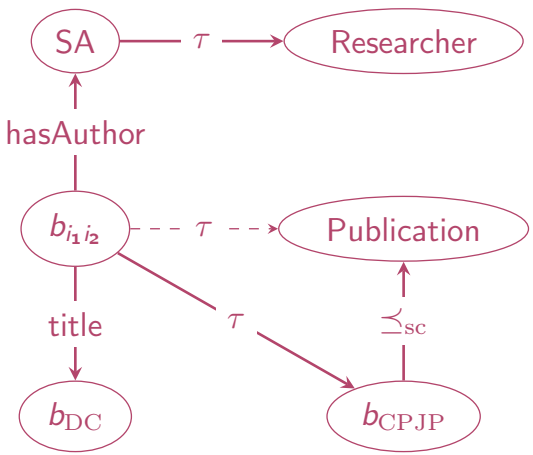


$\mathcal{G}_2$



$\mathcal{G}_{\mathtt{lgg}}$

How to compute this graph ?

The cover graph of RDF graphs

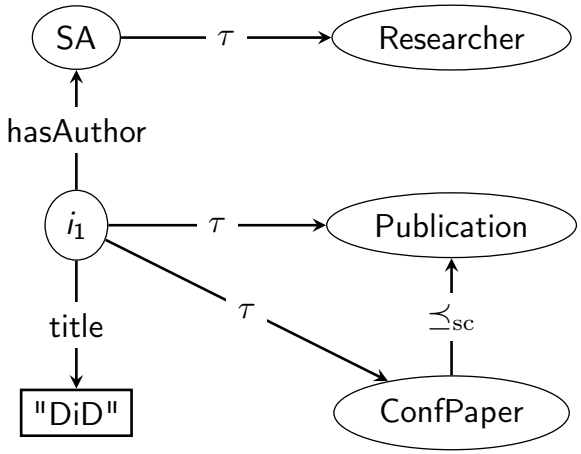
Definition (Cover graph)

The *cover graph* $\mathcal{G}$ of two RDF graph $\mathcal{G}_1$ and $\mathcal{G}_2$ is the RDF graph such that for every property p in both $\mathcal{G}_1$ and $\mathcal{G}_2$:

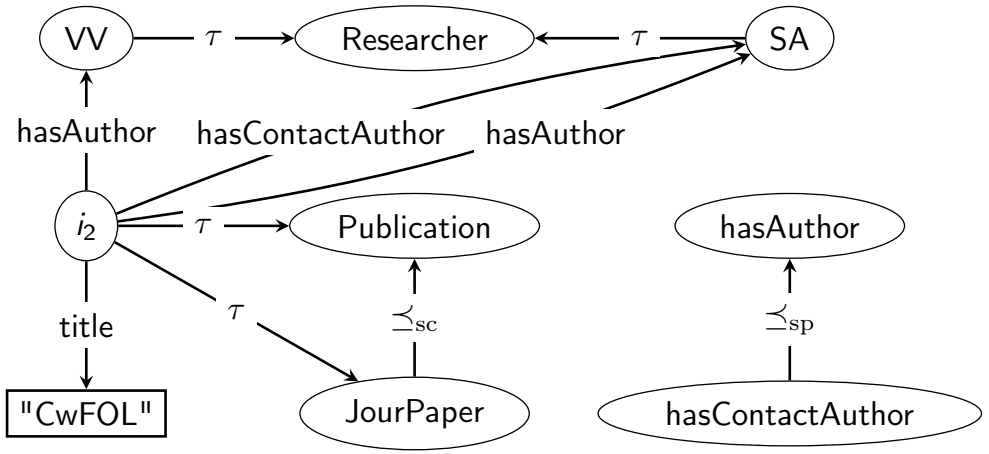
$$(t_1, p, t_2) \in \mathcal{G}_1 \text{ and } (t_3, p, t_4) \in \mathcal{G}_2 \text{ iff } (t_5, p, t_6) \in \mathcal{G}$$

with $t_5 = t_1$ if $t_1 = t_3$ and $t_1 \in \mathcal{U} \cup \mathcal{L}$, else t_5 is the blank node $b_{t_1 t_3}$, and, similarly $t_6 = t_2$ if $t_2 = t_4$ and $t_2 \in \mathcal{U} \cup \mathcal{L}$, else t_6 is the blank node $b_{t_2 t_4}$.

The cover graph of RDF graphs



$\mathcal{G}_1$



$\mathcal{G}_2$

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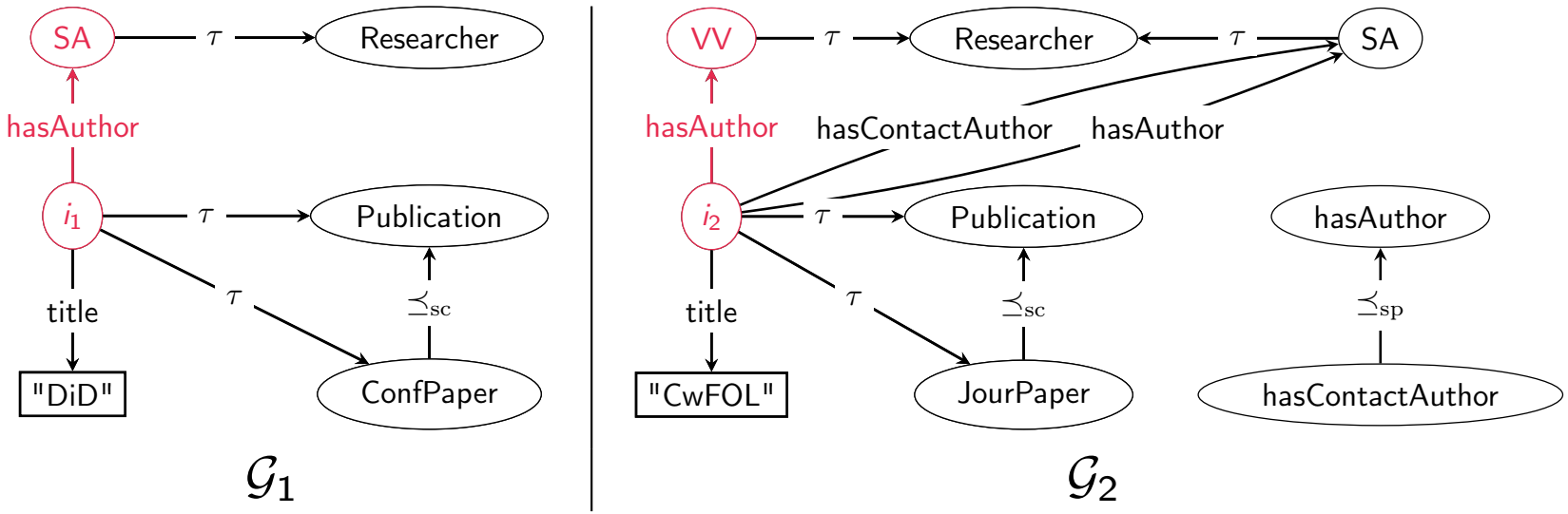
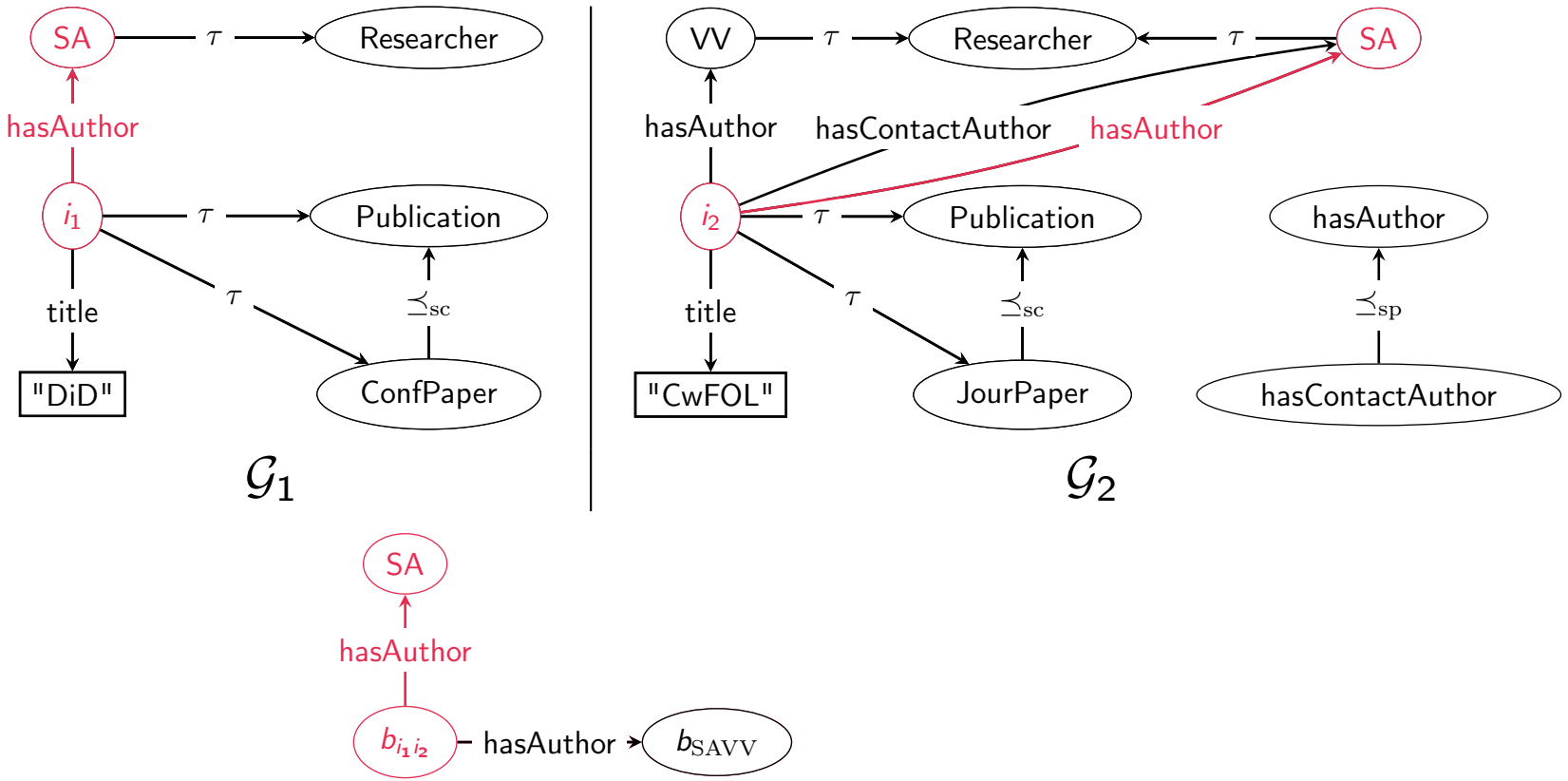


The cover graph of RDF graphs



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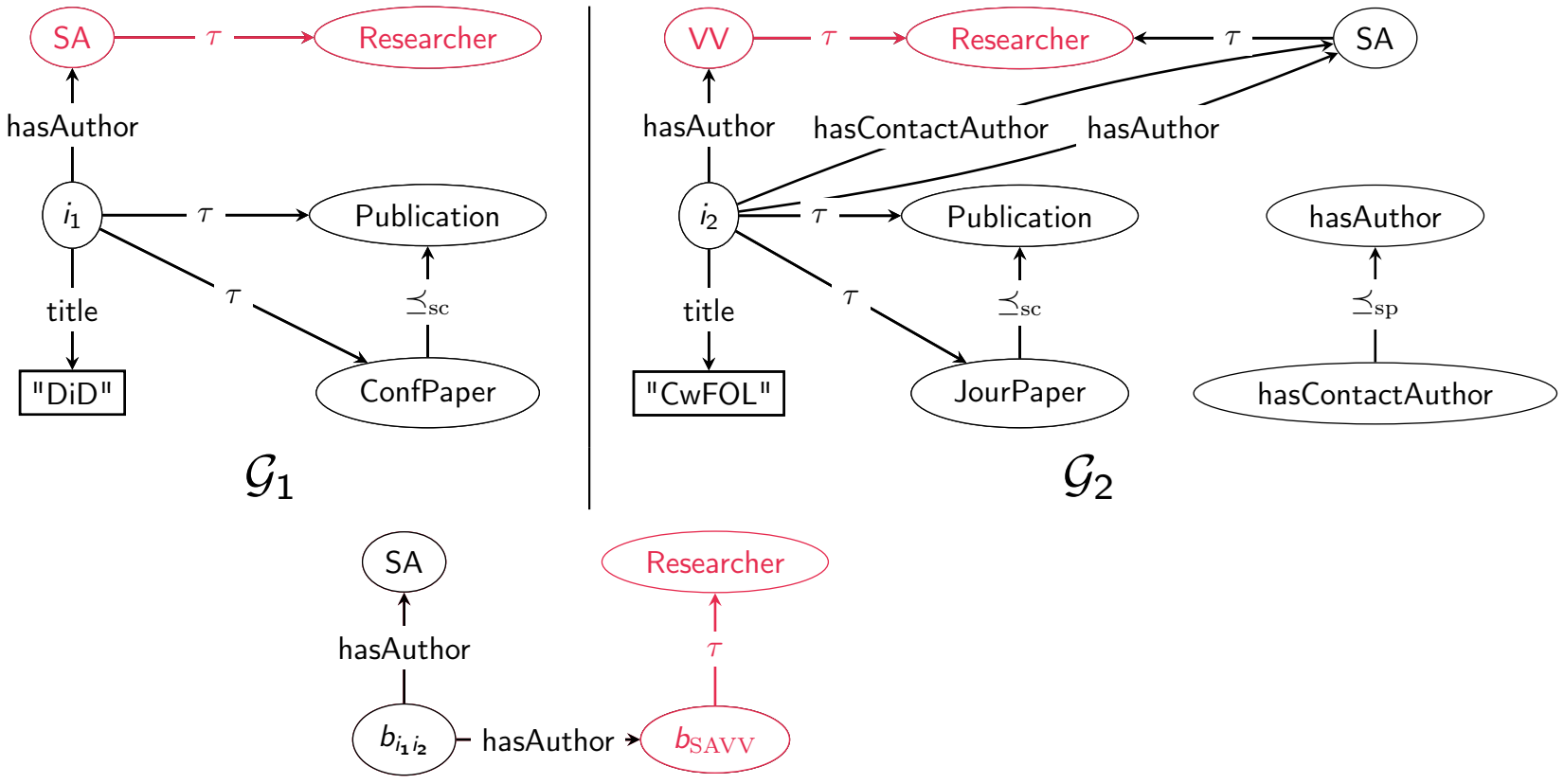
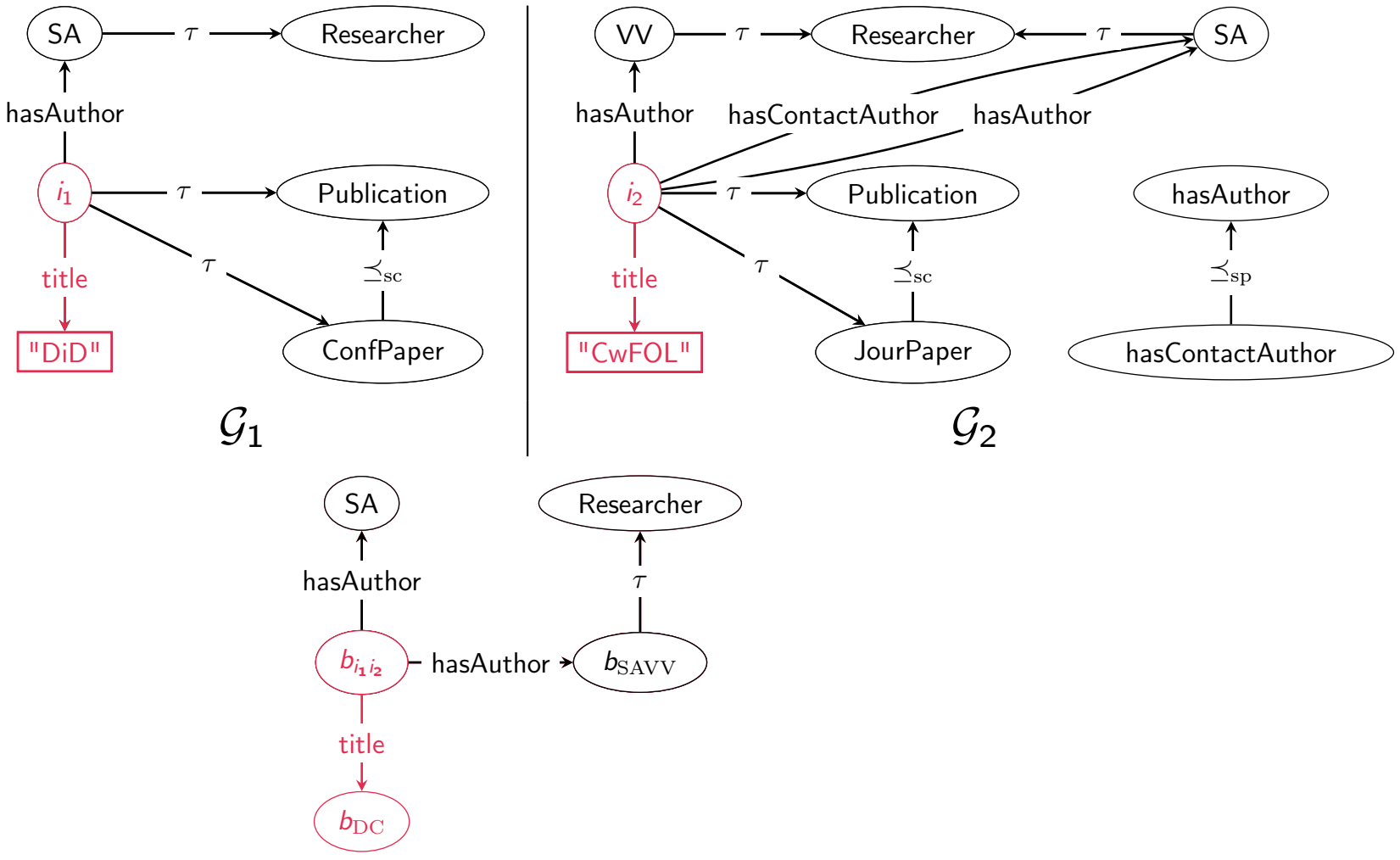
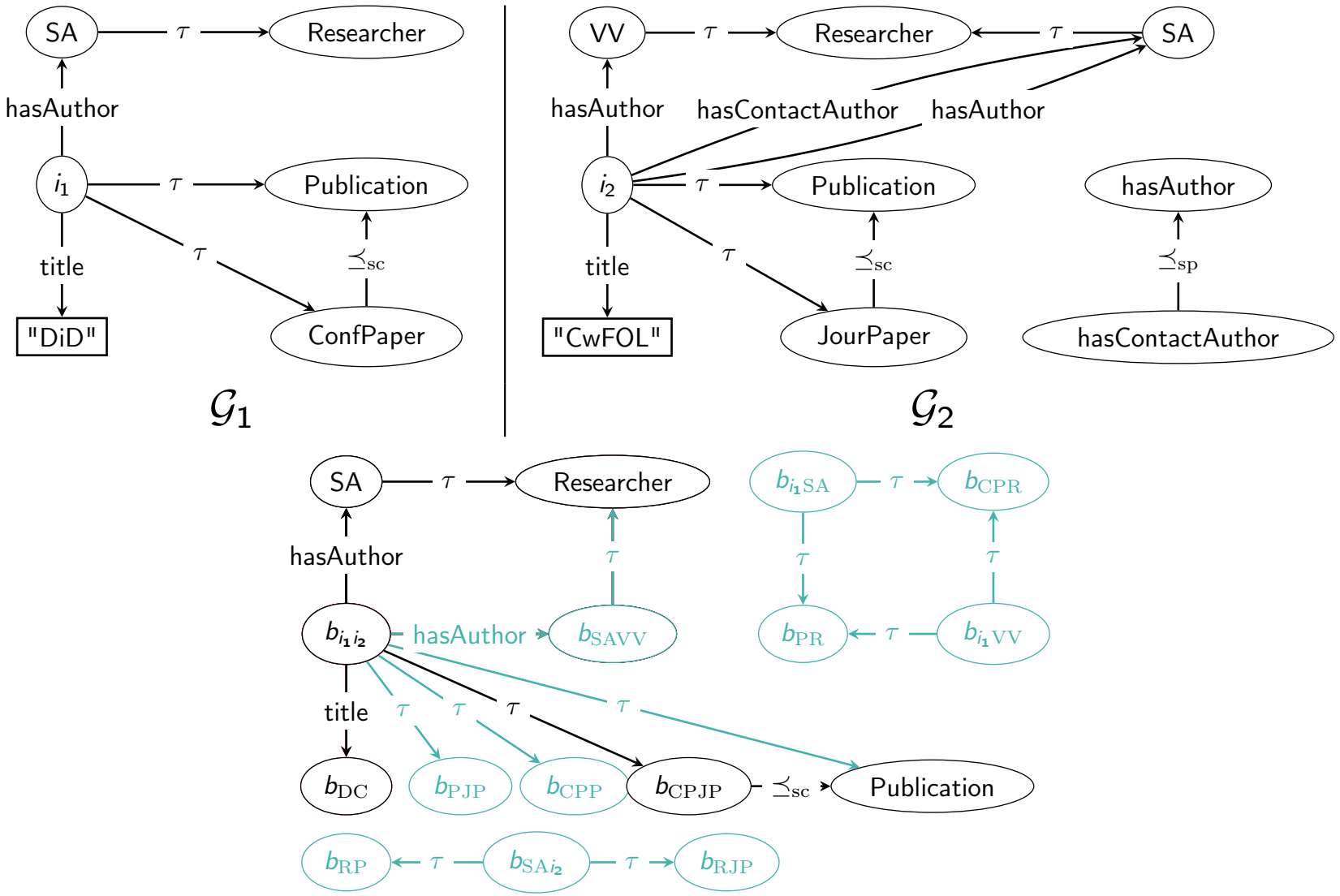


The cover graph of RDF graphs



The cover graph of RDF graphs



Cover graph vs 1gg

Theorem ($\mathcal{R} = \emptyset$)

The *cover graph* $\mathcal{G}$ of the RDF graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ is an 1gg of them for the empty set $\mathcal{R}$ of RDF entailment rules (i.e., $\mathcal{R} = \emptyset$).

Cover graph vs lgg

Theorem ($\mathcal{R} = \emptyset$)

The *cover graph* $\mathcal{G}$ of the RDF graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ is an lgg of them for the empty set $\mathcal{R}$ of RDF entailment rules (i.e., $\mathcal{R} = \emptyset$).

Proposition ($\mathcal{R} = \emptyset$)

The cover graph of two RDF graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ can be computed in $O(|\mathcal{G}_1| \times |\mathcal{G}_2|)$; its size is bounded by $|\mathcal{G}_1| \times |\mathcal{G}_2|$.

Cover graph vs lgg

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Theorem ($\mathcal{R} \neq \emptyset$)

Let $\mathcal{G}_1$ and $\mathcal{G}_2$ be two RDF graphs, and $\mathcal{R}$ a set of RDF entailment rules. The *cover graph* $\mathcal{G}$ of $\mathcal{G}_1^\infty$ and $\mathcal{G}_2^\infty$ is an lgg of $\mathcal{G}_1$ and $\mathcal{G}_2$.

Cover graph vs lgg

Theorem ($\mathcal{R} = \emptyset$)

The *cover graph* $\mathcal{G}$ of the RDF graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ is an lgg of them for the empty set $\mathcal{R}$ of RDF entailment rules (i.e., $\mathcal{R} = \emptyset$).

Proposition ($\mathcal{R} = \emptyset$)

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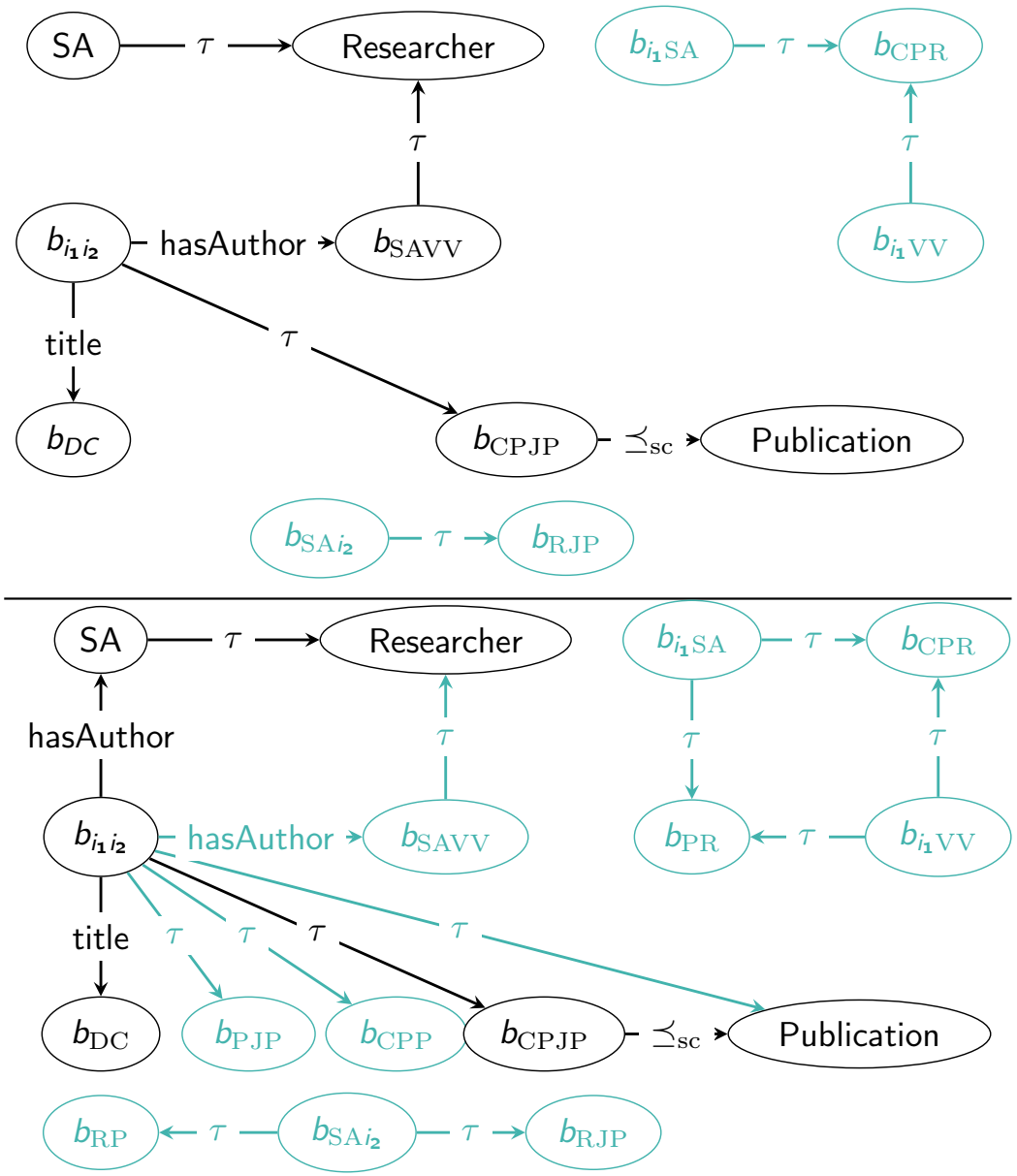
Theorem ($\mathcal{R} \neq \emptyset$)

Let $\mathcal{G}_1$ and $\mathcal{G}_2$ be two RDF graphs, and $\mathcal{R}$ a set of RDF entailment rules. The *cover graph* $\mathcal{G}$ of $\mathcal{G}_1^\infty$ and $\mathcal{G}_2^\infty$ is an lgg of $\mathcal{G}_1$ and $\mathcal{G}_2$.

Corollary ($\mathcal{R} \neq \emptyset$)

An lgg of two RDF graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ can be computed in $O(|\mathcal{G}_1^\infty| \times |\mathcal{G}_2^\infty|)$ and its size is bounded by $|\mathcal{G}_1^\infty| \times |\mathcal{G}_2^\infty|$.

Cover graph vs lgg



Outline

Related work

Structural based approach

- Description Logics $\mathcal{EL}$
  - F. Baader and al. :*Computing least common subsumers in description logics with existential restrictions*.In IJCAI, 1999.
  - B. ZarrieB and al. :*Most specific generalizations w.r.t. general EL-TBoxes*.In IJCAI, 2013.
- RDF
  - SPARQL : tree queries
    - J. Lehmann and L. Buhmann. *Autosparql : Let users query your knowledge base*. In ESWC, 2011.
  - Rooted graphs, ignore RDF entailment :
    - S. Colucci and al. :*Defining and computing least common subsumers in RDF*.J. Web Semantics, 39(0), 2016.

Independent structure approach

- Conceptual Graphs
  - M. Chein and M. Mugnier.*Graph-based Knowledge Representation - Computational Foundations of Conceptual Graphs*.Springer, 2009.

Conclusion

- Revisit the problem of computing a least general generalization in the entire setting of RDF.
- Algorithms to compute `lggs` of small-to-huge RDF graphs.
  - Memory
  - Data management system
  - MapReduce
- Perspective : Heuristics in order to compute `lgg` without redundants triples.

Thank you !

Questions ?

References I

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In *IJCAI*, 1999.

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Journal of Applied Logic, 5(3), 2007.

[3] M. Chein and M. Mugnier.
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Springer, 2009.

[4] S. Colucci, F. Donini, S. Giannini, and E. D. Sciascio.
Defining and computing least common subsumers in RDF.
J. Web Semantics, 39(0), 2016.

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Common subsumbers in RDF.
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In *IJCAI*, 2013.