Label optimal regret bounds for online local learning

PRANJAL AWASTHI, MOSES CHARIKAR, KEVIN LAI, ANDREJ RISTESKI

Online local learning framework (Christiano '14)

Learning Algorithm (**A**)

Adversary

observe item pair $(x_t, y_t) \in \mathcal{X} \times \mathcal{X}$ ←

predict label pair $(L(x_t), L(y_t)) \in \mathcal{L} \times \mathcal{L}$ →

observe payoff $c_t : \mathcal{L} \times \mathcal{L} \to [0, 1]$ ←

- Minimize regret: OPT - E[payoff(A)]; OPT is payoff of best fixed labeling:

$$OPT = \max_{h \in \mathcal{K}} \sum_{t=1}^{T} c_t(h(x_t), h(y_t))$$

$\mathcal{K}$: sef of all possible labelings

n: number of items
l: number of labels

Prior work

- Optimal algorithm (information theoretically): experts/MW -- regret $O(\sqrt{n \log lT})$
- OCO interpretation of experts:

$$\mu_{t+1} = argmax_{\mu \in \mathcal{D}(\mathcal{K})} \left[\nu \sum_{s=1}^{t} E_{\mu}[\text{payoff}_s] - H(\mu) \right]$$

- $H(\mu)$ - Shannon entropy
- $\mathcal{D}(\mathcal{K})$ - polytope of distributions over labelings.

- Relaxation:
  - OGD over 2nd order pseudo-moments polytope.
  - Regularizer: log-det of second-moment matrix.
- (Christiano '14): Follow the regularized leader with log-det regularizer achieves regret $O(\sqrt{nl^3T})$
- (COLT '14): Close gap?

Our results

- **Theorem 1:** Follow the regularized leader with log-det regularizer achieves regret $O(\sqrt{nlT})$.
- **Theorem 2:** Achieving regret $\sqrt{nl^{1-6\epsilon}T}$ refutes the planted clique conjecture for planted clique of size n $^{½-\epsilon}$.

- G(n, ½) – Erdős–Rényi graph.
- G(n, ½, k) – Erdős–Rényi graph with a randomly planted clique of size k.

Planted clique conjecture:

Distinguishing between G(n, ½) and G(n, ½, n $^{½-\epsilon}$) with const. probability is impossible in polynomial time.

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Online version of (Raghavendra '08) result for CSPs:

❖ For CSPs, under UGC, pseudo-moment relaxation is optimal.

❖ Online: same relaxation is optimal with appropriate regularization.

Robust versions of planted clique

- G(n,p) – Erdős–Rényi graph of density p.
- G(n, p, k, q) – Erdős–Rényi graph of density p with a randomly planted Erdős–Rényi subgraph of size k, density q.

Planted dense subgraph conjecture:

Distinguishing between G(n,p) and G(n,p,k,q) with const. probability is impossible in polynomial time for

$p = n^{-\alpha}$ $k = n^{1/2-\epsilon}$ $q = k^{-\alpha-\epsilon}$

- **Theorem 3:** Achieving regret $\sqrt{nl^{1-\Theta(\alpha,\epsilon)}T}$ refutes the planted dense subgraph conjecture.

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Current state of the art: $2^{n^{\epsilon}}$ algorithms. (Bhaskara et al. 2010).

$$p = n^{-\alpha} \quad k = n^{1/2-\epsilon} \quad q = k^{-\alpha-\epsilon}$$

- **Theorem 3:** Achieving regret $\sqrt{nl^{1-\Theta(\alpha,\epsilon)}T}$ refutes the planted dense subgraph conjecture.

Open problems

- Better regret in sub-exponential time?
- Hardness under more standard assumptions?
- More computational hardness results in online learning?

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Thanks!