Second-order Quantile Methods for Experts and Combinatorial Games

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Keep it simple: expert setting

Online sequential prediction with expert advice

Core instance of advanced online learning tasks

- Bandits
- Combinatorial & matrix prediction
- Online convex optimization
- Boosting
- ...
Beyond the Worst Case

Two reasons data is often easier in practice:

- Data complexity:
  - Stochastic data (gap)
  - Low noise
  - Low variance

- Model complexity:
  - Simple model is good
  - Multiple good models

Second-order & Quantiles

- Any combination
Beyond the Worst Case

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All we need is the right learning rate

Existing algorithms
(Hedge, Prod, ...)

with

oracle learning rate $\eta$

exploit

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Can we exploit Second-order & Quantiles on-line?
But everyone struggles with the learning rate

Oracle $\eta$

- **not** monotonic,
- **not** smooth over time.

![Graph showing regret over learning rate](graph.png)

State of the art:
But everyone struggles with the learning rate

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State of the art:

**Second-order**


**Quantiles**

Learning the learning rate

We found a formulation of the algorithm where simply putting a prior on $\eta$ works.

Our algorithm **Squint**

$$w_{t+1}^k \propto \pi(k) \mathbb{E}_{\gamma(\eta)} \left[ e^{\eta R_t^k - \eta^2 V_t^k} \right]$$

guarantees for each subset $\mathcal{K}$ of experts, at each time $T \geq 0$:

$$R_T^{\mathcal{K}} \prec \sqrt{V_T^{\mathcal{K}} \left( -\ln \pi(\mathcal{K}) + \ln \ln T \right)}$$

- Run-time of Hedge
- Only $\ln \ln T$ extra over oracle learning rate.
- Extension to Combinatorial Games

More at our poster