Pushing the Boundaries of Tractable Ontology Reasoning

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Web Ontology Language (OWL)

- Reasoning

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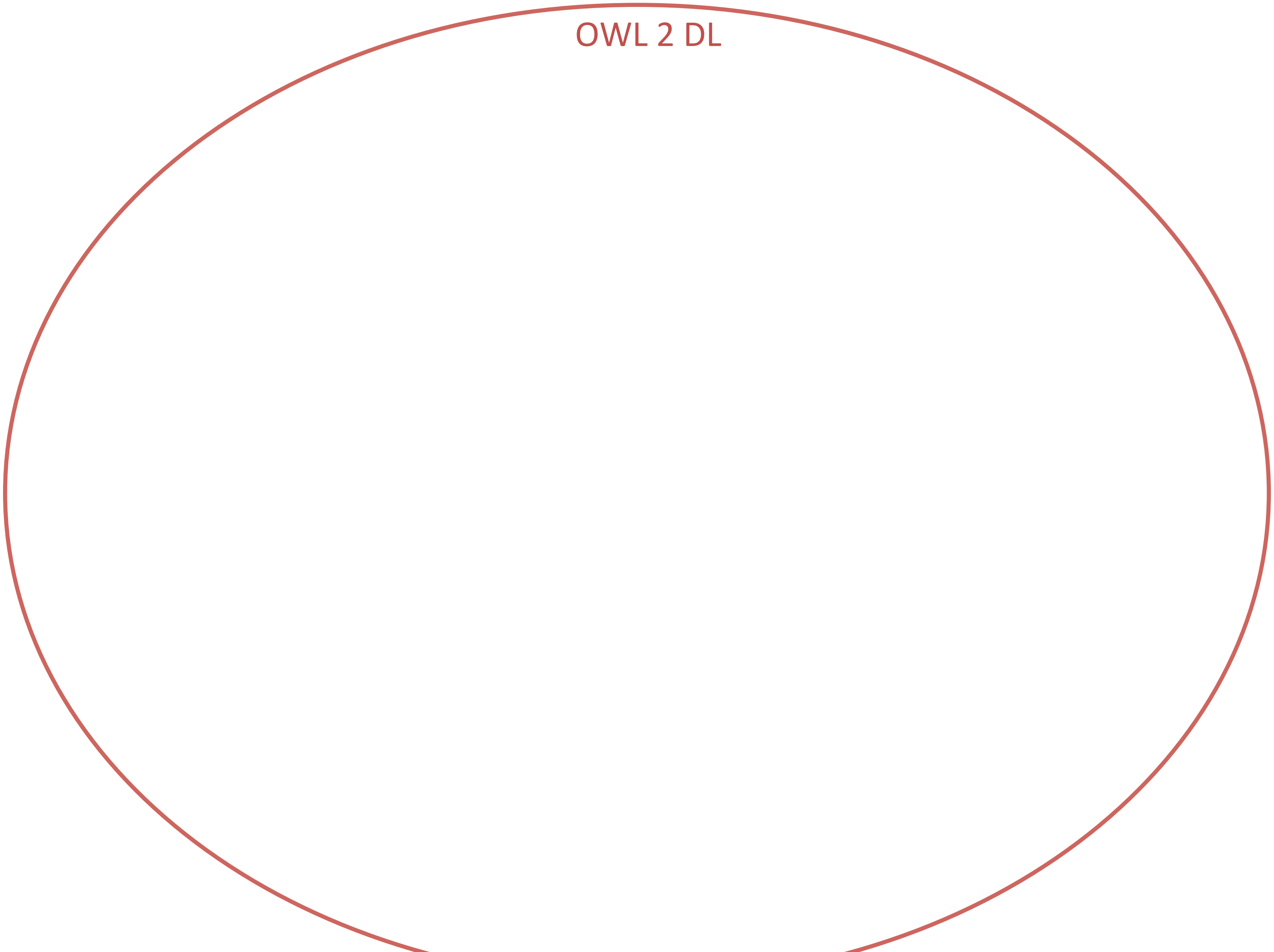
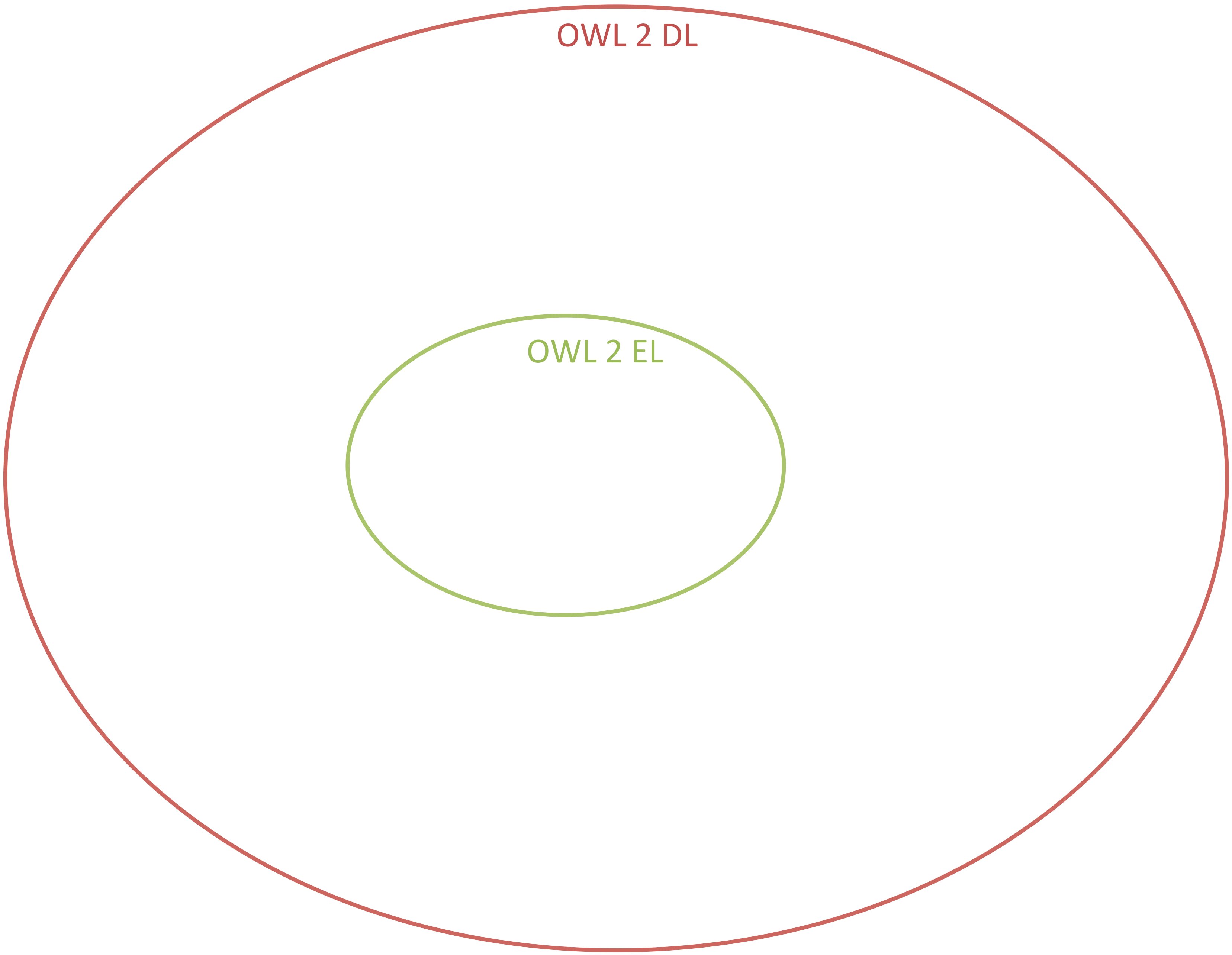
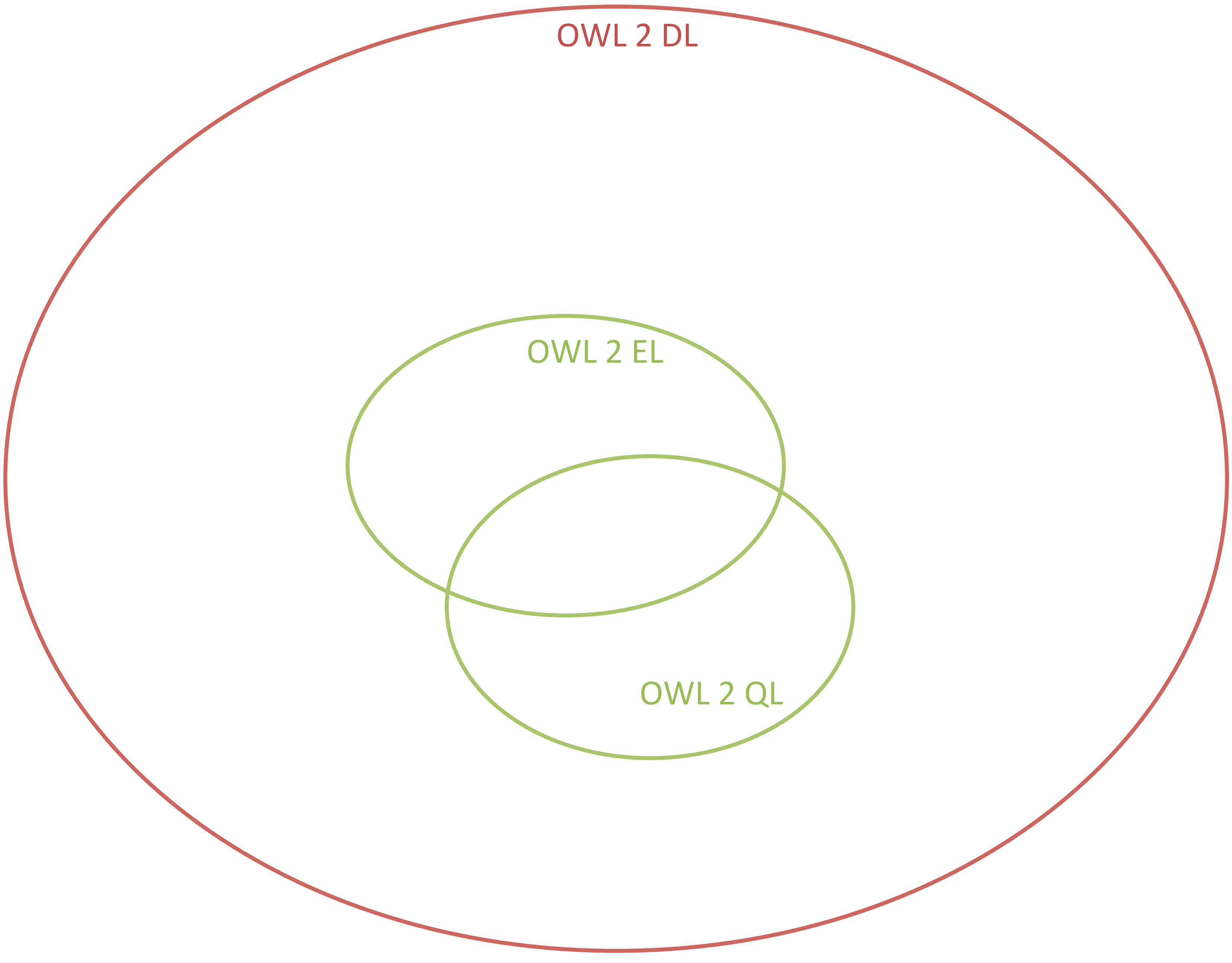
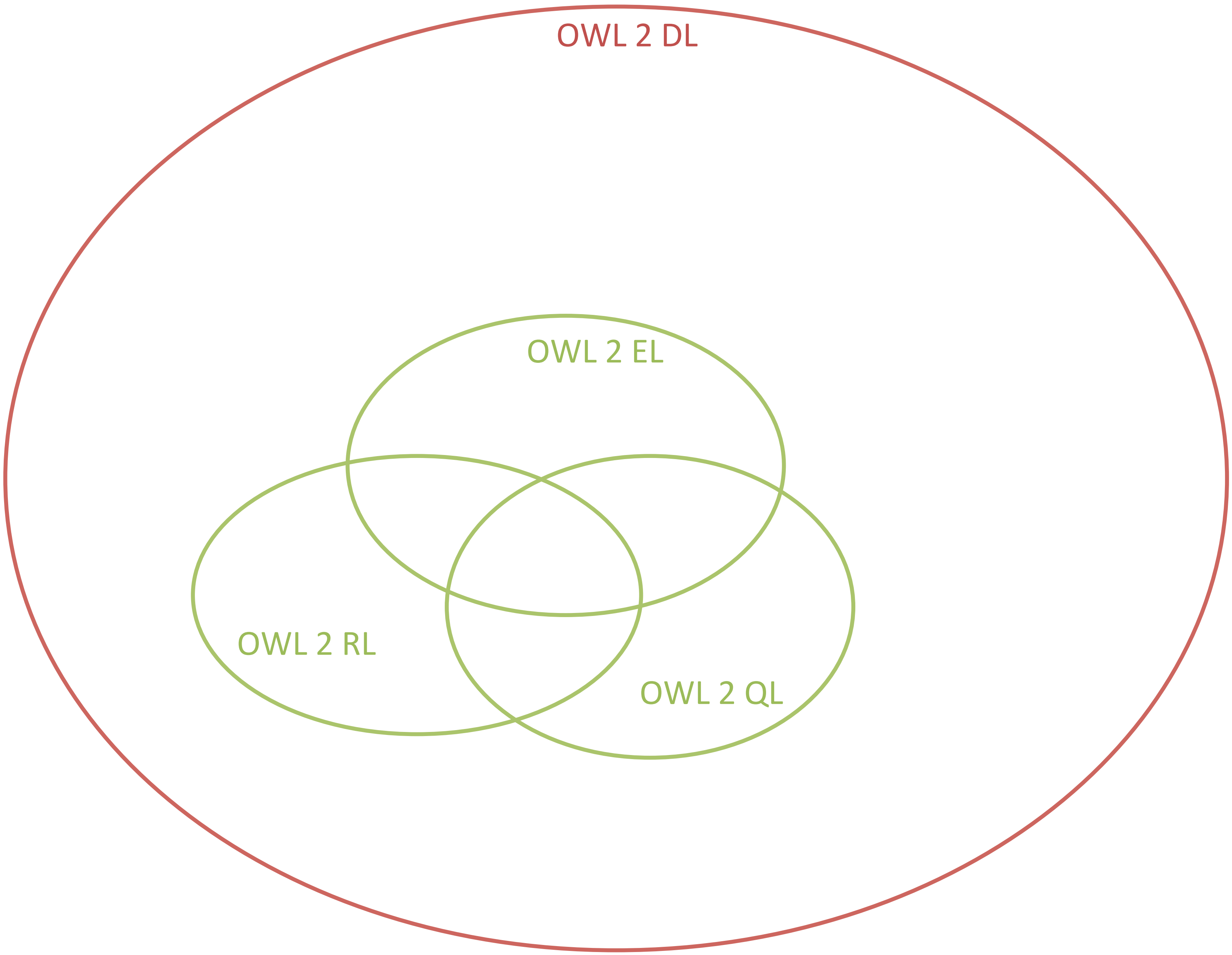
- Reasoning

- OWL Languages
  - Expressivity
  - Complexity of Reasoning


OWL 2 DL


OWL 2 DL
OWL 2 EL


OWL 2 DL
OWL 2 EL
OWL 2 QL


OWL 2 DL
OWL 2 EL
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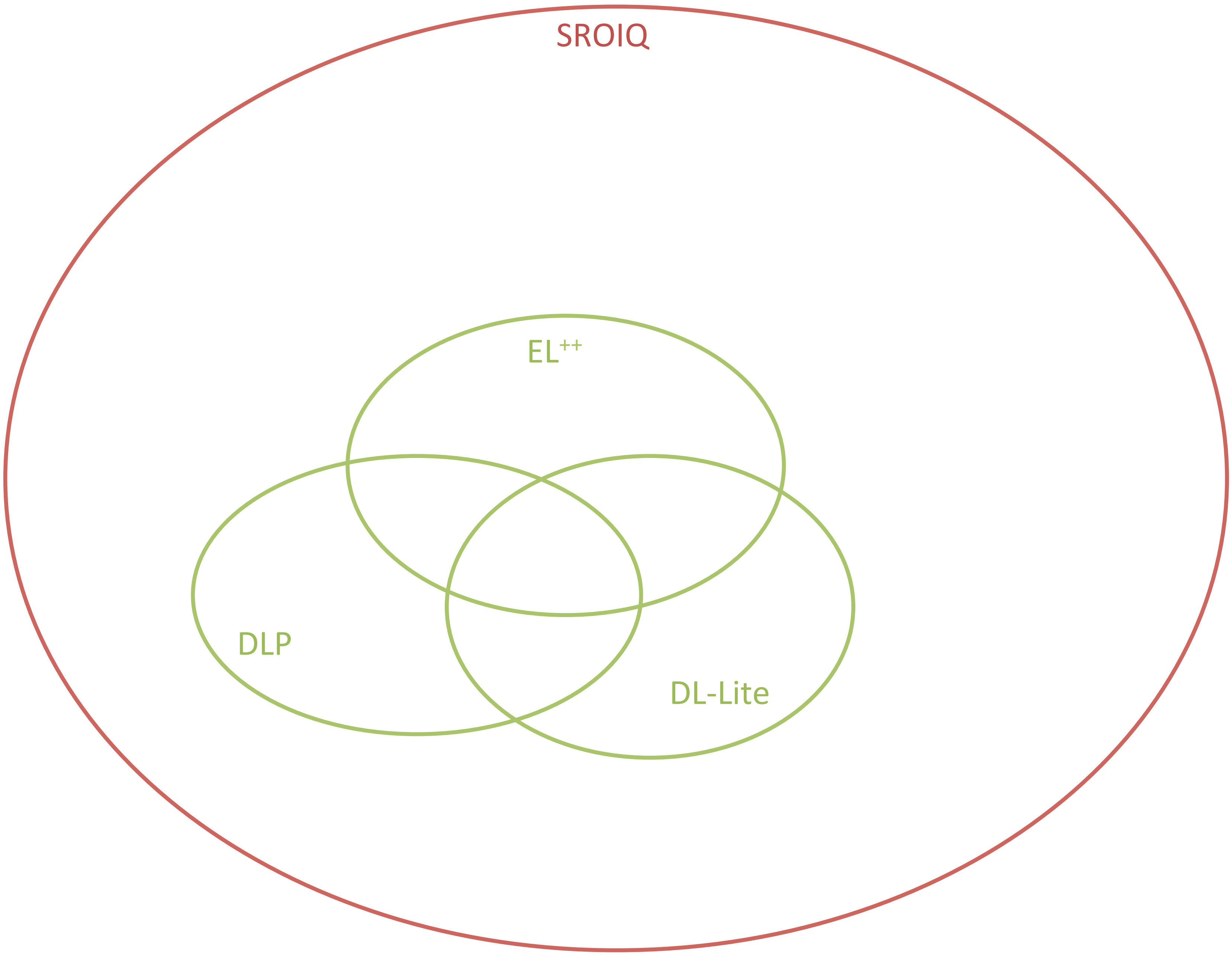
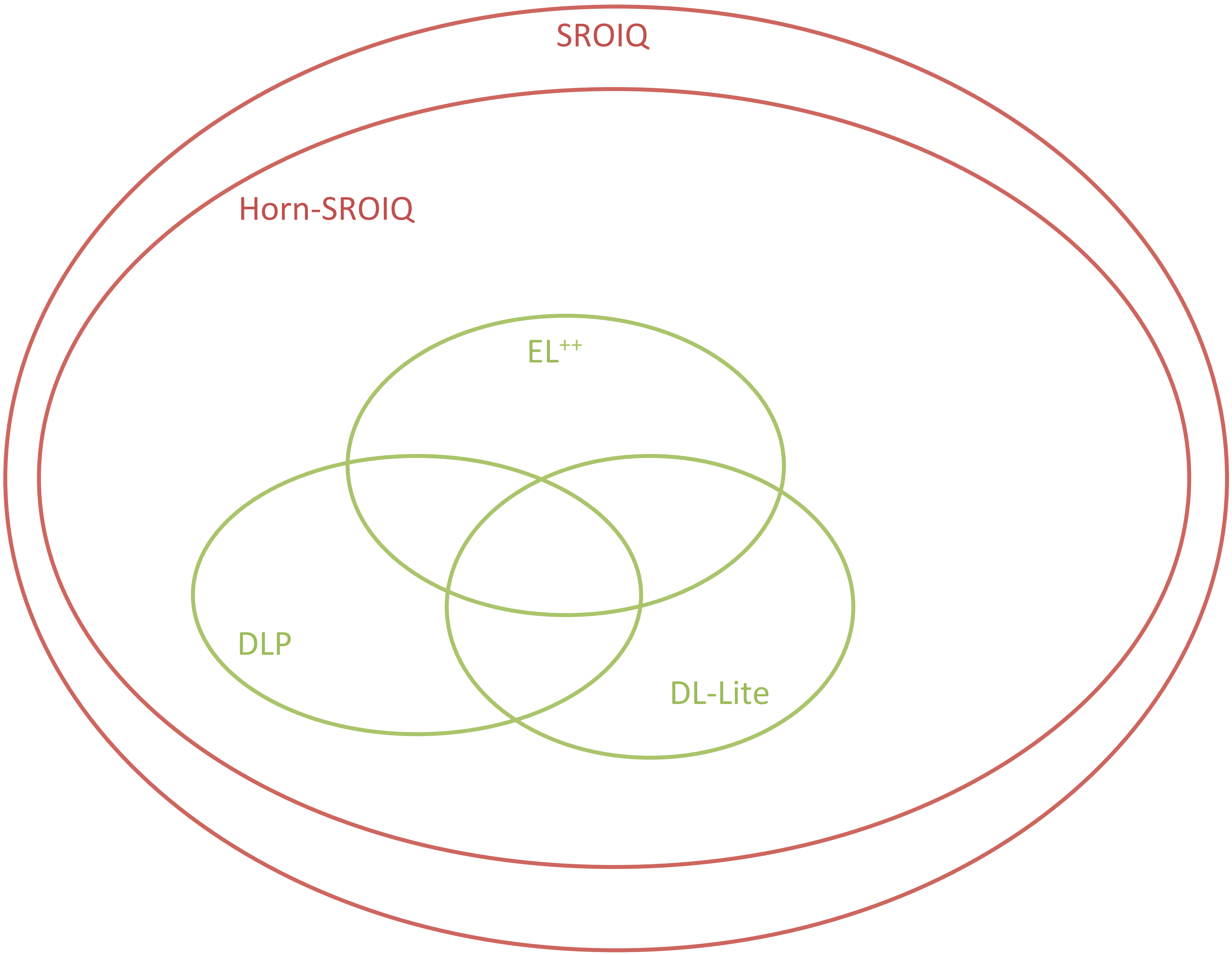
OWL 2 QL


SROIQ
EL++
DLP
DL-Lite


SROIQ
Horn-SROIQ
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DLP
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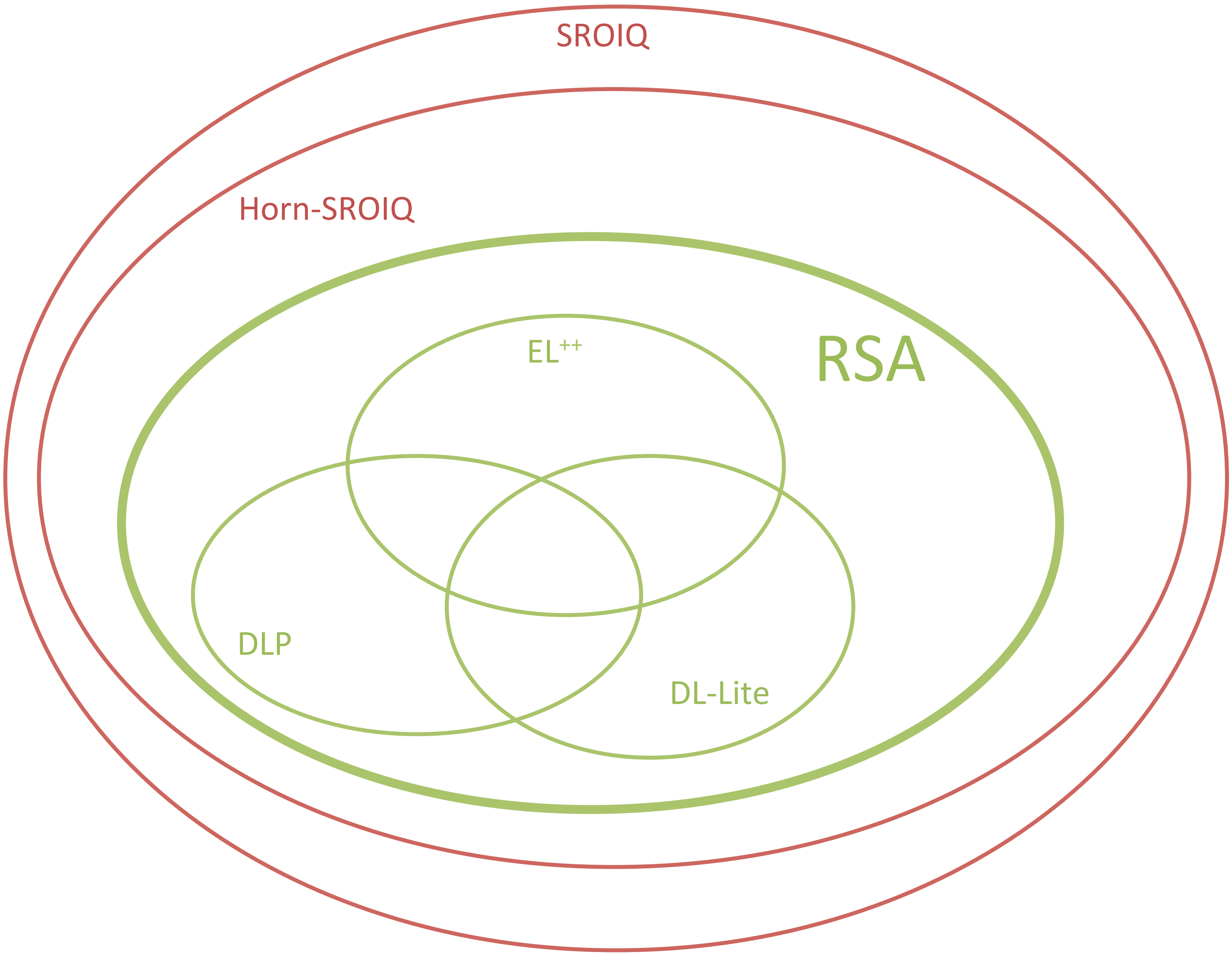
SROIQ
Horn-SROIQ
RSA
EL++
DLP
DL-Lite

Evaluation Results

Repository	Total	RSA
Oxford Ontology Repository	126	68
Ontology Design Patterns Repository	23	22

Evaluation Results

LUBM	HermiT	RSA (RDFox)
1	3.7	0.2
5	OOM	0.8
10	OOM	1.5
20	OOM	7.4

Horn-SHOIQ Normal Form

$$
\begin{aligned}
A_1 \sqcap \ldots \sqcap A_n &\sqsubseteq B & \quad A &\sqsubseteq \leqslant 1 R.B & \quad R &\sqsubseteq S \\
\exists R.A &\sqsubseteq B & A &\sqsubseteq \exists R.B & R^- &\sqsubseteq S \\
\{a\} &\sqsubseteq B & A &\sqsubseteq \{b\} & R \circ R &\sqsubseteq R
\end{aligned}
$$

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Horn-SHOIQ Reasoning

$$\mathcal{O} = \{\mathsf{Human}(\mathsf{david}), \mathsf{Human} \sqsubseteq \exists \mathsf{hasParent}.\mathsf{Human}, \\ \mathsf{hasParent}^{-} \sqsubseteq \mathsf{hasChild}, \exists \mathsf{hasParent}.\mathsf{Human} \sqsubseteq \mathsf{Human}\}$$

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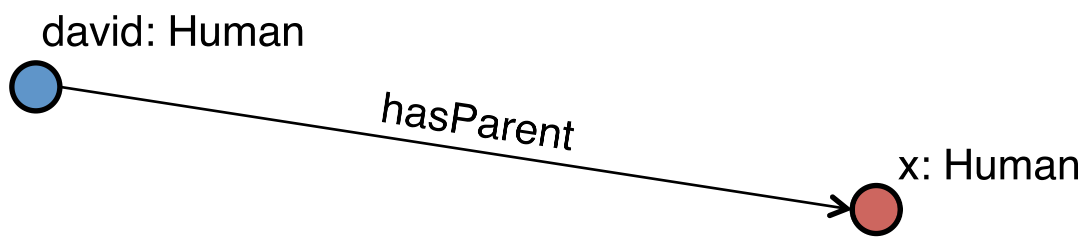

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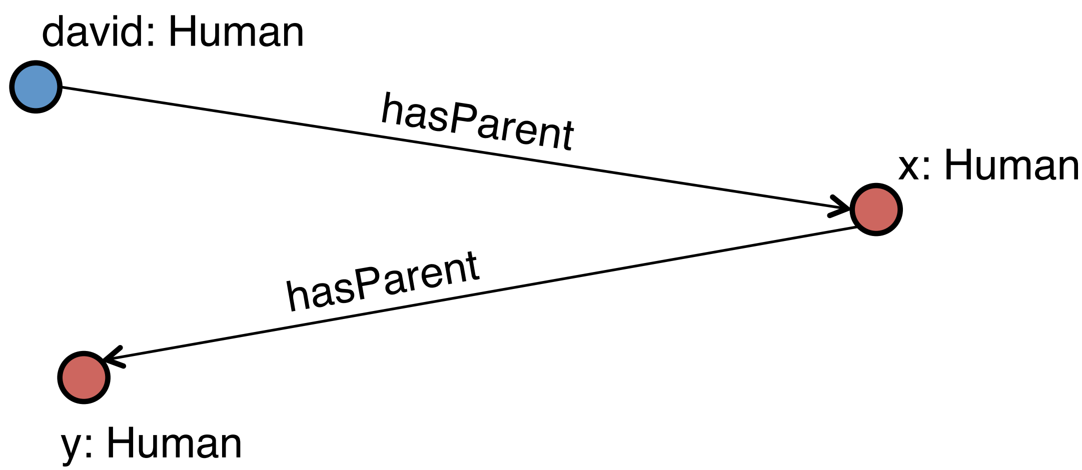

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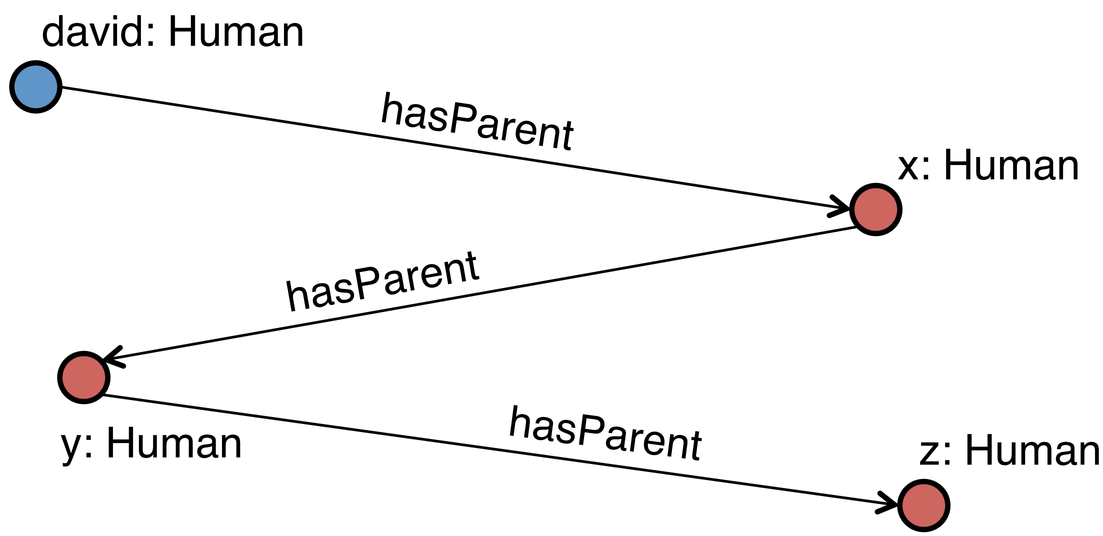

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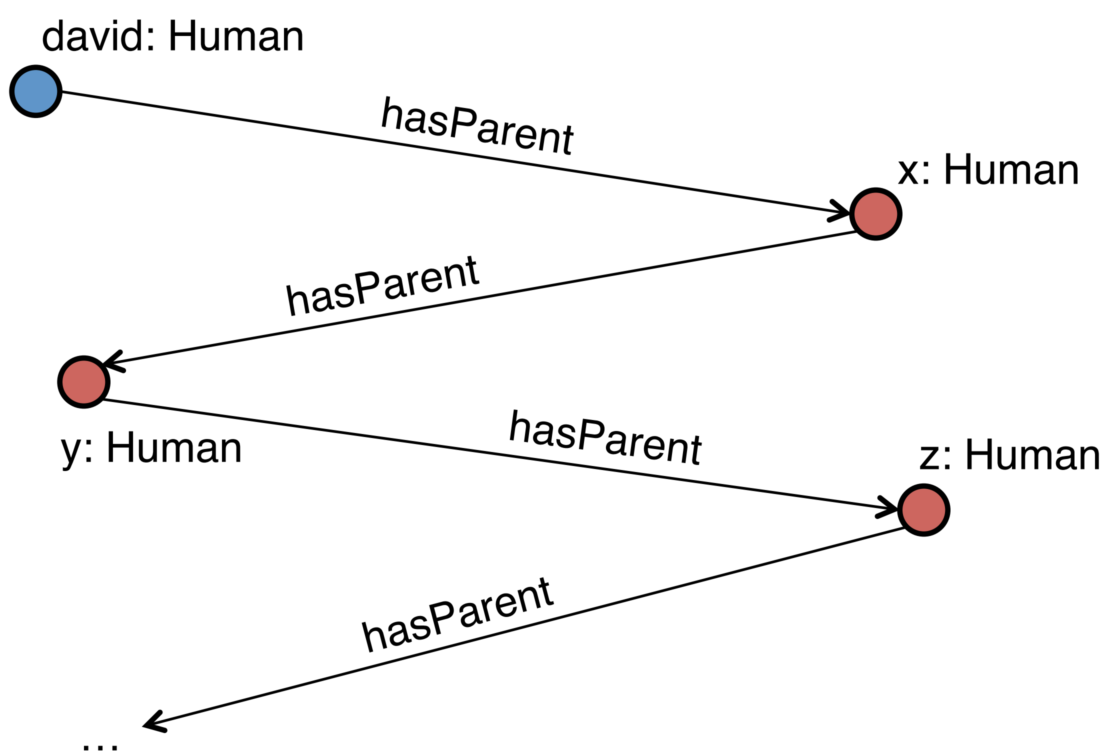

Safe Roles

$$\mathcal{O} = \{\mathsf{Human}(\mathsf{david}), \mathsf{Human} \sqsubseteq \exists \mathsf{hasParent}.\mathsf{Human},$$
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Safe Roles

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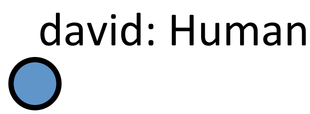

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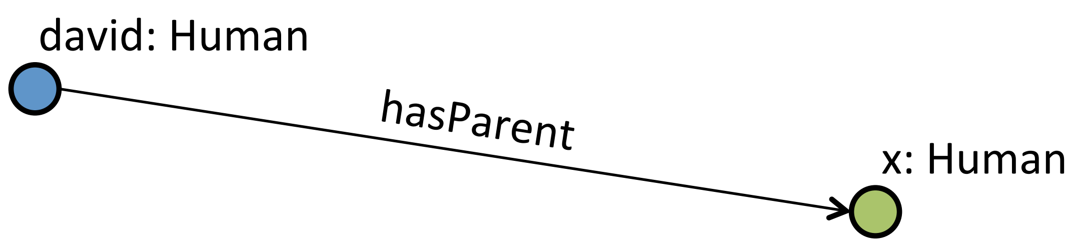

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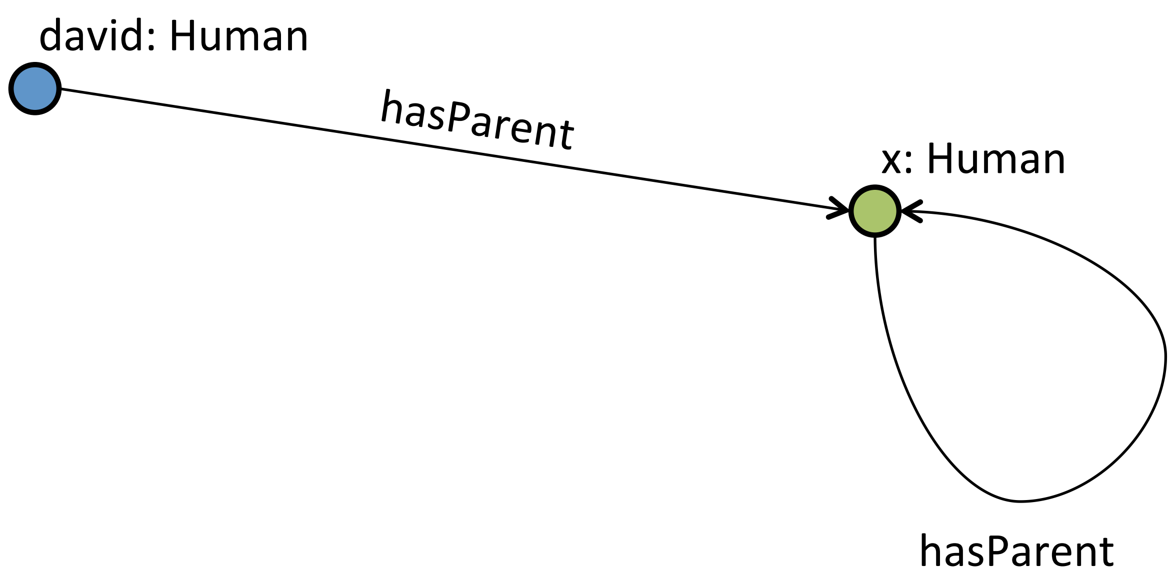

Safe Roles

- Generalizes EL^{++}, DL-Lite and DLP

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- Rewriting to DLP

$$A \sqsubseteq \exists R.B \mapsto \{A \sqsubseteq \exists R.\{c_{RB}\}, B(c_{RB})\}$$

Safe Roles

- Generalizes EL^{++}, DL-Lite and DLP

- Rewriting to DLP

$$A \sqsubseteq \exists R.B \mapsto \{A \sqsubseteq \exists R.\{c_{RB}\}, B(c_{RB})\}$$

- What if not all roles are safe?

Existential Dependencies Graph

$A_1 \sqsubseteq \exists R_1.B_1$

$A_2 \sqsubseteq \exists R_2.B_2$

$A_3 \sqsubseteq \exists R_3.B_3$

$A_4 \sqsubseteq \exists R_4.B_4$

$A_5 \sqsubseteq \exists R_5.B_5$

$A_6 \sqsubseteq \exists R_6.B_6$

$\top \sqsubseteq \leqslant 1 R_1.\top$

$R_2 \sqsubseteq R_1$

$\top \sqsubseteq \leqslant 1 R_3.\top$

$\top \sqsubseteq \leqslant 1 R_4.\top$

$R_5 \sqsubseteq R_4$

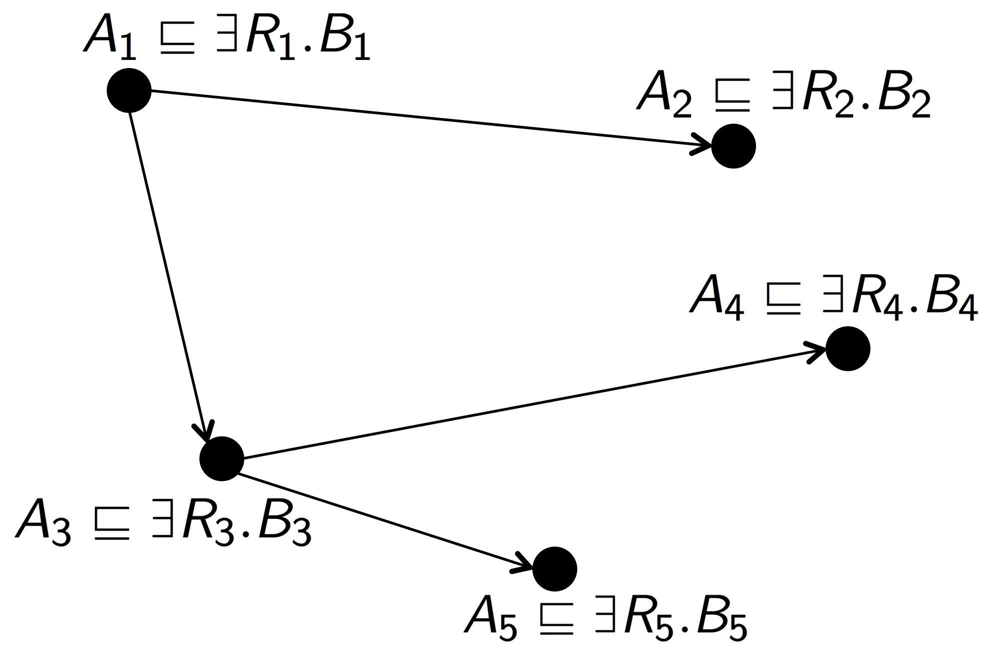
$B_1 \sqsubseteq A_2$

$\exists R_2.\top \sqsubseteq A_3$

$B_3 \sqsubseteq A_4$

$A_4 \sqsubseteq A_5$

$B_1 \sqsubseteq A_6$



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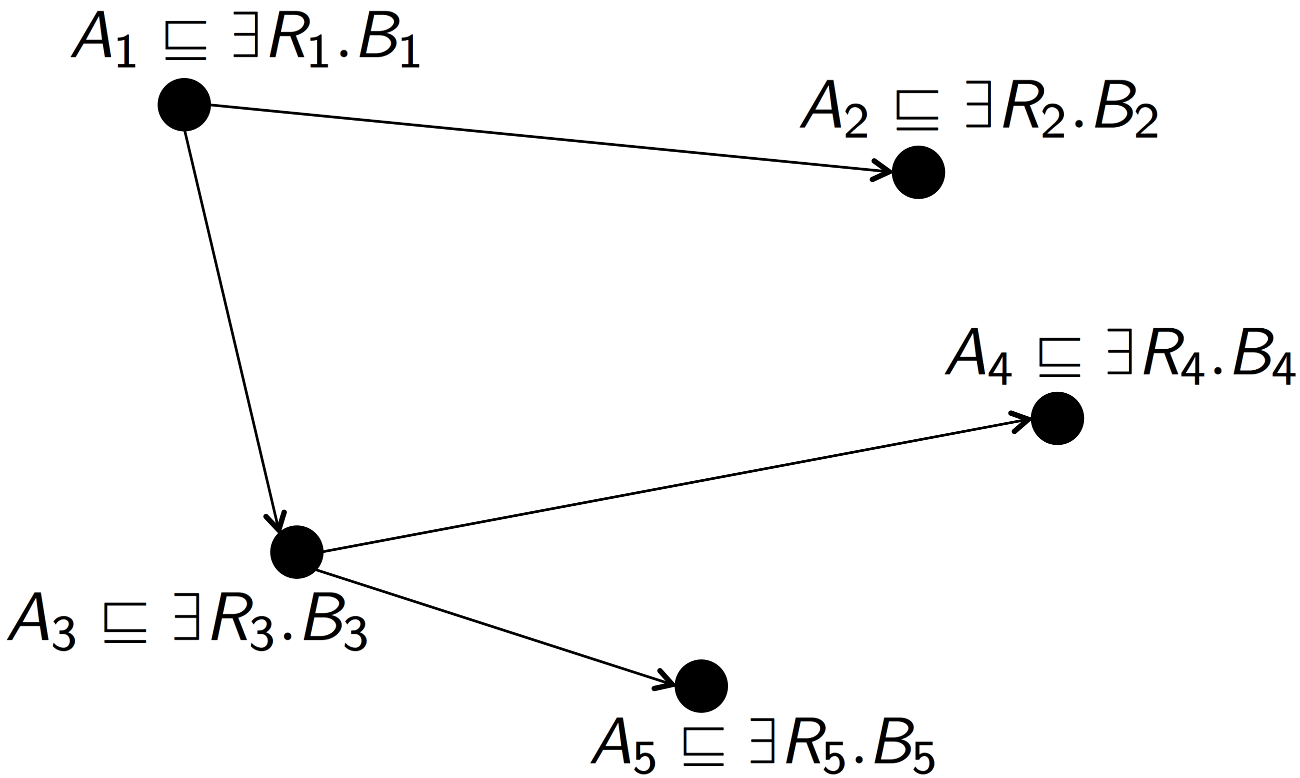
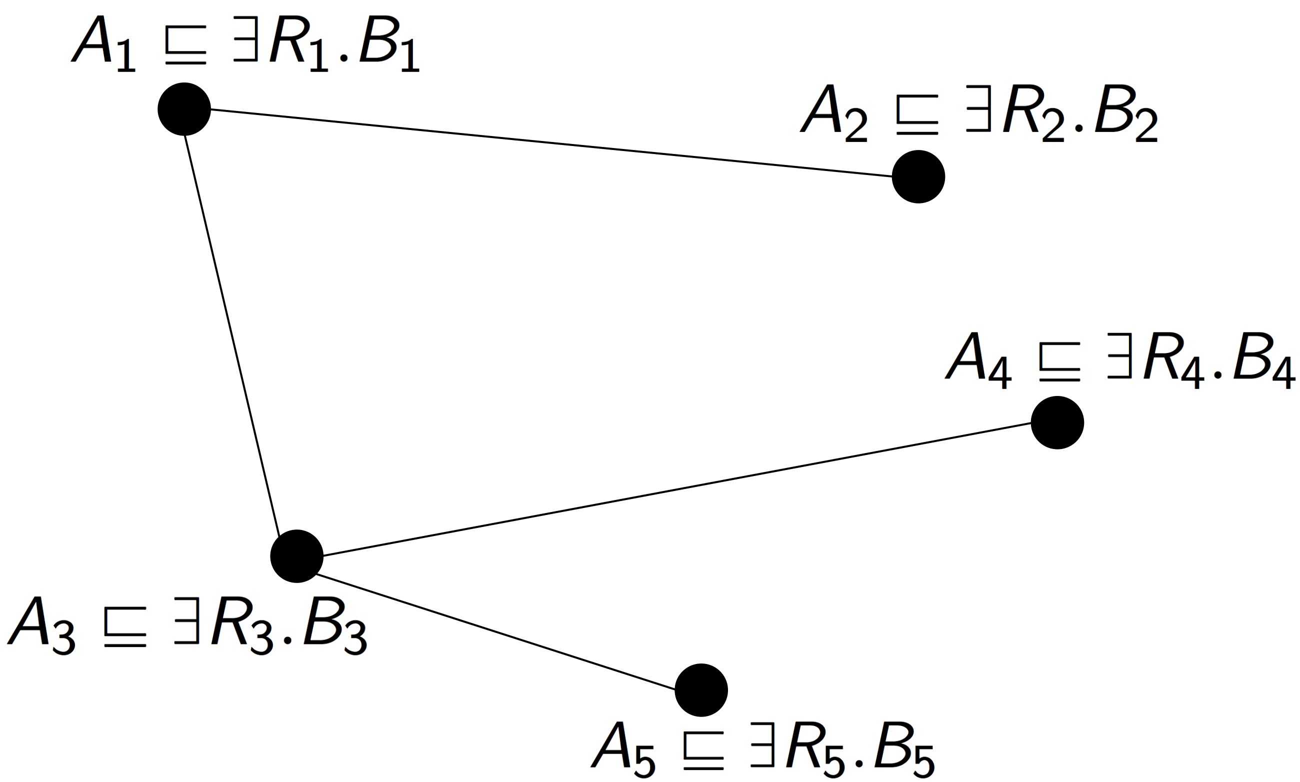
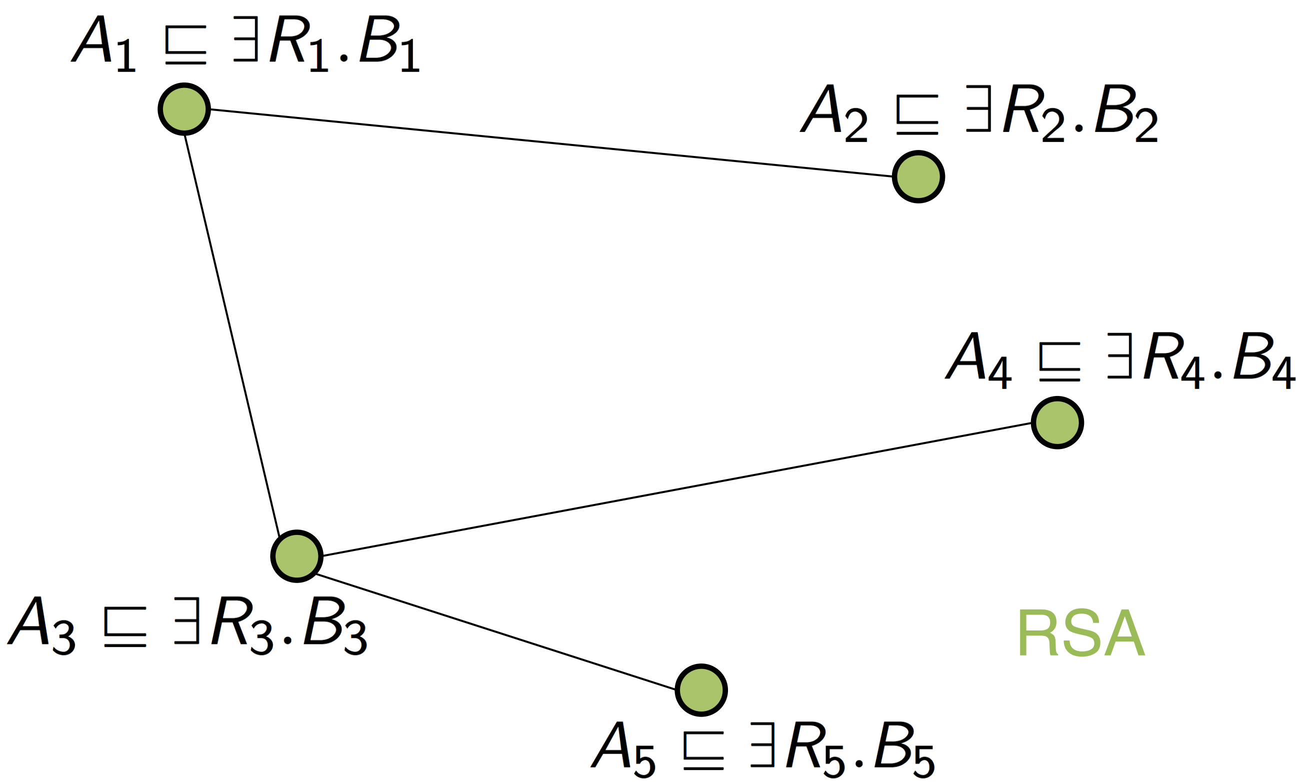


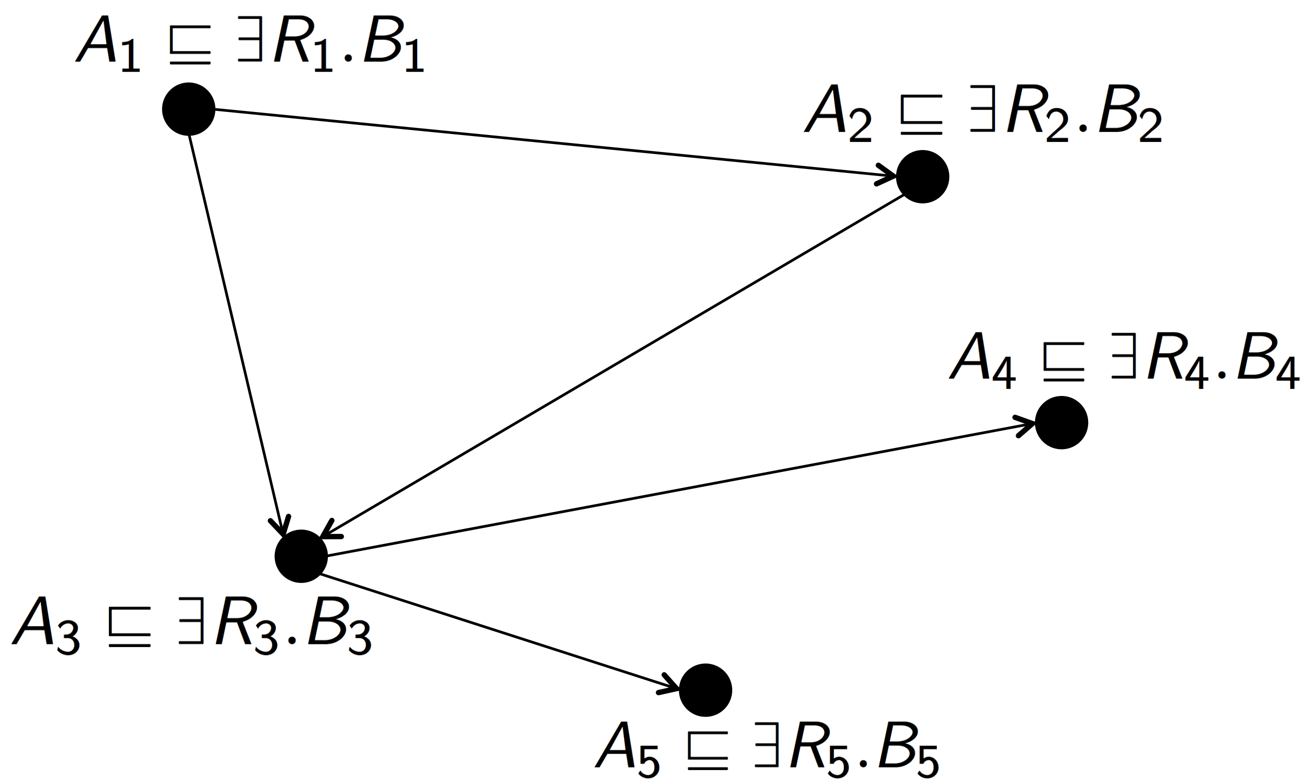
Existential Dependencies Graph



Existential Dependencies Graph



Existential Dependencies Graph



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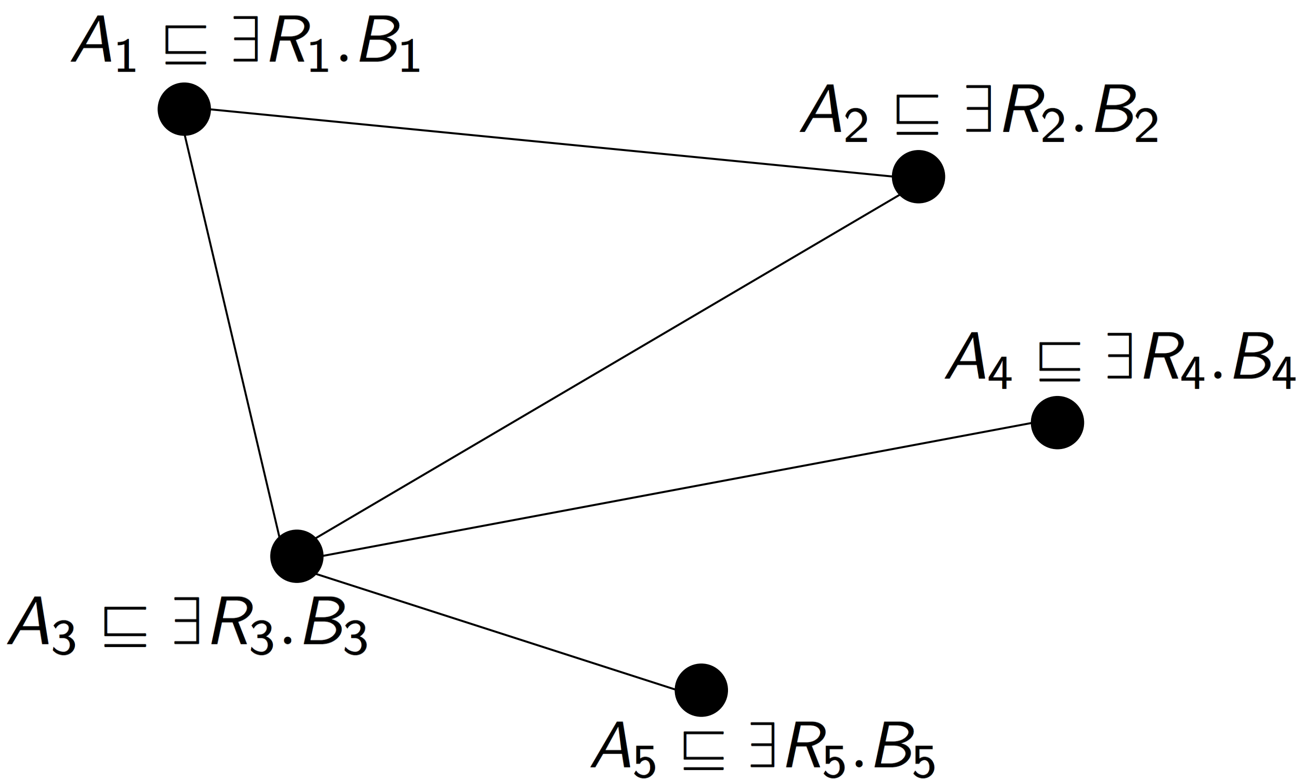

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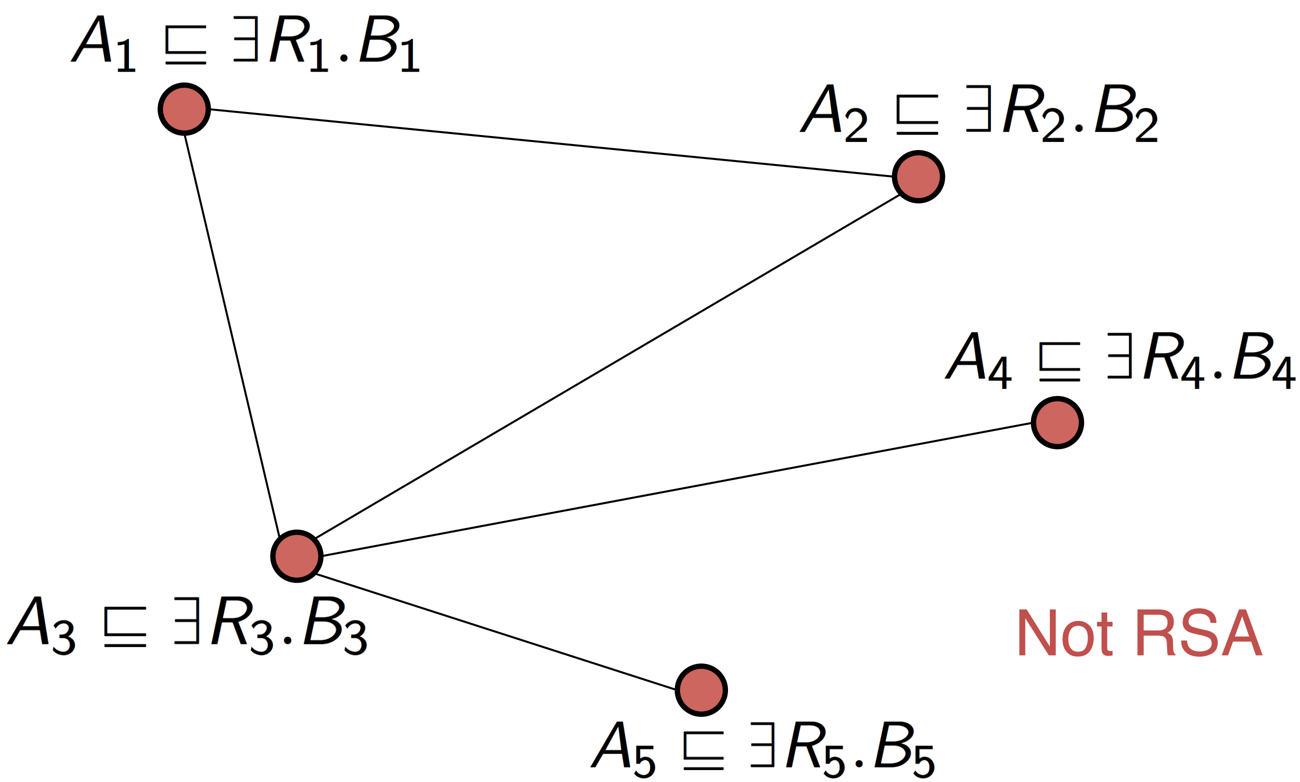

RSA Reasoning Algorithm

$$
\begin{array}{rcl}
A_1 \sqcap \ldots \sqcap A_n \sqsubseteq B & \mapsto & A_1(x) \wedge \ldots \wedge A_n(x) \rightarrow B(x) \\
\exists R.A \sqsubseteq B & \mapsto & R(x,y) \wedge A(x) \rightarrow B(x) \\
\{a\} \sqsubseteq B & \mapsto & B(a) \\
A \sqsubseteq \leqslant 1 R.B & \mapsto & A(x) \wedge R(x,y) \wedge B(y) \wedge R(y,z) \wedge B(z) \rightarrow y \approx z \\
A \sqsubseteq \{b\} & \mapsto & A(x) \rightarrow x \approx b \\
R \sqsubseteq S & \mapsto & R(x,y) \rightarrow S(x,y) \\
R^{-} \sqsubseteq S & \mapsto & R(y,x) \rightarrow S(x,y) \\
 & & \\
A \sqsubseteq \exists V.B & \mapsto & A(x) \rightarrow V(x, c_{VB}) \wedge B(c_{VB}) \\
A \sqsubseteq \exists W.B & \mapsto & A(x) \rightarrow W(x, f^{A}_{WB}(x)) \wedge B(f^{A}_{WB}(x))
\end{array}
$$

where V is safe and W is not!

Future Work

- Conjunctive query answering

Future Work

- Conjunctive query answering
- Reasoner

Q/A?

- Thanks for your attention!