# On the Permanence of Vertices in Network Communities

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20th ACM SIGKDD, New York City, Aug 24-27, 2014

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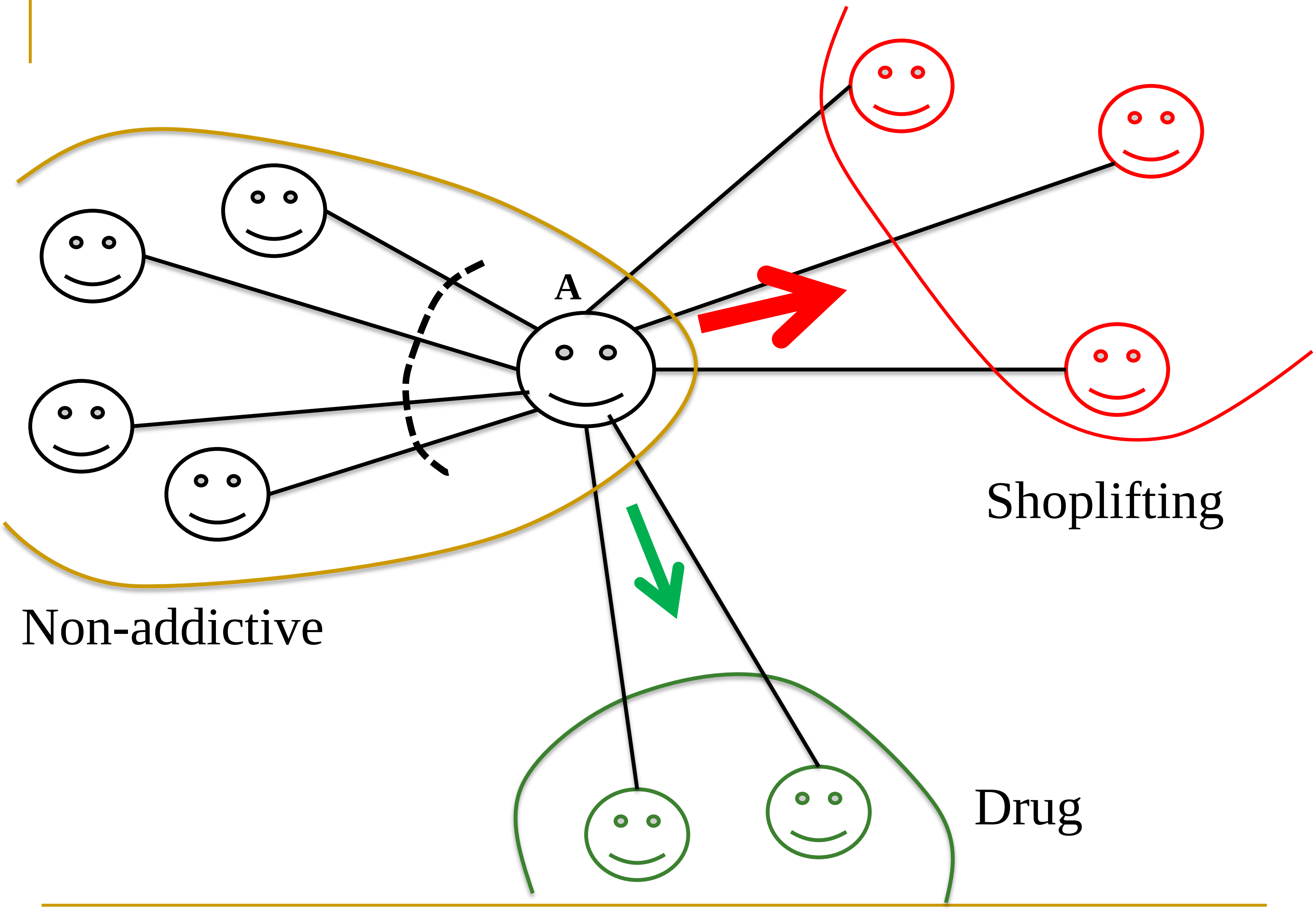
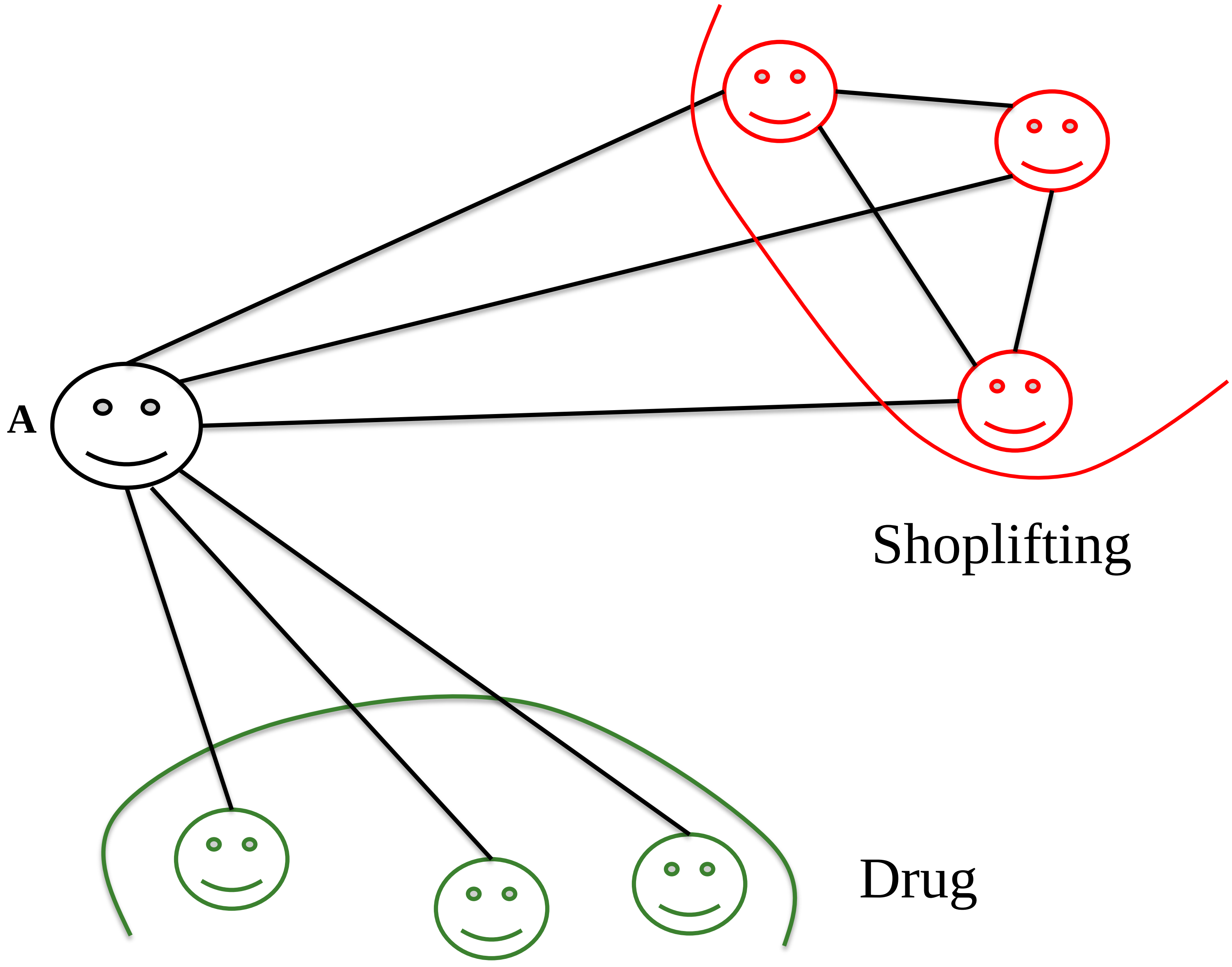

A
Shoplifting
Non-addictive
Drug

# Heuristic I

Total Internal connections > **maximum external connections** to any one of the external communities

Modularity, Conductance, Cut-ratio consider **total external connections**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# Heuristic II

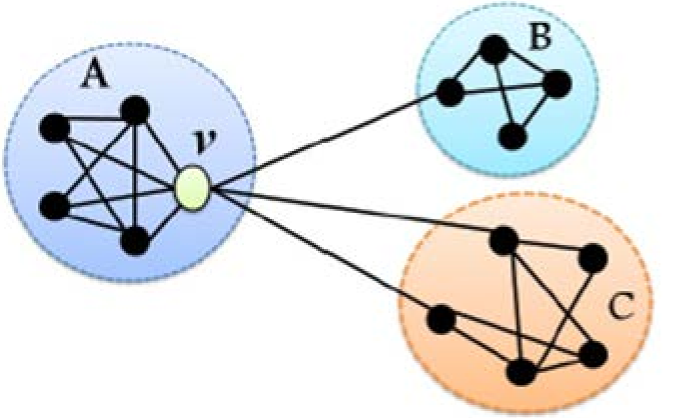
**Internal neighbors** should be **highly connected** => **high clustering coefficient** among **internal neighbors**

✗ Modularity, conductance and cut-ratio do not consider clustering coefficient

# Permanence

$$Perm(v) = \left[\frac{I(v)}{E_{\max}(v)} \times \frac{1}{D(v)}\right] - (1 - C_{in}(v))$$

$I(v)$=internal deg of $v$
$D$(v)=degree of $v$
$E_{max}(v)$=Max connection to an external neighb
$C_{in}(v)$=clustering coefficient of internal neighbors

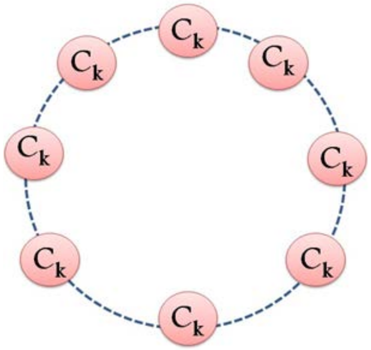



$$Perm(G) = \frac{1}{|V|} \sum_{v \in V} Perm(v)$$

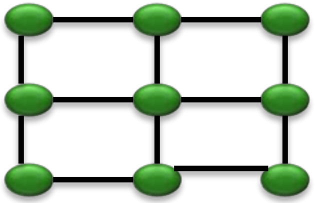
$Perm(v)$=0.12

$I(v)$=4, $D(v)$=7, $E_{max}(v)$=2
$C_{in}(v)$=5/6

# Permanence




Permanence ~ 1



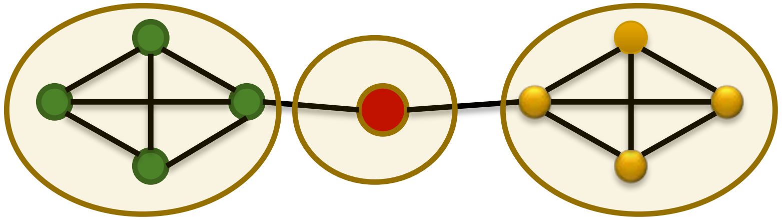
Permanence = 0

Wrong vertex-to-community assignment

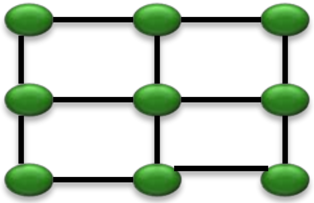
Permanence ~ -1

# Research Questions

1. **Assigning every vertex** to a community is **reasonable**?



2. No one measures the **intensity of belongingness** of a vertex to a community
   - ✓Only try to detect **best community structure** in the network
   - ✗ Never ask for whether a network possesses a strong community



Permanence answers all these questions

# Test Suite of Networks

- **Synthetic Networks**

  LFR networks with different values of mixing parameter (μ)

  *(Lancichinetti & Fortunato, PRE, 11)*

- **Real-world Networks**
  - **Football Network**

    Nodes: teams, Edges: matches, Communities: team-conference

    *(Girvan & Newman, PNAS, 02)*
  - **Railway Network**

    Nodes: station, Edges: train-connections, Communities: state/provinces

    *(Ghosh et al., Acta Physica, 11)*
  - **Coauthorship Network**

    Nodes: authors, Edges: coauthorships, Communities: research field

    *(Chakraborty et al., ASONAM, 13)*

# Baseline Algorithms

- **Modularity based**
    - FastGreedy *(Newman, PRE, 04)*
    - Louvain *(Blondel et al, J. Stat. Mech., 08)*
    - CNM *(Clauset et al, PRE, 04)*
- **Random-walk based**
    - WalkTrap *(Pons & Latapy, J. Graph Algo and Appln, 06)*
- **Compression based**
    - InfoMod *(Rosvall & Bergstrom, PNAS, 07)*
    - InfoMap *(Rosvall & Bergstrom, PNAS, 08)*

# Permanence: A Better Community Scoring Function

# Methodology

- Approach *(Steinhaeuser & Chawla, PRL, 10)*:

1. Consider a network
2. Run *N* community detection algorithms (here N=6)
3. Compute community scoring functions on these outputs
4. Rank the algos based on these values
5. Compare outputs with the ground-truth using validation metrics and rank algos again
6. Find rank-correlation

**Football Network**

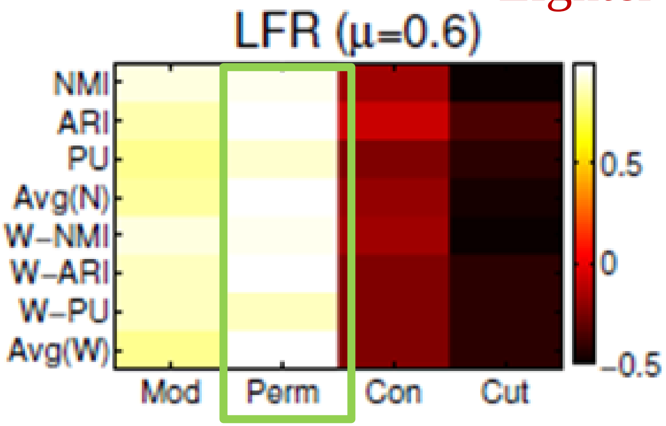
| | Mod \|Per \|Con \|Cut | | NMI\|ARI \| PU | |
|---|---|---|---|---|
| FastGreedy | 0.2 | 5 | 0.3 | 4 |
| Louvain | 0.6 | 2 | 0.5 | 2 |
| CNM | 0.8 | 1 | 0.8 | 1 |
| WalkTrap | 0.4 | 4 | 0.2 | 5 |
| InfoMod | 0.5 | 3 | 0.4 | 3 |
| InfoMap | 0.1 | 6 | 0.1 | 6 |

**correlation**

**Intuition:** Ranking of good scoring function and the validation measures should be high

# Results

Lighter color is better



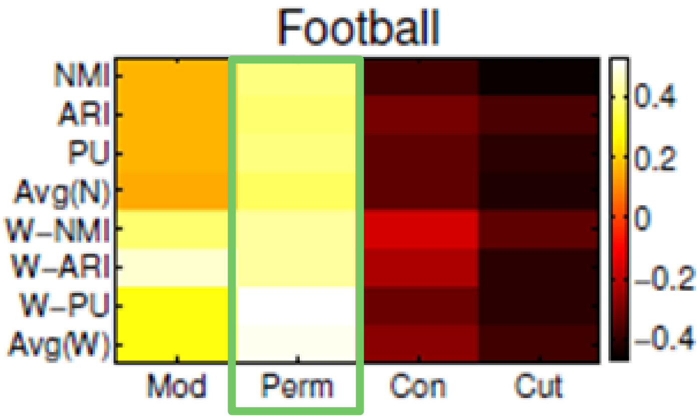




Fig. : Heat maps depicting pairwise Spearman's rank correlation

| Networks | Modularity | Permanence | Conductance | Cut |
|---|---|---|---|---|
| LFR($\mu$=0.1) | **0.88** | **0.88** | **0.88** | 0.02 |
| LFR($\mu$=0.3) | 0.61 | **0.74** | 0.72 | 0.28 |
| LFR($\mu$=0.6) | 0.87 | **0.96** | -0.18 | -0.44 |
| Football | 0.25 | **0.43** | -0.29 | -0.41 |
| Railway | 0.43 | **0.46** | 0.08 | -0.48 |
| Coauthorship | **0.92** | **0.92** | 0.76 | 0.86 |

Table : Performance of the community scoring functions averaged over all the validation measures

# Developing Community Detection Algorithm

# Major Limitations

- **Limitations of optimization algorithms**
  - Resolution limit *(Fortunato & Barthelemy, PNAS, 07)*
  - Degeneracy of solutions *(Good et al., PRE, 10)*
  - Asymptotic growth *(Good et al., PRE, 10)*

# Community Detection Based on Maximizing Permanence

- Follow similar strategy used in Louvain algorithm (a greedy modularity maximization) *(Blondel et al., J. Stat. Mech, 07)*
- Selecting seed nodes helps converge the process faster
- We only consider those communities having size >=3
- Communities having size<3 remain as singleton

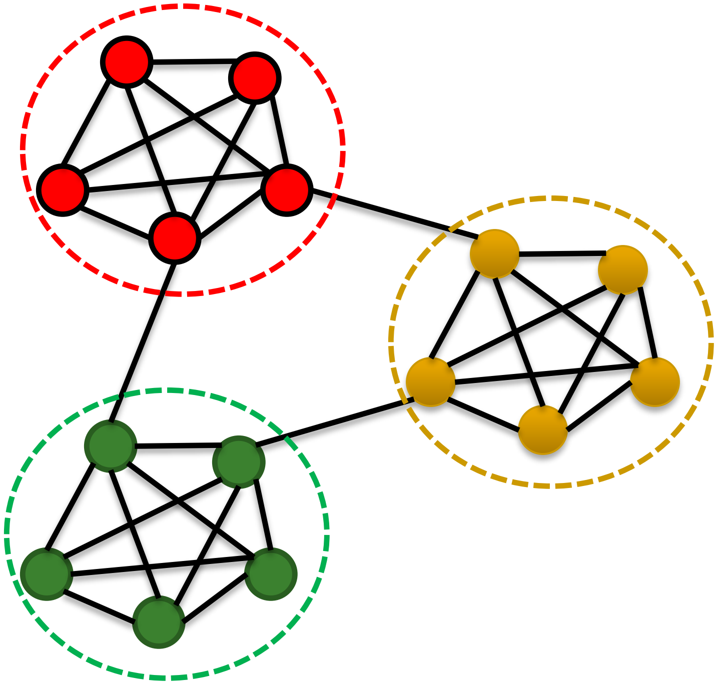
# Experimental Results

Why ????

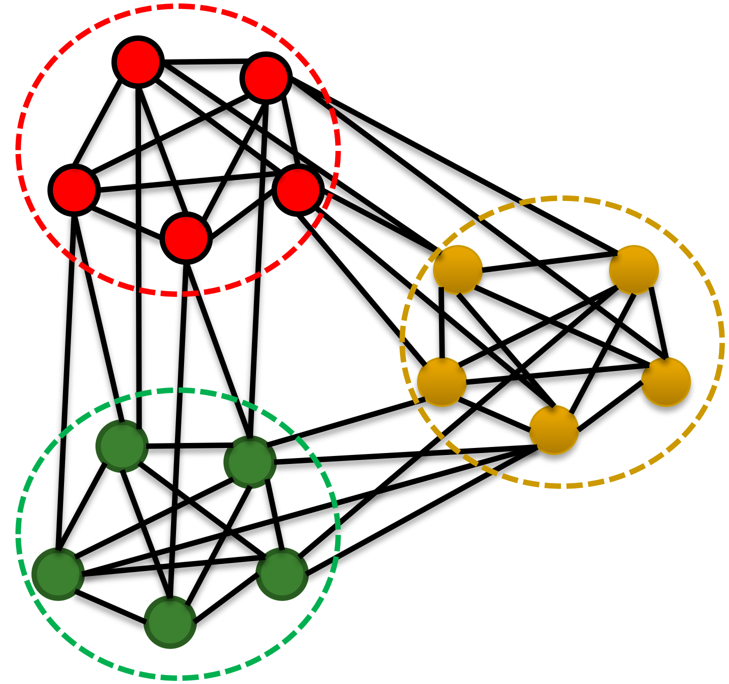
| Algo | LFR (μ=0.1) | LFR (μ=0.3) | LFR (μ=0.6) | Football | Railway | Coauthorship |
|---|---|---|---|---|---|---|
| Louvain | 0.02 | 0.00 | -0.75 | 0.02 | 0.14 | 0.00 |
| FastGrdy | 0.00 | 0.87 | 0.02 | 0.01 | 0.37 | 0.14 |
| CNM | 0.14 | 0.40 | -0.13 | 0.30 | 0.00 | 0.05 |
| WalkTrap | 0.00 | 0.00 | -0.50 | 0.02 | 0.02 | 0.01 |
| Infomod | 0.06 | 0.08 | -0.20 | 0.19 | 0.04 | 0.00 |
| Infomap | 0.00 | 0.00 | -0.72 | 0.02 | -0.02 | 0.03 |

Table: Differences of our algorithm with the other algorithms averaged over all validation measures

# LFR ($\mu = 0.1$) vs. LFR ($\mu = 0.6$)



LFR ($\mu = 0.1$)

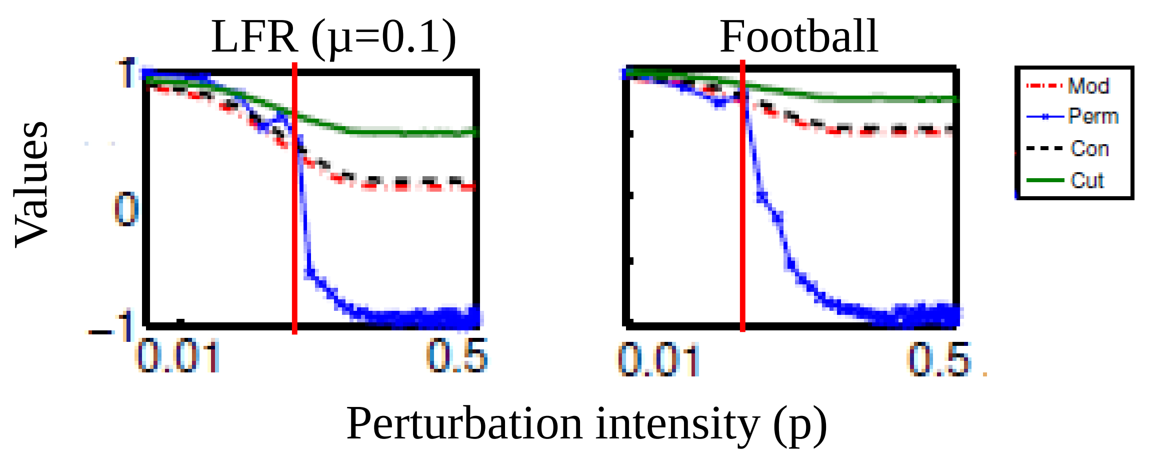


LFR ($\mu = 0.6$)

**$\mu$** = Avg**. ratio** between the number of **external connections** to **its degree**

# Permanence is Nice

- **Permanence** is **not very sensitive** to **minor perturbation**, but **very sensitive** after a **certain threshold**




- **Permanence finds small-size communities**
  - Identify singleton (act as junction in Railway n/w) and small communities (subfields in Coauthorship n/w)

# Issues Related with Modularity Maximization

## Resolution limit

If a vertex is **very tightly connected** to a community and **very loosely connected** to another community, highest permanence is obtained when it joins the community to which it is more connected.

## Degeneracy of solution

if a vertex is **sufficiently loosely connected** to its neighbouring communities and has equal number of connections to each community, then in most cases it will remain as **singleton**, rather than arbitrarily joining any of its neighbour groups.

## Asymptotic growth of value

**All the parameters** of parameters are **independen**t of the **symmetric growth** of network size and the number of communities.

**Analytical proofs:** http://cnerg.org/permanence

# Take Away

- **Permanence**
  - a better community scoring function
  - sensitive to perturbation after a certain threshold
  - indicates the eligibility of a network for community detection

- **Maximizing permanence**
  - a better community detection algorithm
  - can detect small-size communities
  - ameliorates existing limitations

❑ **Future work**

- Recast permanence for overlapping communities
- Recast permanence for weighted graphs
- Recast permanence for dynamic community detection

# Thank you

http://cnerg.org/permanence