

Elicitation and Identification of Properties

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 - ▶ Mean, median, quantiles, ...
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- ▶ Elicitable properties are the only properties for which:
 - ▶ an ERM-type approach can be consistent.
 - ▶ hyper-parameter selection using a validation error can be consistent.
 - ▶ it makes sense to compare learning methods with the help of a test error.

Further Remarks

Importance to other disciplines

- ▶ Climate and Weather Forecast
- ▶ Economy

Questions

- ▶ Which properties are elicatable and why?
- ▶ Find all S for an elicatable property.
- ▶ Find S with additional features such as convexity, calibration, ...

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Unfortunately, proof has a serious bug.

Main Results

- ▶ Generalize Lambert's result (no topological notions!) with correct proof.
- ▶ Characterize all suitable S for an elicitable T .
- ▶ Apply theory to generalized quantiles.
- ▶ Answer a question on expectiles raised by Ziegel.

At the poster you see:

- ▶ Osband's proof.
- ▶ Our main results.
- ▶ The main ideas of our new proof.