Elicitation and Identification of Properties

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A property $T$ assigns a real number to a distribution $P$. Examples:
- Mean, median, quantiles, ...
- Variance, mode, ...

Elicitable properties can be described by risk minimization

$T(P) = \arg \min_{t \in \mathbb{R}} \mathbb{E}_{Y \sim P} S(Y, t)$.

Elicitable properties are the only properties for which:
- an ERM-type approach can be consistent.
- hyper-parameter selection using a validation error can be consistent.
- it makes sense to compare learning methods with the help of a test error.
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Steinwart et al. (University Stuttgart et al.)

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Further Remarks

Importance to other disciplines

- Climate and Weather Forecast
- Economy

Questions

- Which properties are elicatable and why?
- Find all $S$ for an elicatable property.
- Find $S$ with additional features such as convexity, calibration, ...
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\[ T \text{ is elicitable} \implies \text{all } \{T = r\} \text{ convex.} \]

Thus, variance, mode, \ldots are not elicitable.
Characterization of Elicitable Properties

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  These case are
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  - $Y$ is compact, $P$ has continuous density bounded away from 0.
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  Unfortunately, proof has a serious bug.
Main Results

- Generalize Lambert’s result (no topological notions!) with correct proof.
- Characterize all suitable $S$ for an elicitable $T$.
- Apply theory to generalized quantiles.
- Answer a question on expectiles raised by Ziegel.
At the poster you see:

- Osband’s proof.
- Our main results.
- The main ideas of our new proof.