Most correlated arms identification

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Unknown parameter: $\Sigma \in \mathbb{R}^{d\times d}$ s.t. $\Sigma \succeq 0, \Sigma_{i,i} = 1, \Sigma_{i,j} \geq 0$

Goal: upon limited observation of an i.i.d. sequence $X^{(1)}, X^{(2)}, \dots$ from $\mathcal{N}(0, \Sigma)$, find

$$S^* = \operatorname*{argmax}_{S\subset[d], |S|=k} \sum_{i,j\in S} \Sigma_{i,j}$$

Observation model: At each time step $t = 1, 2, \dots$ select $S_t \subset [d]$ and observe

$$X^{(t)}_{S_t} = (X^{(t)}_i)_{i\in S_t}$$

Stop when

$$|S_1| + \ldots + |S_t| > n$$

where n is the observation budget. Choose a candidate optimal subset:

$$\hat{S} \subset [d] \text{ s.t. } |\hat{S}| = k.$$

Theorem: Successive Rejects for Correlation (SR-C) satisfies:

$$\mathbb{P}(\hat{S} \neq S^*) \leq d^3 \exp\left(-\frac{n}{\log(d)\mathbf{H}_{\mathbf{C}}(\Sigma)}\right)$$

Fact: Let $\alpha(x) = \log(x) + \frac{1-x}{x}, x > 1$, and $\rho_0 > \rho_1$, then

$$\mathrm{KL}\left\{\mathcal{N}\left(0, \begin{pmatrix} 1 & 1-\rho_0 \\ 1-\rho_0 & 1 \end{pmatrix}\right), \mathcal{N}\left(0, \begin{pmatrix} 1 & 1-\rho_1 \\ 1-\rho_1 & 1 \end{pmatrix}\right)\right\} = \Theta\left(\alpha\left(\frac{1-\rho_1}{1-\rho_0}\right)\right)$$

Definition: Suboptimality ratio of $S \subset [d]$:

$$\Delta(S) = \sum_{(i,j) \in S^2 \setminus (S \cap S^*)^2} (1 - \Sigma_{i,j}) \Big/ \sum_{(i,j) \in (S^*)^2 \setminus (S \cap S^*)^2} (1 - \Sigma_{i,j})$$

Suboptimality of coordinate $i \in [d]$: $\Delta_i = \min_{S \subset [d], |S| = k, i \in S} \Delta(S)$

Hardness measure of a correlation matrix:

$$\mathbf{H_C} = \frac{k}{\alpha(\Delta_{(k+1)})} + \sum_{i=k+1}^{d} \frac{1}{\alpha(\Delta_{(i)})}$$