Generalizing Analytic Shrinkage for Arbitrary Covariance Structures

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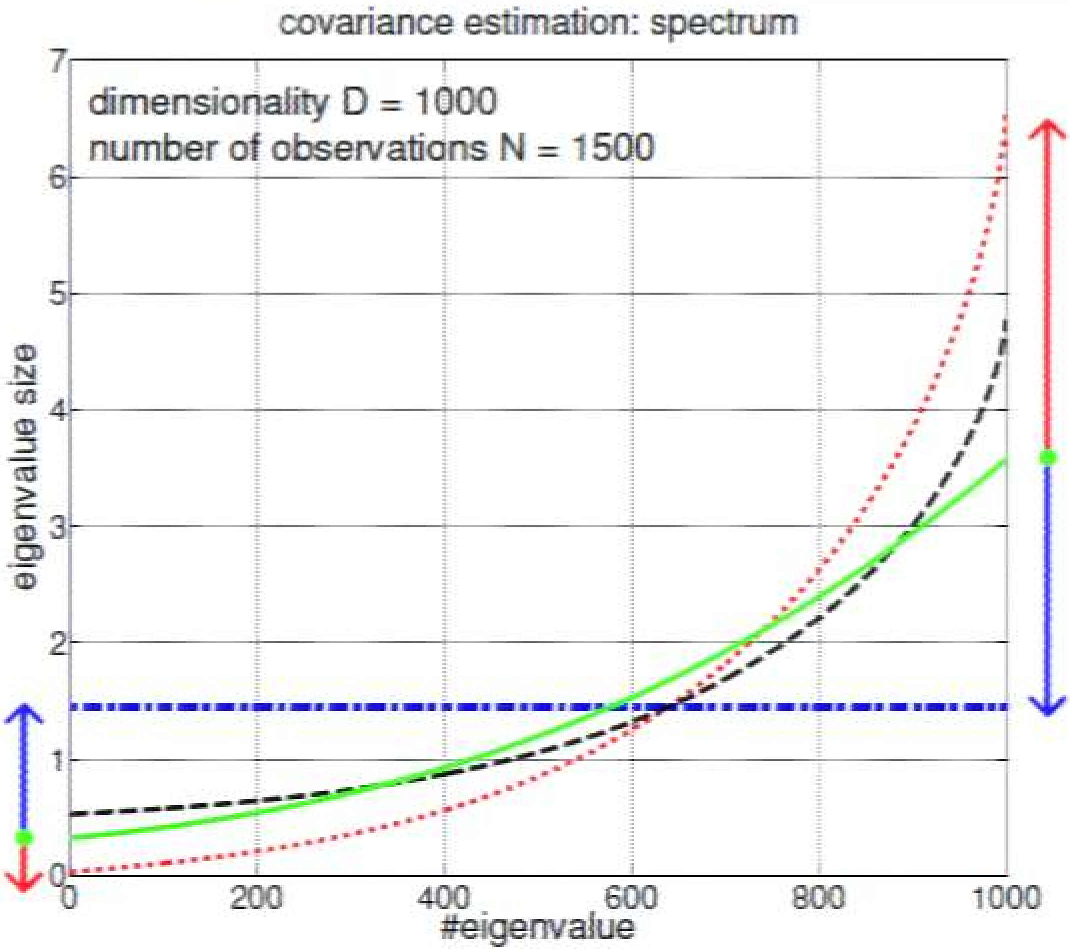

sample covariance **S**
unbiased entries
biased spectrum
high variance

$$\hat{\mathbf{C}}^{shrink}(\lambda) = (1-\lambda)\,\mathbf{S} + \lambda\,\mathbf{T}$$

Target $\mathbf{T} = \frac{\text{trace}(\mathbf{S})}{p} \cdot \mathbf{I}$
biased
low variance

analytic shrinkage:
Ledoit/Wolf (2004)

$$\lambda^* = \underset{\lambda}{\text{argmin}}\, \mathbb{E}\left[\left\| \mathbf{C}^{true} - \hat{\mathbf{C}}^{shrink}(\lambda)\right\|^2\right] \Rightarrow \hat{\lambda}$$

Violation of assumptions in real world data

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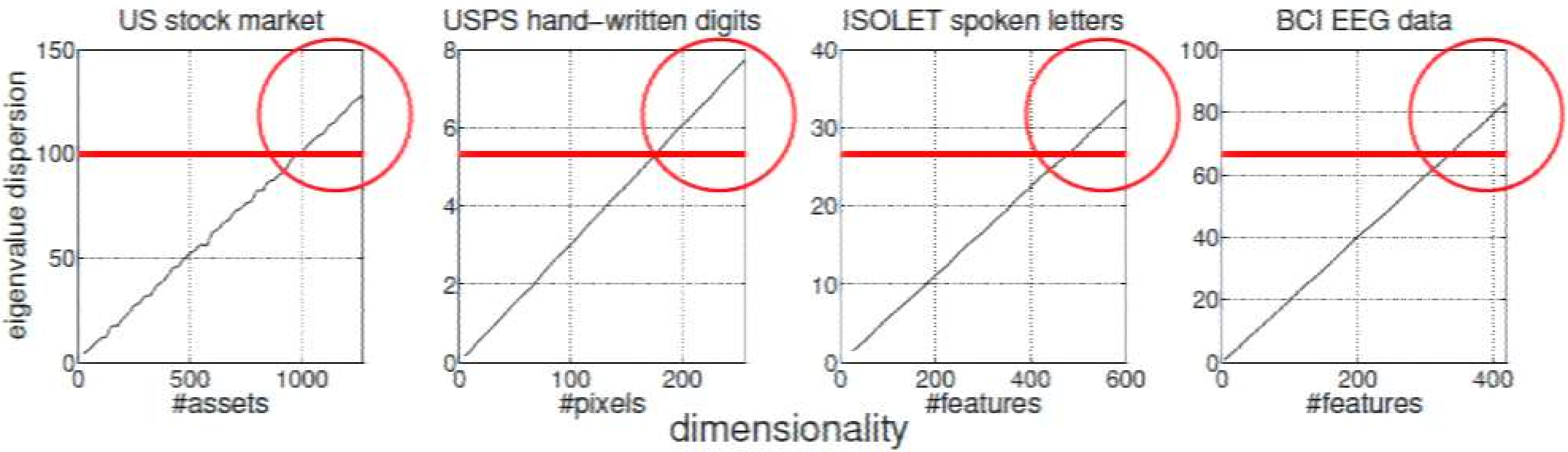
Assumption on 8^{th} moments in the eigenbasis:

$$D_N^{-1} \sum_{d=1}^{D_N} \mathbb{E}[(y_{d1}^N)^8] \leq K_2.$$

$\Rightarrow \hat{\lambda}$ consistent in the limit of $D, N \to \infty$

We show: this requires a bound on the eigenvalue dispersion

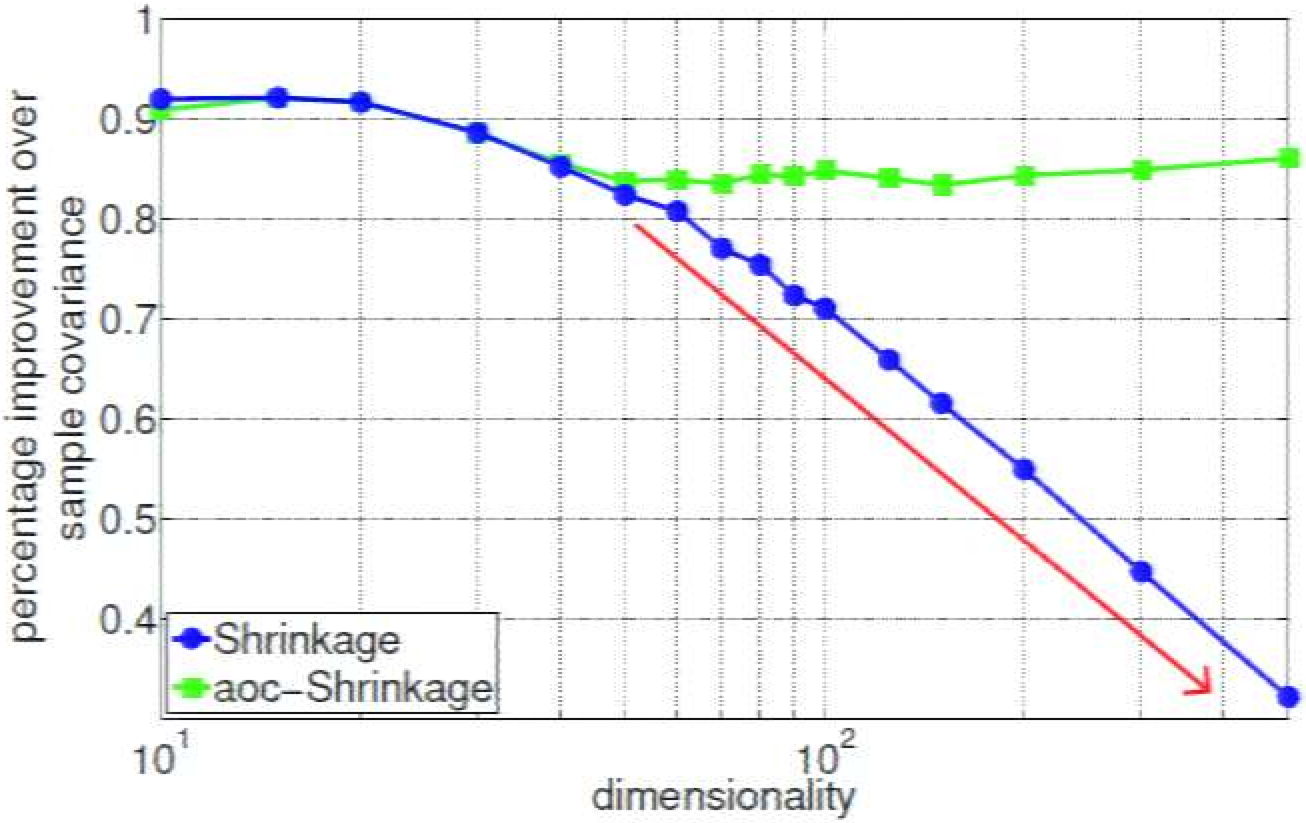
Reality Check: Assumptions are violated!



Is this a problem? Our contribution:

on the one hand: NO!
Theorem: shrinkage is consistent under wider assumptions

on the other hand: YES!
Theorem: there is no shrinkage and no improvement in the limit



automatic orthogonal complement shrinkage

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Visit the poster for

- details on how to overcome zero shrinkage in the limit
- beautiful visualizations
- Results on real world data sets
- mathematical details