Speeding up Permutation Testing in Neuroimaging

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Permutation Testing

Setting

- High-dimensional measurements;
- Highly correlated covariates;
- Comparing distinct phenotype populations, statistically.

Under the Global (Joint) Null Hypothesis, the max observed test statistic is distributed as a function of the # of covariates:

Permutation Testing is an unbiased way of estimating this distribution from the sampled data.
Modelling Assumptions and Approach

Low-rank matrix completion

\[ P = UW + S; \quad P, UW, S \in \mathbb{R}^{v \times t} \]

\[ S_{i,j} \sim \mathcal{N}(0, \sigma^2). \]

\[ P: \text{ Permutation test matrix; } v: \text{ voxels; } t: \text{ tests} \]

\[ UW: \text{ Low-rank component;} \]

\[ U \in \mathbb{R}^{v \times r}, W \in \mathbb{R}^{r \times t}; \quad r \text{ is small} \]

\[ S: \text{ Approx. iid Normal residual} \]

Optimization

\[ \min_{P, U, W} \|P_\Omega - \tilde{P}_\Omega\|_F^2 \quad \text{s.t. } \tilde{P} = UW; \quad U \text{ is column-wise orthogonal} \]

Theoretical guarantees

- Under realistic assumptions, we can model \( PP^T \) as a low-rank perturbation of a Wishart matrix, \( SS^T \).
- The desired sample Null max distribution can be recovered with bounded error.
Thresholds can be recovered with high fidelity with a 50× speedup.

Look for us at poster Sun34, and on the web at
http://pages.cs.wisc.edu/~vamsi/pt_fast.html