Bayesian entropy estimation for binary spike train data using parametric prior knowledge

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Experiment recordings

Goal: Estimate entropy of binary spike data from samples

$$H(\pi) = - \sum_{i=1}^{2^m} \pi_i \log(\pi_i)$$

- far more words than samples

Spike distribution $\pi$

$\pi_1 = p(0 \ 0 \ 1)$
$\pi_2 = p(1 \ 1 \ 0)$

(high dimensional!)
Bayesian approach:

\[ \pi \sim \text{Dirichlet}(\alpha) \]

Bayes Least Squares Estimator:

\[
\hat{H} = \mathbb{E}[H|\text{data}, \alpha] = \int H(\pi)p(\pi|\text{data}, \alpha)\,d\pi
\]

**problem:** Dirichlet priors weight each word equally

- synchronous spikes are unlikely!

\[ p(1 1 1 1 1) \neq p(0 0 0 0 0) \]

**solution:** we choose priors that,

- exploit spike train structure
- make computation tractable
Our approach: center with simple parametric model

\[ \pi \sim \text{Dirichlet}(\alpha G) \]

closed-form moments of entropy

Simple parametric model for spike patterns

Problem: sum intractable for most base measures

\[ \text{E}[H|\text{data}] = \psi_0 \left( \sum_j \alpha_j + 1 \right) - \frac{1}{\sum_j \alpha_j} \sum_{i=1}^{2^M} \alpha_i \psi_0(\alpha_i + 1) \]

digamma

Solution: models with equivalence classes of words

- e.g., probability depends on # spikes: \( G = \text{Bernoulli}(p) \)

| m neurons | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
|           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|           |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

Most likely | less likely | even less likely | less likely still | least likely

Retinal Ganglion Cell data (1ms bins)

27 ON and OFF parasol cells [Chichilnisky lab]

Poster 43 tonight for more details (44 next door)