What is Online Learning?

- Sequential prediction over rounds $t = 1, 2, \ldots$
- At each round $t$
  - Learner receives a question $x_t \in X$
  - Predicts answer $\hat{y}_t$
  - Receives “correct” answer $y_t \in Y$ and suffers loss $\ell(\hat{y}_t, y_t)$
- Allows $x_t$ and $y_t$ to be generated by an adversary
- Goal of learner: minimize “Regret” over $T$ rounds
- Online learning “implies” stochastic optimization
Unconstrained Online Learning with Lipschitz Loss

- Learning rate should be $\frac{\eta}{\sqrt{t}}$, e.g. Zinkevich (2003)
- Setting $\eta$ optimally the regret is $O(\|u\|\sqrt{T})$ otherwise $O((\|u\|^2 + 1)\sqrt{T})$
- Unfortunately the optimal $\eta$ depends on the future
An Improved Lower Bound

Bad news: The optimal rate $O(\|u\|\sqrt{T})$ cannot be achieved!

Theorem

- Regret against $u = 0$ is $0 \Rightarrow$ Regret against other $u$ is $\Omega(T)$
- Regret against $u = 0$ is $\epsilon > 0 \Rightarrow$ Regret against other $u$ is $\Omega \left( \|u\|\sqrt{T} \sqrt{\log \frac{\|u\|\sqrt{T}}{\epsilon}} \right)$

Is it possible to obtain this regret without having to tune any parameter?

See also Streeter and McMahan (2012)
FTRL with a new regularizer, and new analysis
Parameter-free
Same complexity of SGD
Dimension-Free $\Rightarrow$ Kernelizable

**Theorem**

For any sequence of Lipschitz convex losses

$$\text{Regret}(T) \leq O \left( \|u\| \sqrt{T} \left( \ln \left( T^{1.5} \|u\| \right) - 1 \right) \right)$$

See how at poster Sun13

Do you like adaptive algorithms? Read my other paper too: Kpotufe&Orabona, “Regression-tree Tuning in a Streaming Setting”