# Exact learning curves for Gaussian process regression on large random graphs

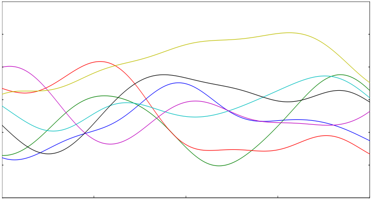
**T84**

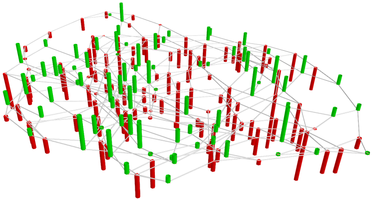
Matthew Urry, Peter Sollich

KING'S College LONDON

- Gaussian Processes are a Bayesian method for regression
  - Prior on function values: joint Gaussian distribution
  - Specified by mean and covariance
  - For Gaussian noise & observations, predictions are straightforward (conditional Gaussian distributions)

- Study learning curves: average (over data set and teacher) mean square error as function of number of examples $N$
- Relatively well understood in continuous spaces but what about discrete spaces?
- Use GPs to learn a function defined on the vertices of a graph





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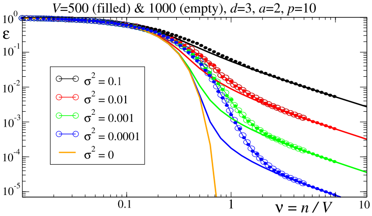
- In previous work (NIPS 2009) we studied how learning curve approximations from continuous spaces performed in the discrete graph case, for:
  - Random regular graphs (all vertices have same degree)
  - Random walk kernel
- While approximation worked o.k. for continuous spaces, in our discrete space:
  - Learning curve approximation performs badly in the midsection of the learning curves
  - Becomes worse as noise level decreases
- This approximation does not account for the additional structure in random graphs: locally tree-like



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- In this paper we
  - exploit graph structure to produce far more accurate results
  - derive approximations for a wide range of graph ensembles
  - obtain predictions that become exact in large graph limit
- Write mean square error as $\epsilon_g = -\frac{2}{V}\lim_{\lambda\to 0}\frac{\partial}{\partial\lambda}\langle \ln Z\rangle_{\{n_i\},G}$

$$Z = \int d\boldsymbol{f}\exp\left(-\frac{1}{2}\boldsymbol{f}^T\boldsymbol{C}^{-1}\boldsymbol{f} - \frac{1}{2\sigma^2}\sum_{i=1}^{V} n_i f_i^2 - \frac{\lambda}{2}\sum_{i=1}^{V} f_i^2\right)$$

- Want to apply the cavity method but first need to:
  - Eliminate the inverse of the covariance matrix $\boldsymbol{C}$
  - $\boldsymbol{C} \propto ((1-a^{-1})\boldsymbol{I} + a^{-1}\boldsymbol{D}^{-1/2}\boldsymbol{A}\boldsymbol{D}^{-1/2})^p$ correlates vertices $p$ steps apart – reduce to nearest neighbour correlations by introducing $2p$ additional variables at each vertex
- End up with a *complex valued* Gaussian graphical model

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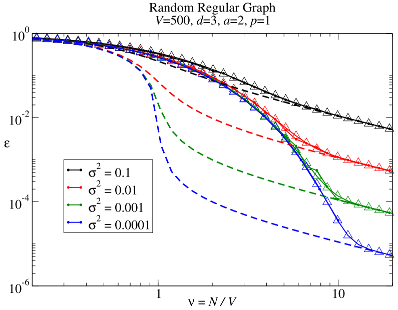


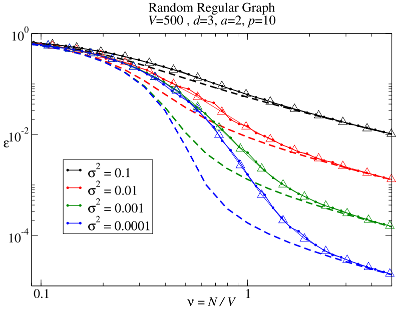


- Even for graphs with $V = 500$ vertices graphs our new approximation (triangles) appears exact
- More on the poster: other types of random graph, comparisons with previous approximation, . . .