Worst-case bounds on the quality of max-product fixed-points (T75)

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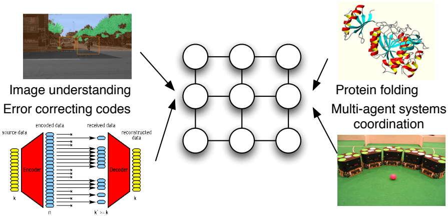
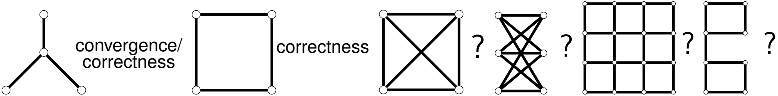
Motivation

Many **real-world problems** can be represented using a **graphical model** and formulated as **finding** the maximum a posteriori assignment (**MAP**).



Loopy max-sum belief propagation is a **popular MAP algorithm** with **good empirical performance** but with **few theoretical guarantees**.

[Open question:] can we **provide theoretical guarantees** for max-sum algorithm **over further structures upon convergence**?

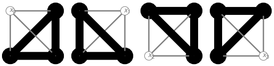


Contribution

We provide **quality guarantees for max-sum fixed-points** in **general graphs calculated prior to the execution** of the algorithm.

Our results build upon two main components:

- The **characterization of any max-product assignment**, x^{MS}, as a **neighbourhood maximum** in an specific region of the graphical model **[Weiss and Freeman, 2001]**



- **Worst-case bounds** on the **quality** of a **neighbourhood maximum**, generalization of **[Pearce and Tambe, 2007]**.

$$\theta(x^{MS}) \geq \frac{cc_*}{|C| - nc_*}\theta(x^*)$$

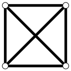
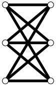
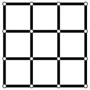
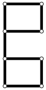
Number of times covered the less covered edge (cc_*)

Number of sets in the region ($|C|$)

Number of times non-covered the less non-covered edge (nc_*)

(All bounds provided are independent of the specific MRF parameters)

$$\theta(x^{MS}) \geq \alpha \cdot \theta(x^*).$$

	Complete	$(n,m)-$ Bipartite	2-D n^2-Grids	Variable-disjoint cycles
α	$\frac{1}{\lvert V\rvert-2}$	$\frac{n}{3n-4}$	$\alpha(n,m)=\begin{cases}\frac{1}{n} & m\geq n+3\\ \frac{2}{n+m-2} & m<n+3\end{cases}$	$\left(1-\frac{2(d-1)}{d\cdot l}\right)$
e.g.	 $\frac{1}{2}$	 $\frac{1}{2}$	 $\frac{1}{2}$	 $\frac{3}{4}$