Global Analytic Solution for Variational Bayesian Matrix Factorization

S. Nakajima (Nikon), M. Sugiyama (Tokyo Tech.), R. Tomioka (Tokyo Univ.)

Matrix Factorization:

\[ V \approx L \]

Applications
- Multivariate analysis (CCA, RRR)
- Missing entries prediction (CF)

Our approach assumes no missing entry.

\[ y = B A^\top x. \]
Bayesian matrix factorization

\[ p(V | A, B) \propto \exp \left( -\frac{\| V - BA^T \|_{\text{Fro}}^2}{2\sigma^2} \right), \]

\[ \phi_A(A) \propto \exp \left( -\sum_{h=1}^{H} 2c_{a_h}^2 \| a_h \|^2 \right), \]

\[ \phi_B(B) \propto \exp \left( -\sum_{h=1}^{H} 2c_{b_h}^2 \| b_h \|^2 \right), \]

Variational Bayesian (VB) method [Lim&Teh2007, Raiko et al.2007]:

Factorizable posterior as \( r(A, B) = \prod_{h=1}^{H} r(a_h)r(b_h) \).

Empirical VB (EVB) method:

Hyperparameters \( \{c_{a_h}^2, c_{b_h}^2\} \) also estimated from observation.

Iterative algorithms for VBMF and EVBMMF were derived.
We analytically derived ...

- **Global solution for VBMF,** which is written as *the 2nd largest real solution of a quartic equation.*

- **Global solution for EVBMF,** which *theoretically proves automatic relevance determination.*

No iteration is needed.

Practical advantage is demonstrated in our poster.
Model-induced regularization (MIR)

Analytic solution reveals details of *unintentional regularization*, which is induced (not by prior but) by model likelihood.

Jeffreys prior explains why MIR occurs.

Evidence view for *non-identifiable* models.