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Poster # : W82

Global Analytic Solution for Variational Bayesian Matrix Factorization

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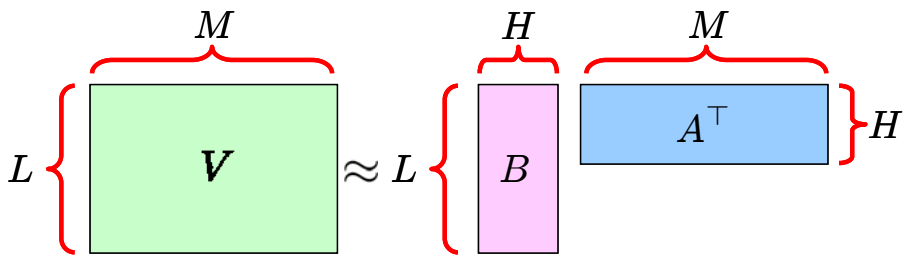
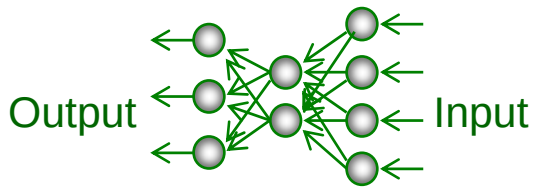
Matrix Factorization :



Applications
- Multivariate analysis (CCA, RRR)
- ~~Missing entries prediction (CF)~~

Our approach assumes no missing entry.



$$\boldsymbol{y} = BA^\top \boldsymbol{x}.$$

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Bayesian matrix factorization

[Salakhutdinov&Mnih2008]

$$\text{Likelihood}: p(\overline{V}|A,B) \propto \exp\left(-\frac{\|\overline{V}-BA^\top\|_{\text{Fro}}^2}{2\sigma^2}\right),$$

$$\text{Priors}: \phi_{\text{A}}(A) \propto \exp\left(-\sum_{h=1}^{H}\frac{\|\boldsymbol{a}_h\|^2}{2c_{a_h}^2}\right),$$

$$\phi_{\text{B}}(B) \propto \exp\left(-\sum_{h=1}^{H}\frac{\|\boldsymbol{b}_h\|^2}{2c_{b_h}^2}\right),$$

Not easy to obtain Bayes posterior.

Variational Bayesian (VB) method [Lim&Teh2007, Raiko et al.2007] **:**

Factorizable posterior as $r(A,B) = \prod_{h=1}^{H} r(\boldsymbol{a}_h) r(\boldsymbol{b}_h)$.

Empirical VB (EVB) method :

Hyperparameters $\{c_{a_h}^2, c_{b_h}^2\}$ also estimated from observation.

Iterative algorithms for VBMF and EVBMF were derived.

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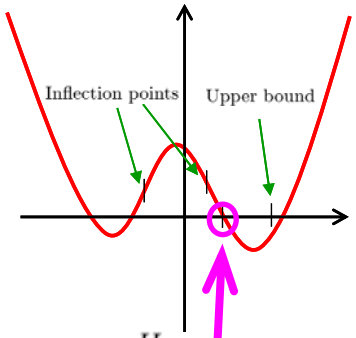
We analytically derived ...

- Global solution for VBMF,

which is written as *the 2nd largest real solution of a quartic equation.*

$$f(\widehat{\gamma}) = \widehat{\gamma}^4 + \xi_3\widehat{\gamma}^3 + \xi_2\widehat{\gamma}^2 + \xi_1\widehat{\gamma} + \xi_0$$



- Global solution for EVBMF,

which *theoretically proves automatic relevance determination.*

$$\widehat{B}\widehat{A}^\top = \sum_{h=1}^{H} \widehat{\gamma}_h^{\mathrm{VB}} \boldsymbol{\omega}_{b_h} \boldsymbol{\omega}_{a_h}^\top$$

No iteration is needed.

Practical advantage is demonstrated in our poster.

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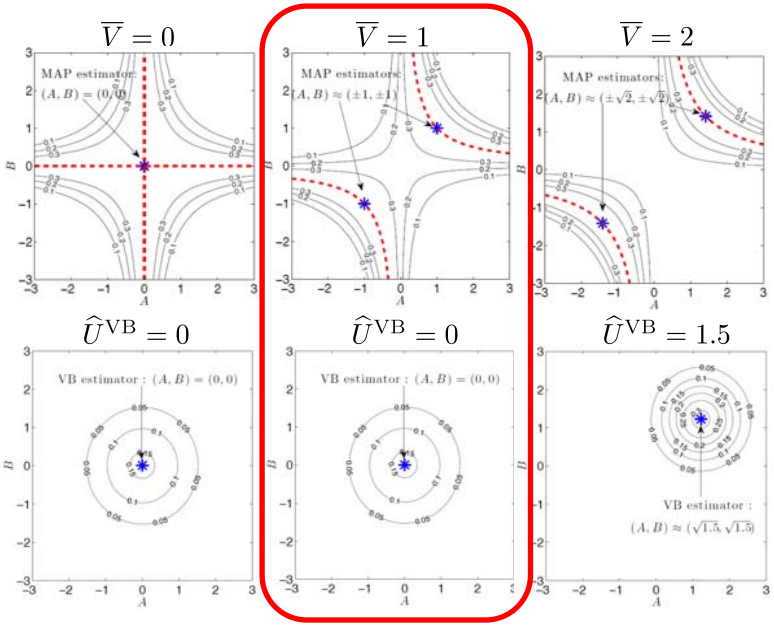
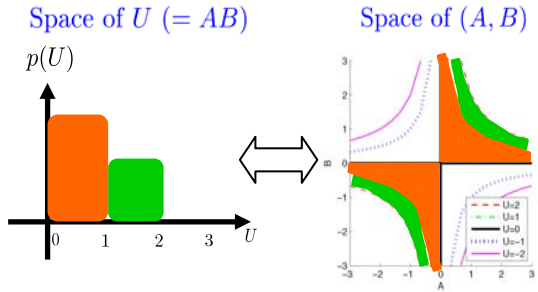
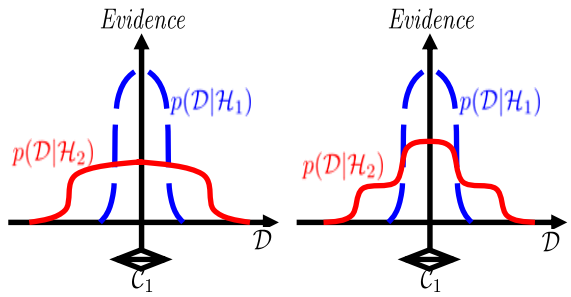
Model-induced regularization (MIR)



VBMF solution is strongly regularized even with almost flat prior.



Jeffreys prior explains why MIR occurs.



Evidence view for *non-identifiable* models.

Analytic solution reveals details of ***unintentional regularization***, which is induced (not by prior but) ***by model likelihood.***