b-Bit Minwise Hashing For Estimating Three-Way Similarities

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- **Minwise hashing is a standard technique in the context of search**

  Numerous practical problems (e.g., duplicate detection, query optimization) can be formulated as set intersection/similarity problems. Minwise hashing is an efficient algorithm for approximating set similarities/intersections.

- **Conventional minwise hashing stores each hashed value using 64 bits**

- **We suggest to store only the lowest b-bits, e.g., b = 2**

- **Our method often achieves about 20-fold improvement in storage space and similar improvement in computation, to attain the same accuracy**

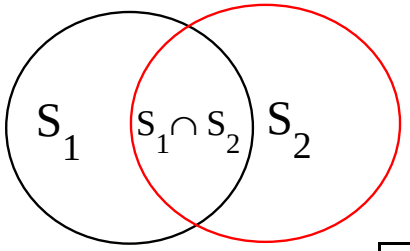
- **Intuitive idea and practical method, but the analysis is technical.**

Shingling, Resemblance, Minwise Hashing

Shingling: A document (Web page) can be represented by a set of w-shingles (w contiguous words, $w \geq 5$). The set space is often assumed to be $|\Omega| = 2^{64}$.

Document similarity $\Rightarrow$ Set intersection in ultra-high dimensions.

Resemblance (R): $\boxed{R = \frac{|S_1 \cap S_2|}{|S_1 \cup S_2|}}$, $S_1, S_2 \in \Omega = \{0, 1, ..., 2^{64} - 1\}$.



Minwise Hashing: Apply a random permutation $\boxed{\pi : \Omega \longrightarrow \Omega}$ on S_1, S_2.

Then, the two minimums are equal with a probability R.

$$\mathbf{Pr}\left(\min(\pi(S_1)) = \min(\pi(S_2))\right) = \frac{|S_1 \cap S_2|}{|S_1 \cup S_2|} = R.$$

b-Bit Minwise Hashing (WWW'10)

Define: $z_1 = \min(\pi(S_1))$, $z_2 = \min(\pi(S_2))$. i.e., $\mathbf{Pr}(z_1 = z_2) = R$

Theorem:

$$\mathbf{Pr}\left(\text{Lowest b bits of } z_1 = \text{Lowest b bits of } z_2\right) = C_{1,b} + (1 - C_{2,b})\, R$$

$$C_{1,b} = A_{1,b}\frac{|S_2|}{|S_1|+|S_2|} + A_{2,b}\frac{|S_1|}{|S_1|+|S_2|}, \quad C_{2,b} = A_{1,b}\frac{|S_1|}{|S_1|+|S_2|} + A_{2,b}\frac{|S_2|}{|S_1|+|S_2|}.$$

$A_{1,b} = fun(|S_1|, b)$ and $A_{2,b} = fun(|S_2|, b)$ are pre-computed.

Consequence:

1. Most favorable case: using $b = 1$ achieves 64-fold improvement in storage.
2. Least favorable case: using $b = 1$ achieves 21.3-fold improvement in storage if resemblance $R \geq 0.5$, the common threshold used in practice.
3. Similar improvements in computation time.

b-Bit Minwise Hashing for Estimating Three-Way Similarities

Many problems are multi-way: Word associations/co-occurrences (e.g., search engine query "Machine, Learning, NIPS"); Learning beyond pairwise relations; Tensor algebra, ...

Three-Way Resemblance: $R_3 = \frac{|S_1 \cap S_2 \cap S_3|}{|S_1 \cup S_2 \cup S_3|}$

b-bit Hashing for estimating R_3

- The collision probability formula is extremely complicated.
- However, the probability formula is greatly simplified by using data sparsity.
- One must use $b \geq 2$ bits for R_3. (Using $b = 1$ leads to ∞ variance.)
- Still significant (e.g., 20-fold) improvement over using 64 bits.