

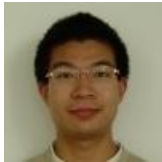
# Shadow Dirichlet for Restricted Probability Modeling



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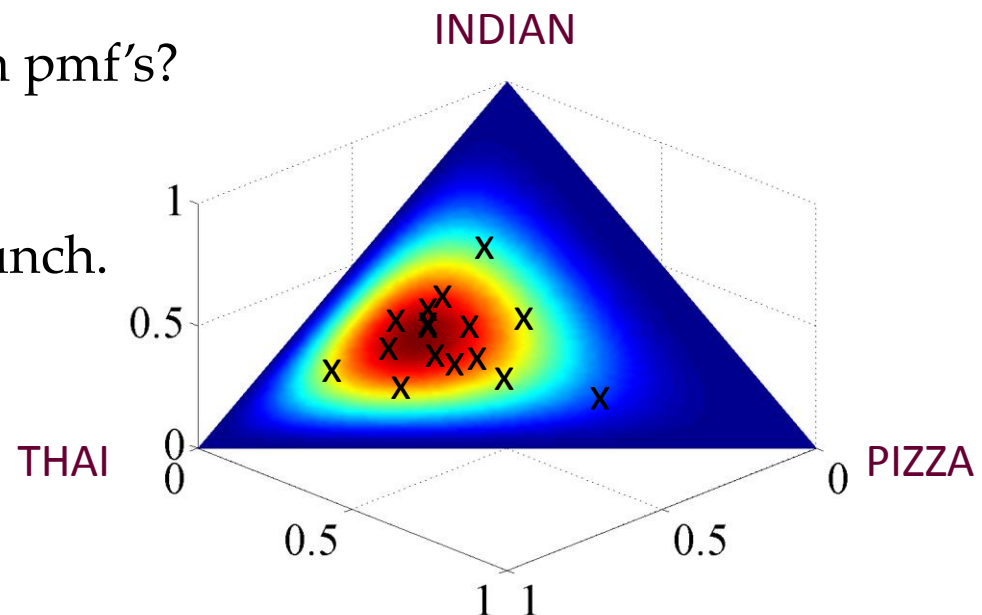
Yihua (James) Chen, Google

**Problem:** How do we model random pmf's?

**Ex:** people have different pmfs of choosing Indian, Thai, or Pizza for lunch.

**Common Model:**

Dirichlet distribution ( $\alpha$ ),  
where  $\alpha$  is the scaled mean pmf.

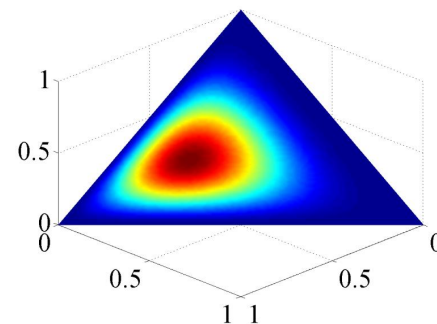


# Dirichlet vs Shadow Dirichlet

Dirichlet:

one parameter  $\alpha \in \mathbb{R}_+^d$

every pmf possible



Shadow Dirichlet:

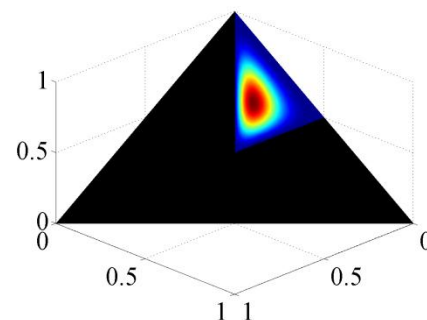
two parameters  $\alpha \in \mathbb{R}_+^d$  and left-stochastic  $M \in [0, 1]^{d \times d}$

new parameter  $M$  controls the support (which pmfs possible)

The random pmf  $\Theta \sim \text{shadow Dirichlet}(\alpha, M)$ ,

$$\text{if } \Theta = M\tilde{\Theta}$$

where  $\tilde{\Theta} \sim \text{Dirichlet}(\alpha)$ .



# Restricting the Support Can Be Used to Ensure:

Ordered probabilities: if  $\theta[i] \geq \theta[j]$

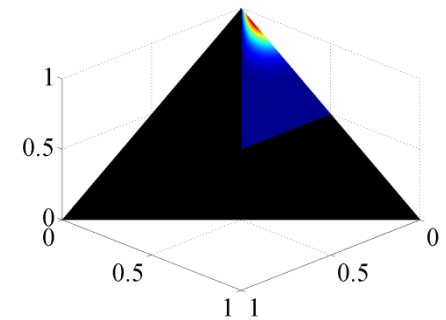
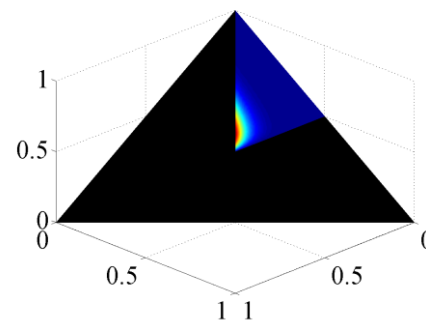
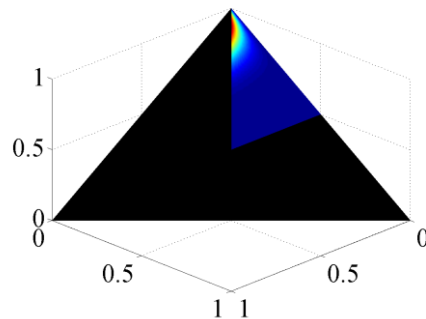
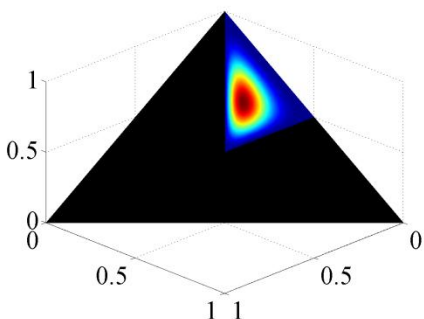
Certain Probabilities Must Be Close: if  $\|\theta[i] - \theta[j]\| < \epsilon$

Regularized pmfs:

if every realization must obey  $\theta = (1 - \lambda)q + \lambda q_0$  for fixed  $q_0$

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Example shadow Dirichlet distributions for  $\Theta[\text{Indian}] \geq \Theta[\text{Pizza}] \geq \Theta[\text{Thai}]$



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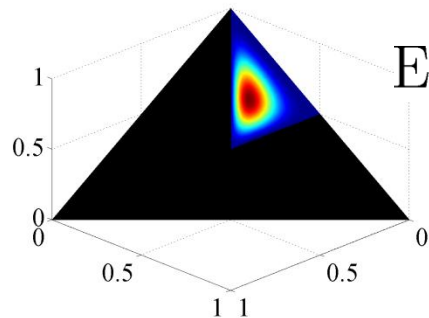
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Check out the paper/poster for:

Maximum entropy constructions of  $M$  given above constraints



EM algorithm for estimating parameters

Comparisons with Real Data

and more!