Distributed Dual Averaging in Networks

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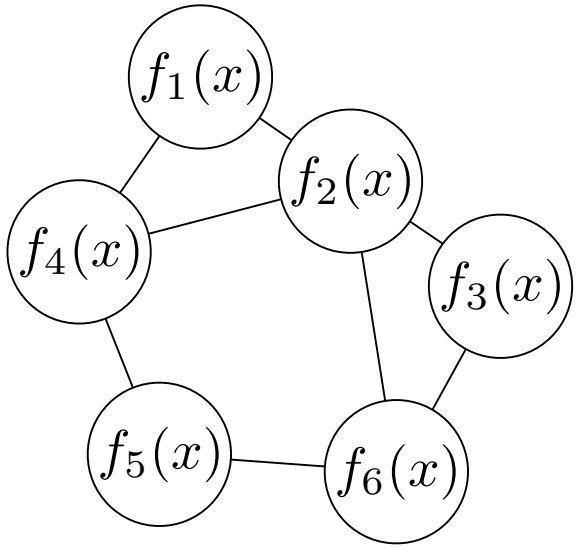


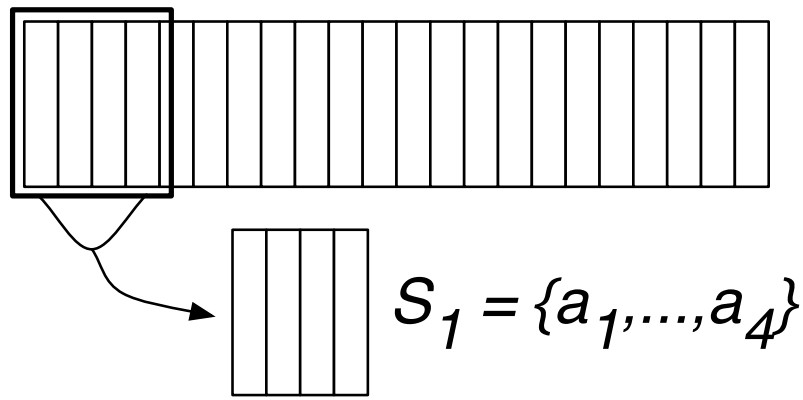
Setup: Function f_i at node i

Goal: Solve

$$\min_{x \in \mathcal{X}} f(x) := \frac{1}{n} \sum_{i=1}^{n} f_i(x)$$

Example: Data $\{a_j\}_{j=1}^N$, shard onto processors

$$f_i(x) = \sum_{j \in S_i} \ell(\langle x, a_j \rangle)$$



Distributed Dual Averaging Algorithm

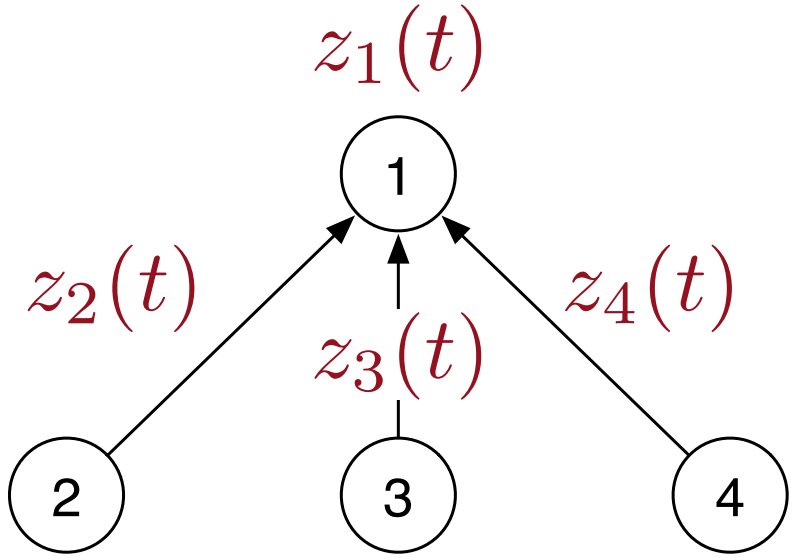
At node i: Primal $x_i(t) \in \mathcal{X}$, dual $z_i(t) \in \mathbb{R}^d$.

Dual Update:

$$z_i(t+1) = \sum_j P_{ji} z_j(t) + \nabla f_i(x_i(t))$$

Primal Update:

$$x_i(t+1) = \arg\min_{x \in \mathcal{X}} \left\{ \alpha \langle z_i(t+1), x \rangle + \frac{1}{2} \|x\|_2^2 \right\}.$$



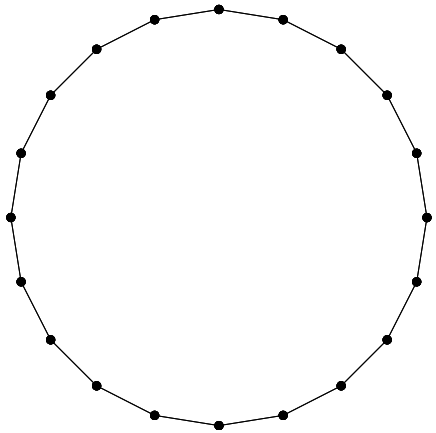
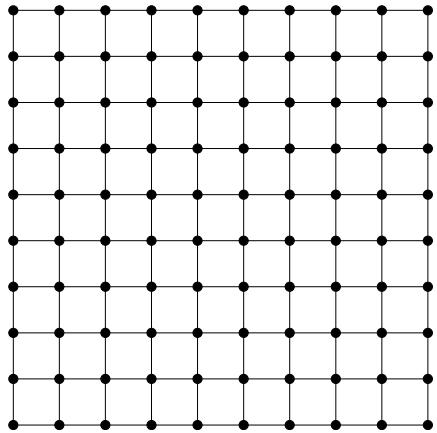
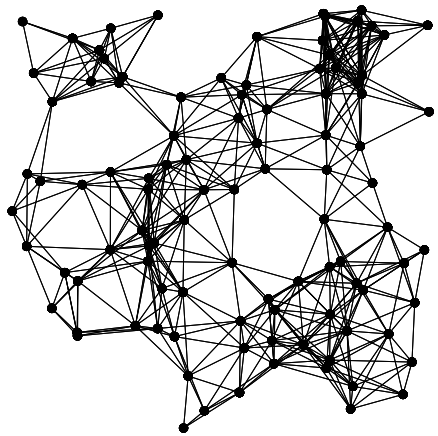
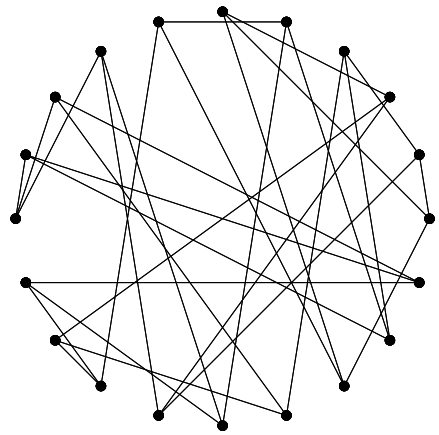
Main Results

Theorem: At each node i, optimization error at most

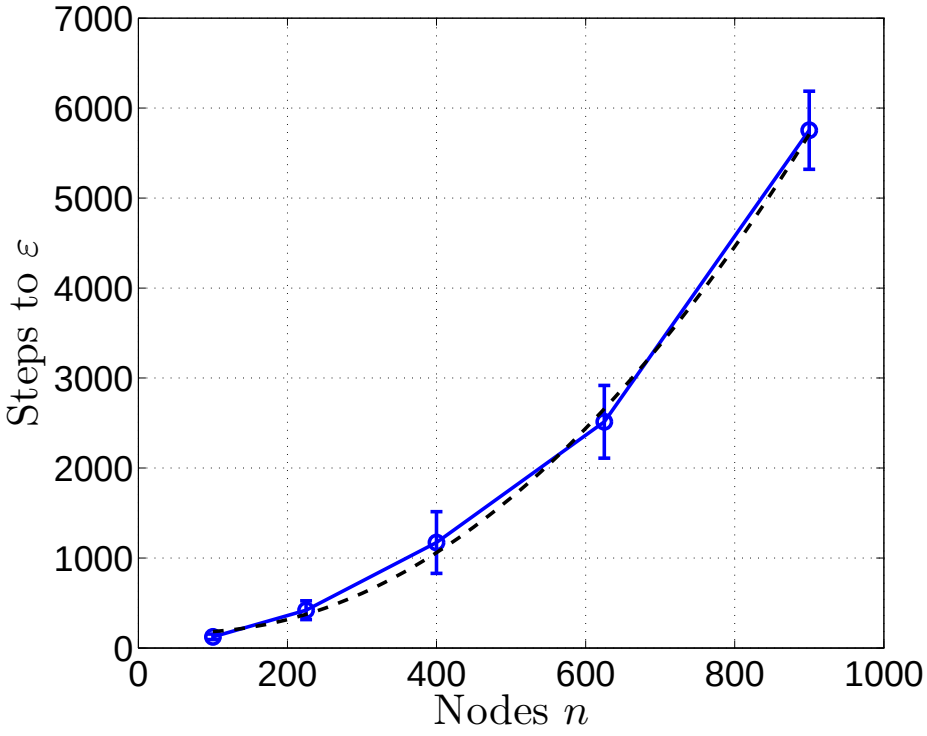
$$f(x_i(T)) - f(x^*) = \mathcal{O}\left(\frac{1}{\sqrt{T}\sqrt{1-\sigma_2(P)}}\right).$$

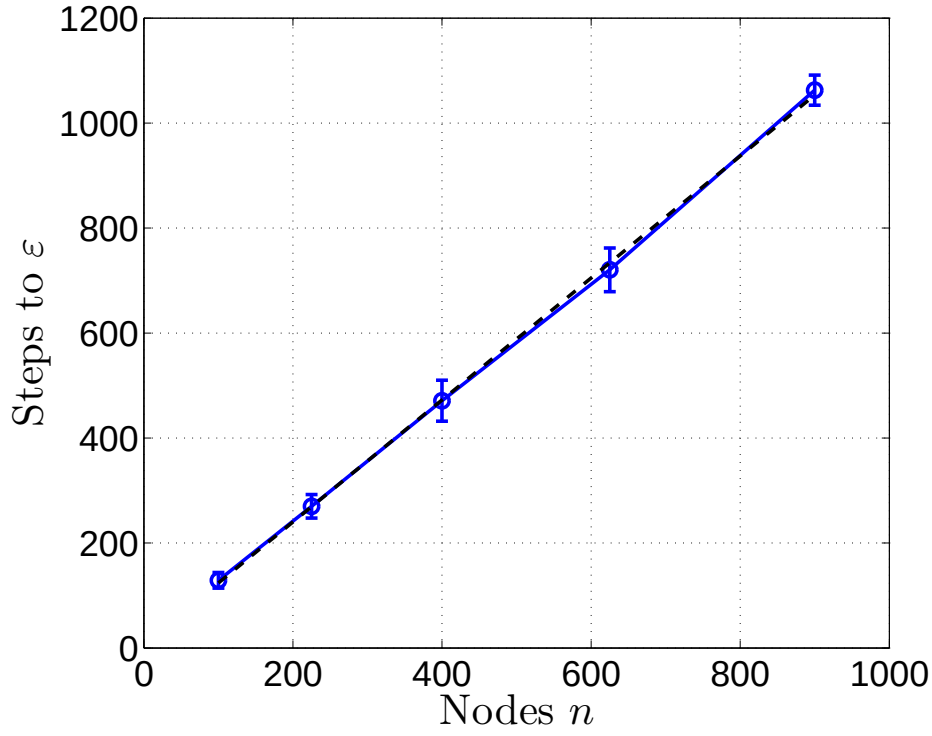
Dependence on convergence time of random walk on graph.

Examples:

Cycle	Grid	Geometric	Expander
			
$\frac{n}{\sqrt{T}}$	$\frac{\sqrt{n}}{\sqrt{T}}$	$\sqrt{\frac{n}{\log n}}\frac{1}{\sqrt{T}}$	$\frac{\sqrt{\log n}}{\sqrt{T}}$

Network scaling and extensions





- **Random Communication:** robustness to edge failures, high probability convergence even with randomness
- **Stochastic optimization:** streaming and online applications, see stochastic versions of each f_i
- **Simulations and lower bound:** match theoretical predictions