A decision procedure for $\mathcal{SHOIQ}$ with Transitive Closure of Roles

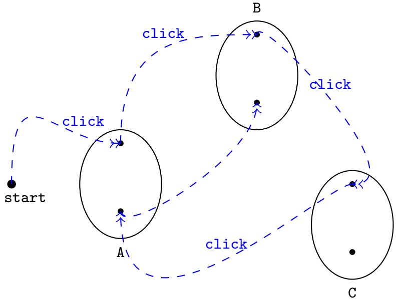
Chan LE DUC, Myriam LAMOLLE and Olivier CURE

Université Paris 8 - IUT de Montreuil, Université Marne La Vallée

International Semantic Web Conference, 2013

Why transitive closure of roles is needed ?

- Example 1 :



O_1 : Trans(reachable)

click $\sqsubseteq$ reachable

$A \sqcup B \sqcup C \sqsubseteq \exists \text{reachable}^{-}.\{\text{start}\}$

$C \sqsubseteq \forall \text{click}^{-}.\bot$

$\Rightarrow$ Consistent !

O_2 : $A \sqcup B \sqcup C \sqsubseteq \exists (\text{click}^{-})^{+}.\{\text{start}\}$

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$\Rightarrow$ Inconsistent : design error is detected

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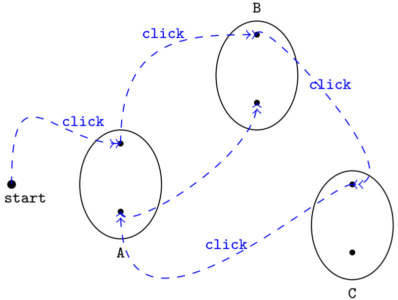

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- Example 2 :
Human $\sqsubseteq \exists \text{hasParent}^{+}.\{\text{Eva}\}$ versus
Human $\sqsubseteq \exists \text{hasAncestor}.\{\text{Eva}\}$ where “hasAncestor” transitive

$\mathcal{SHOIQ}$ and Transitive Closure of Roles

- The logic $\mathcal{SHOIQ}$:
  - Finite sets of concept, role and (nominals) individual names
  - Concept descriptions : $C \sqcap D, C \sqcup D, \neg C, \exists R.C, \forall R.C, \leq nS.C, \geq nS.C$ where C, D are concepts ; R is a role (possibly inverse and transitive) ; S is a simple role
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  - Concept axioms : $C \sqsubseteq D$ and role axioms : $R \sqsubseteq S$: ontology
- Transitive closure of roles : $(Q^+)^{\mathcal{I}} = \bigcup_{n>0} (Q^n)^{\mathcal{I}}$ with an interpretation $\mathcal{I}$
- In $\mathcal{SHOIQ}_{(+)}$ ($\mathcal{SHOIQ}$ with transitive closure), one can say : $\exists R^+.C$ or $\forall R^+.C$ but not

  $\leq nS^+.C$ [Horrocks, Sattler and Tobies, 1999], or
  $R \sqsubseteq S^+$ [Le Duc and Lamolle, 2010]

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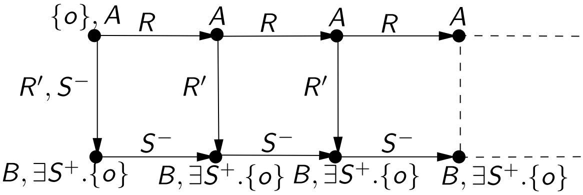
- $\mathcal{SHOIQ}$ has a forest-like model whose the infinite part is tree-like [Horrocks and Sattler, 2005]
- $\mathcal{SHIQ}_{(+)}$ has a tree-like model [Le Duc, Lamolle and Curé, 2011]
- There exists a consistent ontology in $\mathcal{SHOIQ}_{(+)}$ whose all models are non-tree-like
  $\{o\} \sqsubseteq \mathsf{A}$; $\mathsf{A} \sqcap \mathsf{B} \sqsubseteq \bot$; $\mathsf{A} \sqsubseteq \exists \mathsf{R}.\mathsf{A} \sqcap \exists \mathsf{R}'.\mathsf{B}$; $\mathsf{B} \sqsubseteq \exists \mathsf{S}^+.\{o\}$
  $\{o\} \sqsubseteq \forall \mathsf{X}^-.\bot$; X is functional with $\mathsf{X} \in \{\mathsf{R}, \mathsf{R}', \mathsf{S}\}$



Overview of the algorithm

- Goal : constructing a model of a $\mathcal{SHOIQ}_{(+)}$ ontology
- Key structures :
  - A star-type for representing a set of individuals
  - A frame and sections for representing a model
  - A new blocking condition based on sections
- Algorithm :
  Tiling a valid frame with star-types until a blocked section is detected

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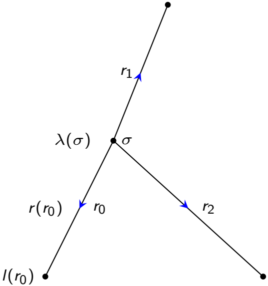
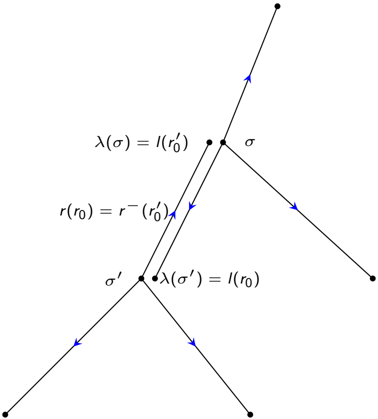
Star-type



Star-types and linking



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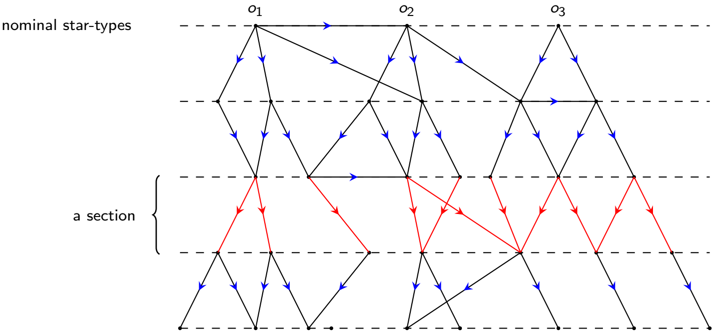
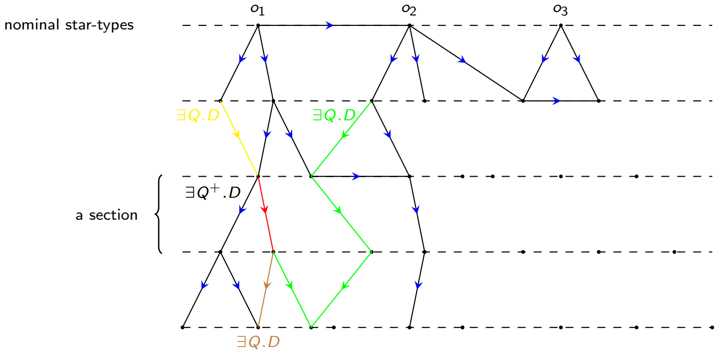
Frame and sections



Frame and sections (2)



With sections, one can say that a concept $\exists Q^+.C$ is satisfied :

- in the past
- in the future
- somewhere from the future
- somewhere from the past

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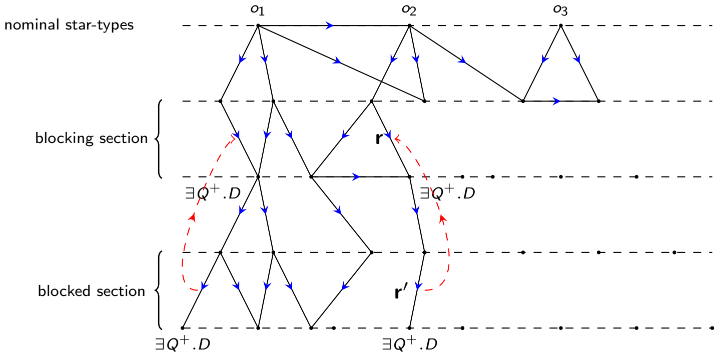
Blocking condition



- Each ray $\mathbf{r}$ in the blocking (blocked) section blocks (is blocked by) a ray $\mathbf{r}'$ in the blocked (blocking) section such that (i) $\mathcal{L}(\mathbf{r}) = \mathcal{L}(\mathbf{r}')$, and (ii) each $\exists Q^+.C$ in both $\mathbf{r}$ and $\mathbf{r}'$ is satisfied in "the same way"
- Each concept $\exists Q^+.C$ in the blocking section is satisfied

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Termination of the algorithm

Termination is a consequence of the following facts :

- The algorithm never removes a star-type
- The number of sections from nominal to blocked one is bounded by $\mathcal{O}(2^{2^{|(\mathcal{T},\mathcal{R})|}})$
- Checking satisfaction of a concept $\exists Q^+.D$ over a frame is bounded by a polynomial function in the size of the frame

Soundness : from a valid frame to a model

- A frame is valid if
  - all star-types are valid
  - each nominal star-type is not duplicated
  - each concept $\exists Q^+.C$,
    - either it is directly satisfied in the frame
    - or there is a Q-sequence leading to the blocked section containing a $\exists Q^+.C$

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    - or there is a Q-sequence leading to the blocked section containing a $\exists Q^+.C$
- Adapting the unravelling technique for frames :
  - defining a set of paths over the frame
  - extending infinitely the set of defined paths through blocked and blocking sections
  - satisfying a concept $\exists Q^+.C$ (extended path) by a Q-sequence leading to an extended path or to an initial path containing $\exists Q.C$

Completeness : from a model to a valid frame

Main ideas :

- A model can be reduced to a frame with valid star-types :

  The first section contains only nominal star-types
  It contains a section whose all concepts $\exists Q^{+}.D$ are satisfied (blocking section)
  Parts between two "blockable" sections can be removed until a blocked section is detected

- The reduced model can guide the algorithm to build a valid frame.

Conclusion and Future Work

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  - A first decision procedure for $\mathcal{SHOIQ}_{(+)}$
  - A structure, namely frame, with a new blocking condition for representing infinite non-tree-like parts of a model
  - The complexity of the algorithm is high (triply nondeterministic exponential)

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  - A first decision procedure for $\mathcal{SHOIQ}_{(+)}$
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  - The complexity of the algorithm is high (triply nondeterministic exponential)
- Future work :
  - Reducing the size of frames
  - A more goal-oriented algorithm (tableau algorithm)
  - An implementation in progress
  - Hardness of $\mathcal{SHOIQ}_{(+)}$
  - The technique could be used for other logics ($\mathcal{ZOIQ}$?)