The dynamics of opinion formation in social networks

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What is an opinion?
Opinion

- Is a subjective belief, and is the result of emotion or interpretation of facts
  [Wikipedia]
prevent global warming
reduce military spending
fight poverty
People living in a society are influencing each other

- behavior
- opinions
- decisions they make
- products they buy
- clothes they wear
Opinions change with time
- opinions assume a continuous range of values
- opinions are influenced by our social environment
- opinions change over time
Repeated averaging [DeGroot’74, DeMarzo et al. ’03, Golub&Jackson’10, Jackson’08 ]

- Opinion modeled as a value in $[0, 1]$.
- Every person $i$ has an opinion $z_i$.
- Opinion gets updated at every step

$$z_i = \frac{z_i + \sum_{j \in N(i)} w_{ij} z_j}{1 + \sum_{j \in N(i)} w_{ij}}$$

- Beliefs converge to a consensus.
Do we always say our true opinion?
your loyalty to Red Sox?
Predisposition and expressed opinions

- opinion modeled as a value in $[0, 1]$
- person $i$ has
  - predisposition $S_i$
  - expressed opinion $Z_i$
  - personal cost expressing conflict

$$c(z_i) = (z_i - s_i)^2 + \sum_{j \in N(i)} w_{ij} (z_i - z_j)^2$$
Forming expressed opinions

- rational agents minimizing their costs

\[ c(z_i) = (z_i - s_i)^2 + \sum_{j \in N(i)} w_{ij} (z_i - z_j)^2 \]

gives

\[ z_i = \frac{s_i + \sum_{j \in N(i)} w_{ij} z_j}{1 + \sum_{j \in N(i)} w_{ij}} \]
Interpretation of opinion $\mathcal{Z}_i$

- value in the equilibrium state of the spring model
Opinion formation as a game [Bindel et al. 11]

- Nash optimum: \( z_i \) that optimizes

\[
c(z_i) = (z_i - s_i)^2 + \sum_{j \in N(i)} w_{ij} (z_i - z_j)^2
\]

- Social optimum: \( y_i \) that optimizes

\[
c(y) = \sum_{i \in V} c(y_i)
\]

price of anarchy = \( \frac{9}{8} \)
Can we influence social opinions?
The opinion-maximization problem

[Gionis, T. ,Tsaparas 2013]

- In a setting of constantly changing opinions
- select \( k \) initial nodes to convince 100% about your idea
- to maximize the overall positive opinion of the crowd on this idea
DON’T BUY
THIS JACKET

It’s Black Friday, the day in the year retail turns from red to black and starts to make real money. But Black Friday, and the culture of consumption it reflects, puts the economy of natural systems that support all life firmly in the red. We’re now using the resources of one-and-a-half planets on our one and a half

Because Patagonia wants to be in business for a good long time – and leave a world inhabitable for our kids – we want to do the opposite of every other business today. We ask you to buy less and to reflect before bankruptcy, can happen very slowly, then all of a sudden. This is what we face unless we slow down, then reverse the damage. We’re running short on fresh water, topsoil, fisheries, wetlands – all our planet’s natural systems and resources that support the environment.

The environmental cost of everything we make is

Jacket shown, one of our best sellers. To make it required 135 liters of

REDUCE
WE make useful gear that lasts a long time
YOU don’t buy what you don’t need

REPAIR
WE help you repair your Patagonia gear
YOU pledge to fix what’s broken

REUSE
WE help find a home for Patagonia gear you no longer need
YOU sell or pass it on*

RECYCLE
WE will take back your Patagonia gear that is worn out
YOU pledge to keep your stuff out of the landfill and incinerator

REIMAGINE
TOGETHER we reimagine a world where we take only what nature can replace

*Under our Common Threads Initiative.
The opinion-maximization problem (formally)

- find $k$ users to set $z_i = 1$
- maximize the overall expressed opinion (or average opinion)

$$g(z) = \sum_{i \in V} z_i$$
Random walks with absorbing states

- **Graph:** $H = (X, R)$
- **Absorbing states:** $A \subseteq X$
- **Transient states:** $U = X \setminus A$
- **Transition matrix:** $P = \begin{pmatrix} P_{UB} & P_{UU} \\ I & O \end{pmatrix}$
- **Fundamental matrix**
  \[ F = \sum_{\ell=0}^{\infty} (P_{UU})^\ell = (1 - P_{UU})^{-1} \]
- **Absorption matrix**
  \[ Q_{UB} = F P_{UB} \]
Random walks with absorbing states

- Value $b_j$ for $j \in B$ (absorbing state)

- Expected value of $i \in U$

  $$f_i = \sum_{j \in B} Q_{UB}(i, j) b_j$$

- Or,

  $$f = Q_{UB} b$$
Random-walk interpretation of $Z_i$

- Absorbing states (B): true selves
- Transient states (U): expressed selves
- Absorption matrix: $Q$

- Expressed opinions

$$z = Qs$$
Absorbing random walks: a tool for studying opinion-maximization
Properties of the opinion-maximization problem

- **NP-hard**
- function \( g(z) = \sum_{i \in V} z_i \) is monotone and submodular
- **GREEDY** algorithm is an \((1 - 1/e)\) approximation algorithm

\[
g(z_{\text{GREEDY}}) \geq \left(1 - \frac{1}{e}\right) g(z^*)
\]
Example: monotonicity

- objective \( g(z) = \sum_{i \in V} z_i \)
- where \( z_i = \sum_{j \in B} Q(i, j) b_j \)
Algorithms

- **GREEDY** (matrix inversion vs power iteration)

Karate club

- **DEGREE**
- **MINS**
- **RWR**
On a larger dataset

Bibsonomy — data mining

- RWR
- free.degree
- degree
- min.s
- min.z

On a larger dataset, the campaign manager should aim to alter the information formation process. To this end, it seems reasonable to propagate the endorsement of the information item in question throughout the network through the opinion network, with the ultimate goal of propagating this positive opinion.

The difference between the problems is seemingly small, the problems we are asking to fix the internal opinions expressed opinions of individuals such that the overall opinion would be convinced to change their free. degree is improved as much as possible. Formally, the goal is to such that the resulting overall opinion of individuals such that the overall opinion is improved as much as possible. Formally, the goal is to such that the resulting overall opinion of individuals such that the overall opinion is improved as much as possible.

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Opinion-maximization variants
i-opinion maximization

- Find nodes to set \( s_i = 1 \)

- Can be solved in polynomial time
i-maximization: remarks

- Property of undirected graphs

\[ g(z) = \sum_{i=1}^{n} z_i = \sum_{i=1}^{n} s_i \]

- Find the nodes with the highest $s_i$ values and set them to 1
Opinion maximization

vs

Influence maximization
Influence maximization [Kempe ’03...]

- select $k$ initial adopters in a social network

- to maximize the spread of adoption of a product / action / ...
Both problems

- try to identify influential nodes
When influential nodes

- buy products or adopt opinions.....
- others follow them
Influence maximization

- is about products/actions
Actions in social networks (re-tweets)
independent-cascade model
Opinion maximization

- is about opinions
- end-result is less binary and more continuous
Opinion maximization

- is about opinions
- end-result is less binary and more continuous
- is about rational nodes
Selection of *rational* agents

- opinion maximization
- influence maximization
- team formation
- .....
Thank you!