Honest Compressions and Their Application to Compression Schemes

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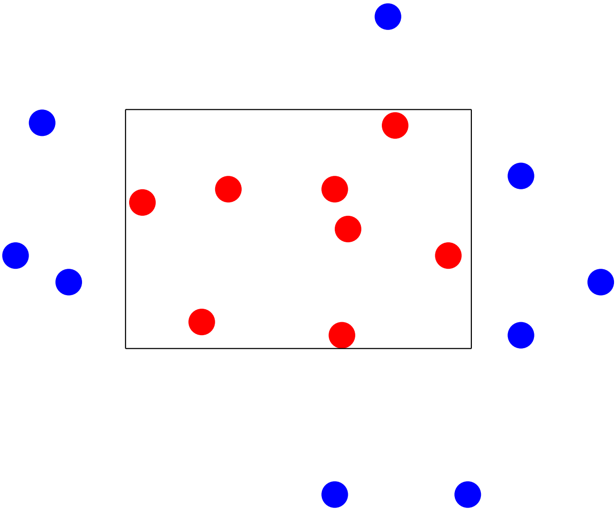

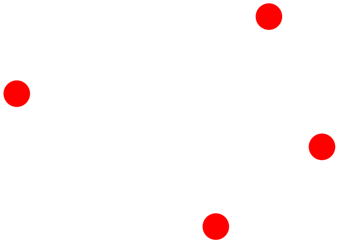

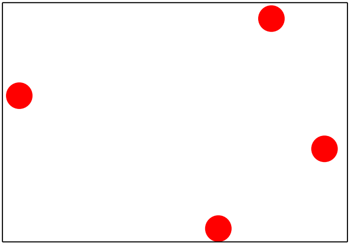

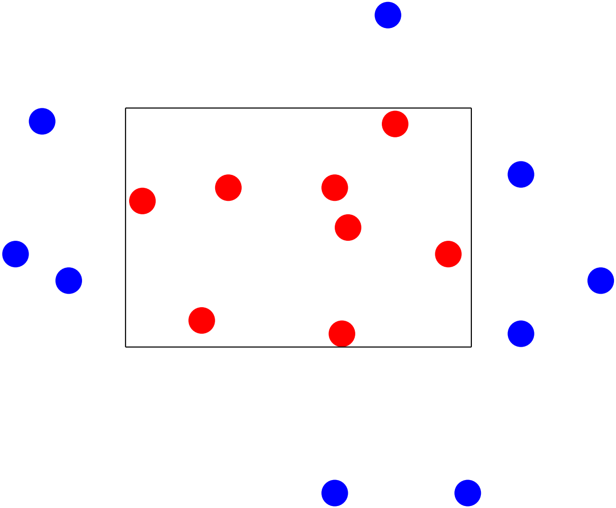

Let $(X, \mathcal{C})$ be a concept class with VC-dimesnion d. There exists an $O(d)$-compression scheme.

?

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Let $(X, \mathcal{C})$ be a concept class with VC-dimesnion d. There exists an $O(2^d)$-compression scheme.

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Let $(X, \mathcal{C})$ be a concept class with VC-dimesnion d. There exists an $O(f(d))$-compression scheme.

?

For every concept class $\mathcal{C}$ we associate a family of concept classes: $\mathcal{ALT}_m(\mathcal{C})$.

We don't need model theory to *state* our statement.
We don't need model theory to *prove* our statement.

An $\langle M; (R_i), (f_j), (c_k)\rangle$-structure

- A set M.
- Relations (R_i) over M.
- Functions (f_j) whose domain and image are M.
- Distinguished elements (c_k) of M referred to as *constants*.

$$\varphi(\boldsymbol{x};\boldsymbol{y}) :=$$

$$(\exists z_1, z_2)(z_1 = \times(x_1, y_1) \wedge z_2 = \times(x_2, y_2) \wedge (\exists z_3)(z_3 = +(z_1, z_2)) \wedge \geq (z_3, 0)).$$

Says "The scalar product of $\boldsymbol{x}$ and $\boldsymbol{y}$ is positive".

$$\phi(M;\boldsymbol{c}) = \{\boldsymbol{a} \in M^{|\boldsymbol{x}|} : M \models \phi(\boldsymbol{a},\boldsymbol{c})\}.$$

This is a definable concept.

If $\text{VC-dim}(\{\varphi(M, \boldsymbol{c}); \boldsymbol{c} \in M^{\boldsymbol{c}}\}) < \infty$, we say $\varphi(\boldsymbol{x}, \boldsymbol{y})$ has NIP.

Let $\phi(\boldsymbol{x};\boldsymbol{y})$ be an NIP-formula. There exists a formula $\psi(\boldsymbol{x};\boldsymbol{z})$ such that for any finite subset $A \subset M$ and tuple $\boldsymbol{c} \in M$, there is some $\boldsymbol{d} \in A^{|\boldsymbol{z}|}$ such that $\phi(A;\boldsymbol{c}) = \psi(A;\boldsymbol{d})$.

?

A definable compression scheme for definable concept classes:

Given A and $\boldsymbol{c} \in M$ return $\boldsymbol{d} \in A^{|\boldsymbol{z}|}$ such that $\phi(A; \boldsymbol{c}) = \psi(A; \boldsymbol{d})$.

For any concept class $(X, \mathcal{C})$:
Let $M = X \cup \mathcal{C}$ and let $R(x, C)$ hold whenever $x \in C$.

Let M be an NIP structure. The UDTFS holds for all formulas over M.

$$\langle \mathbb{R}, \leq, +, \cdot, 0, 1 \rangle$$

Ben-David and Litman (1995)

$$\langle \mathbb{R}, \leq, \exp, +, \cdot, 0, 1 \rangle$$

Johnson and Laskowski (2010)

$\langle \mathbb{Q}_p, |, +, \cdot, 0, 1 \rangle$

Guingona (2012)

$\langle \mathbb{Z}[X], \prec, +, 0 \rangle$
Chernikov and Simon (2012)

VC classes were introduced in 1971 by Vapnik and Chervonenkis

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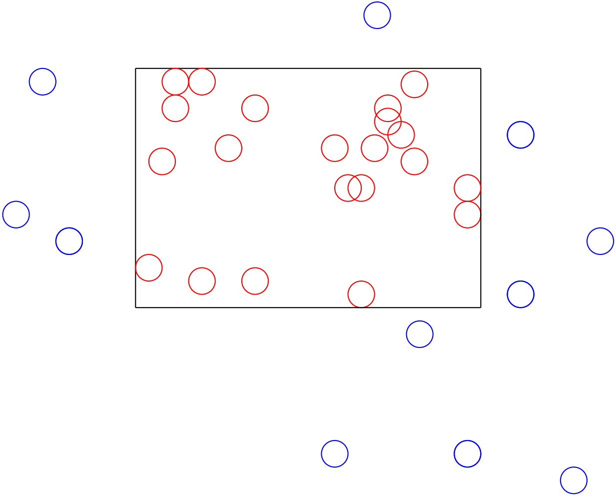
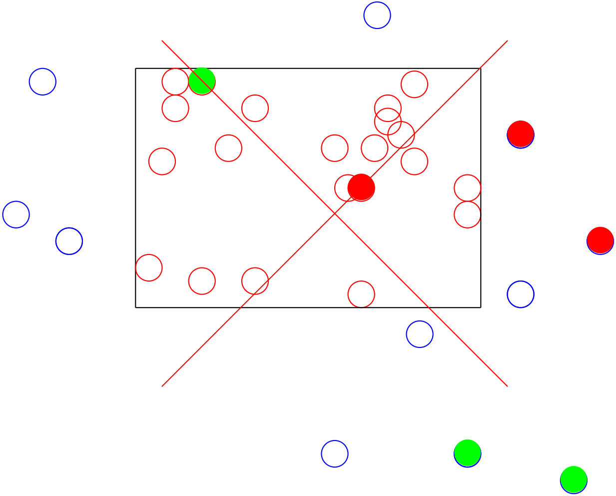
$\varphi(x, \boldsymbol{y})$ says:

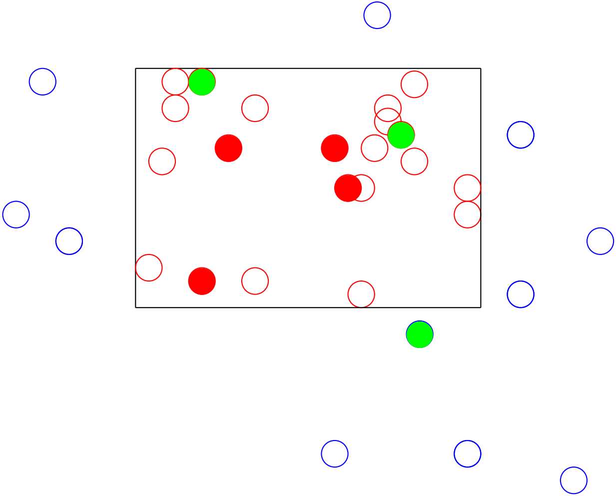
”The sequence $< x, \boldsymbol{y} >$ doesn’t have a subsequence of size $2d + 1$ with alternating labels for some concept in $\mathcal{C}$”

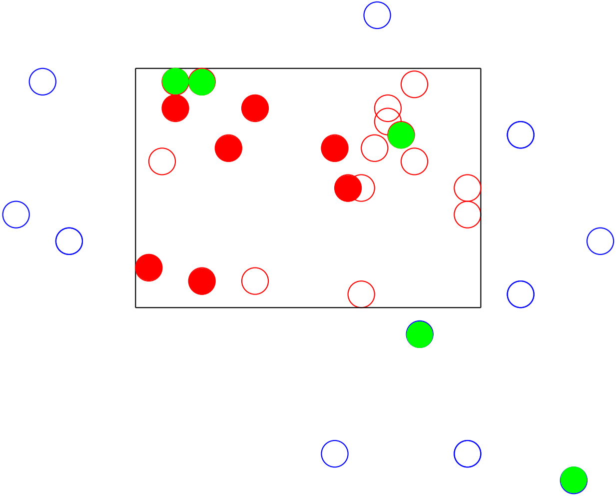
$$\mathcal{ALT}_m(\mathcal{C}) := \{\varphi(X, \bar{\boldsymbol{y}}) : \bar{\boldsymbol{y}} \in X^m\}$$

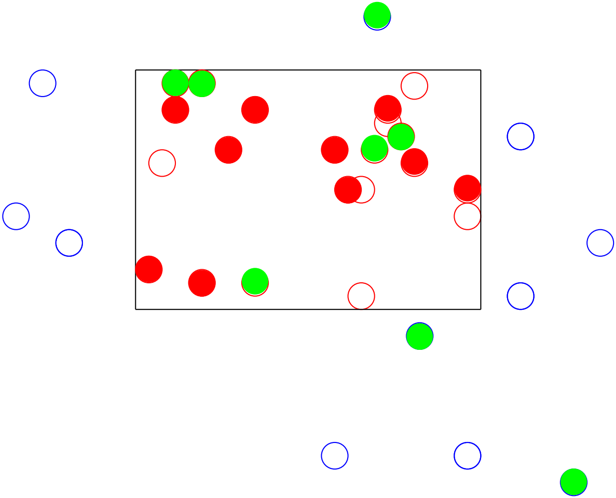
The alternating concept class with respect to sequences of length m from X:

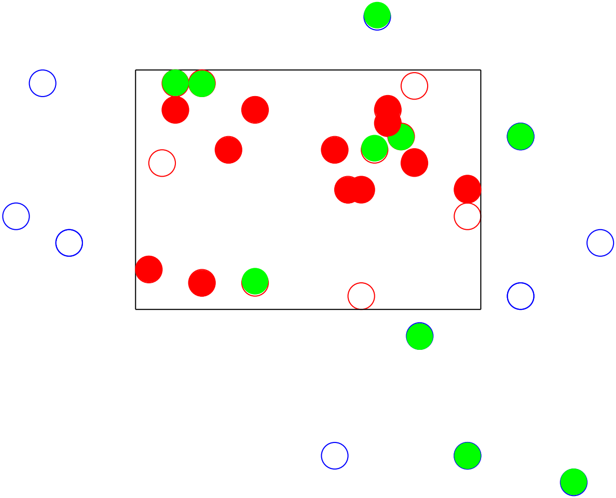


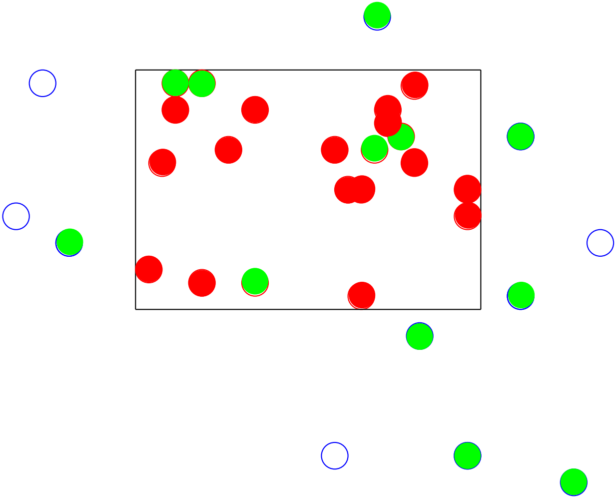












Let $\mathcal{C}$ be a concept class with VC-dimension $d < k$. There exists an $N(d)$ such that if $A \subseteq X$ has the (p, k) property, it can be covered by N concepts in $\mathcal{C}$.

- Input: sequence $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N$ and a point to be labeled x

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- Output: label

If UDTFS: What can be said on $\psi(\boldsymbol{x}, \boldsymbol{y})$?

Do we need finite VC assumption?

(p,k) Theorem for convex sets (Alon and Kleitman (1992)).

Are we going to let model theorists take our jobs???

http://www.normalesup.org/~simon/