Modeling Diffusion in Social Networks using Network Properties

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Why Diffusion in Social Networks?

Diffusion of items (videos, news, photos, etc) is important and ubiquitous in social networks.

Proper models of diffusion can predict:

- Rate of adoption at a particular time
- The time of peak demand
- The magnitude of peak demand

Applications of Diffusion Models in Telecommunications, Nigel Meade

Why Modeling Diffusion using Network Properties?

For item diffusion we have micro and macro models.

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\(^1\) Goldenberg et al. (2001) *Talk of the Network: A Complex Systems Look at the Underlying Process of Word-of-Mouth*

\(^2\) Granovetter, M. (1978) *Threshold Models of Collective Behavior*

\(^3\) Bass, F. M. (1969) *A new Product Growth Model for Consumer Durables*
Research Questions

How can macro models exploit network properties (e.g. degree distribution)?

In this work:
Q1) How to model diffusion in a network given its degree distribution?
Q2) How to combine parameters of diffusion and of degree distribution to give a better model?
Concepts & Notations

- $N$: network size.
- $f(t)$: instantaneous fraction of adopters at time $t$
- $F(t)$: cumulative fraction of adopters at time $t$
  
  \[ f(t) = F'(t) \]

- $a_t$: adoption at $t$, $A_t$: adoption before $t$.

Observe that:

\[
\frac{f(t)}{F(t)} = (1 - F(t)) \cdot \mathbb{E}[P(a_t | \overline{A_t})]
\]

Ordinary differential equation (ODE) linking $F(.)$, $P(.)$:

\[
F'(t) = (1 - F(t)) \cdot P(a_t | \overline{A_t})
\]

Goal: estimate the adoption probability $P(.)$ as a function of $F(.)$
Estimation of Adoption Probability

General Case

Contributions from external & internal influences are weighted with $w_e$ and $1-w_e$ respectively.

$$P(a_t | \overline{A}_t) = w_e \cdot P_{ext}(a_t | \overline{A}_t) + (1-w_e) \cdot P_{int}(a_t | \overline{A}_t)$$

Internal comes from WOM (word of mouth)
Bass Model (BM)

Assumptions of BM:

B1) Each user can influence every other user.

\[ |u's \text{ adopted neighbors} | = N \cdot F(t), \quad \forall u, \forall t \]

B2) Internal influence is proportional to No. of adopted neighbors:

\[ P_{\text{int}}(a_t | \overline{A_t}) = q_1 \cdot N \cdot F(t) \]

\[ \Rightarrow P(a_t | \overline{A_t}) = p + q \cdot F(t) \quad (*) \]

where \( p = w_e \cdot p_e \) and \( q = (1 - w_e) \cdot q_1 \cdot N \)

(*) combines with the ODE

\[ F(t) = \frac{e^{[(p+q)t]} - 1}{e^{[(p+q)t]} + q / p} \]

\[ f(t) = F'(t) = \frac{(p+q)^2}{p} \cdot e^{[(p+q)t]} \cdot \left\{ e^{[(p+q)t]} + q / p \right\}^2 \]

\[ \rightarrow \text{Bass Model (1969)} \]
Bass Model (cont.)

“...Bass model ignores the network structure...”
[Xiaodan S. et al, WWW 07]

B1) Each user can influence/influenced every other user

Each user can influence/influenced only his friends
→his adoption prob. depends on his degree
Bass Model (cont.)

http://serial.innovatiasystems.eu/cms/
Adoption Probability for Specific Degree Distributions

• Given any degree distribution $P(k)$, we obtained the formula:

\[
P_{\text{int}}(a_t | A_t) = \sum_{k=1}^{N-1} P(k) \sum_{j=0}^{k} \binom{k}{j} F_t^j (1 - F_t)^{k-j} P(a_t | A_t, j)
\]

where $F_t \equiv F(t)$ and $P(a_t | A_t, j)$ is the prob of adopting given that a user has $j$ adopted neighbors.

• Still keep B2), linear influence: \[P(a_t | A_t, j) = c \cdot j\]

where $c$ is a constant.

To complete estimation, needs specific degree distributions!
Specific Degree Distributions

Power-law (scale free): \( P_{sf}(k) = \frac{1}{Z_{sf}} \cdot k^{-\alpha} = \frac{1}{\zeta(\alpha)} \cdot k^{-\alpha} \)

Exponential: \( P_{exp}(k) = \frac{1}{Z_{exp}} \cdot e^{-k/\lambda} = \left(1 - e^{-1/\lambda}\right) \cdot e^{-k/\lambda} \)

Pagel et al. BMC Evolutionary Biology
2007 7(Suppl 1):S16

Parameter of degree distribution: \( \alpha \) (power law) or \( \lambda \) (exponential)
Estimation of Internal Adoption Probability

Linear assumption & specific degree distribution provide estimations:

1) Scale free network:

\[ P_{int}^{sf} (a_t \mid \overline{A}_t) = \frac{\zeta(\alpha - 1)}{\zeta(\alpha)} \cdot c \cdot F_t \]

where \( \zeta(\alpha) = \sum_{k=1}^{\infty} k^{-\alpha} \) is the Riemann Zeta function.

2) Exponential network:

\[ P_{int}^{exp} (a_t \mid \overline{A}_t) = \frac{e^{-1/\lambda}}{1 - e^{-1/\lambda}} \cdot c \cdot F_t \]

These estimations \( \rightarrow \) two models in our work.
Proposed Models

1. SLIM (Scale-free Linear Influence Model): Scale-free network.
2. ELIM (Exponential Linear Influence Model): Exponential network

Remarks:
- Give more rigorous estimation of adoption probability by combining parameters of diffusion and of degree distribution.
- Give the same fitting error as BM though!
What is the problem?

\[ f(t) = (1 - F(t)) \cdot \mathbb{E}[P(a_t | A_t)] \]

- Is it correct to use degree distribution of the whole network for \( P(k) \)?
  
  **NO.** Should use degree distribution over the set of non-adopters (NA).

- NA changes over time → its degree distribution also changes?
  
  **YES.**
Degree Distribution is Dynamic!!

Evolving Degree distribution (among non adopters)

Log(P(degree)) vs Log(degree)

Time 10 - Time 40

Synthetic scale-free network; 27,289 nodes and 27,031 edges ($\alpha_0=3$).

As time proceed, users with high/low degs are more/less likely to adopt and leave/stay NA set. Thus later distributions are more biased to low degrees.
Multi-Stage Model (MLIM)

For different time pts, need to employ different models. How to decide the proper model?

Heuristic approach: in a short duration, degree distribution does NOT significantly change.

- Divide diffusion process into $n$ stages. Each has short duration ($<10$ time steps).
- For each stage, choose between SLIM and ELIM the one that gives smaller fitting error.

→ Multi-Stage Model
Experiments on Synthetic Data

- Network: 28,172 nodes; 34,578 edges ($\alpha=2.5$).
- Evaluation metrics: model-fitting error (LSE) & parameter-learning error.

(a) $w_e$ in $\{0.05, 0.1, 0.2, 0.3, 0.4\}$

(b) $c$ in $\{0.032, 0.064, 0.096, 0.12\}$
Experiments on Synthetic Data

\( n=1 \) corresponds to BM

\((c) \, n \in \{1, 4, 5, 8, 10\}\)
Real-world Dataset
From Goodreads network (www.goodreads.com), \( \approx 87 \)K users; 159,442 follow links

You have no updates from your friends yet.

Recent Updates From the Community

05/15

Alice gave ★★★★★☆ to:
Pudd'nhead Wilson and Other Tales (World's Classics)
by Mark Twain

Alice said: "I really didn't think I would enjoy this book....why did I ever think that?! AMAZING!!!! loved it."

5 minutes ago  • comment  • see review
Experiment Design

• Adopting a book ≈ writing review on it.
• Review data was collected for 73 popular books.
• Period: 05/2007 to 02/2011 (45 months).
• Filter out books with review data spans < 30 months → 20 books remain (Harry Potter 7, Breaking Dawn, …).

• Evaluation metric: ratio of model-fitting errors (LSE of MLIM over LSE of BM)
  → less than 1 shows improvement of our model.

Special thanks to Agus and Anh.T.H
Results for Top-20 Popular Books

- MLIM outperforms BM for all top-20 popular books.
- 75% of books have error ratios less than $\frac{1}{2}$. 
Zoom-in for One Book

Result for book with ID=30183

Fitting result for *City of Ashes* by Cassandra Clare

MLIM provides **significant** improvement over BM in terms of fitting data.
Conclusion

• This work:
  – Proposed two models SLIM, ELIM for diffusion in scale-free and exponential networks respectively.
  – Proposed multi-stage model (MLIM) to deal with dynamic degree distribution.

• Future works:
  – Derive a more rigorous way to deal with dynamic degree distribution.
  – Replace linear influence by other (e.g. quadratic, exponential) influence?
  – Examine the effect of other network quantities on diffusion.
Adoption prob. for scale free and exponential network

\[
P_{sf}(a_t | \overline{A}_t) = w_e \cdot p_e + (1 - w_e) \cdot \frac{\zeta(\alpha - 1)}{\zeta(\alpha)} \cdot c \cdot F_t
\]

\[
P_{exp}(a_t | \overline{A}_t) = w_e \cdot p_e + (1 - w_e) \cdot \frac{e^{-1/\lambda}}{1 - e^{-1/\lambda}} \cdot c \cdot F_t
\]
Formulae of SLIM, ELIM

\[ F_{SLIM}(t) = \frac{\exp[(p + q_{SLIM}) \cdot t] - 1}{\exp[(p + q_{SLIM}) \cdot t] + (q_{SLIM} / p)} \]

where \( p = p_e \cdot w_e \) and \( q_{SLIM} = (1 - w_e) \cdot \frac{\zeta(\alpha - 1)}{\zeta(\alpha)} \cdot c \)

\[ F_{ELIM}(t) = \frac{\exp[(p + q_{ELIM}) \cdot t] - 1}{\exp[(p + q_{ELIM}) \cdot t] + (q_{ELIM} / p)} \]

where \( p = p_e \cdot w_e \) and \( q_{ELIM} = (1 - w_e) \cdot \frac{e^{-1/\lambda}}{1 - e^{-1/\lambda}} \cdot c \)