Minimax Localization of Structural Information in Large Noisy Matrices

Poster: W055

M. Kolar

S. Balakrishnan  A. Rinaldo  A. Singh
Identifying biclusters

**Goal:** De-noise and re-order rows/columns of the matrix to infer biclusters that are activated.

Observation model

\[
A = \beta uv^T + R \\
\text{u} - k_1 \text{ sparse unit vector} \\
\text{v} - k_2 \text{ sparse unit vector} \\
\text{u, v} \sim \{-1, 0, 1\} \\
R \sim \text{i.i.d. zero-mean subgaussian(\(\sigma^2\)) perturbation}
\]
Identifying biclusters

Information Theoretic minimax limit: If

\[
\frac{\beta}{\sigma} \sim \sqrt{\frac{k_1 k_2 \log(n_1 n_2)}{\min(k_1, k_2)}}
\]

then, for any biclustering procedure, the probability of failure remains bounded away from zero by a constant.

Note:

Optimal performance achieved by scanning over all submatrices of size \(k_1 \times k_2\).
Computationally efficient procedures

Elementwise thresholding

Sparse Singular Value Decomposition

Row/Column Averaging
(large clusters only $k \sim n^{1/2+\alpha}$)

SNR

$$\frac{\beta}{\sigma} \sim \sqrt{k_1 k_2 \log(n_1 n_2)}$$

Note:

These procedures do not achieve information theoretic lower bound.