# Large-Scale Learning and Inference: What We Have Learned with Markov Logic Networks


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# Markov Logic Networks

- Basic idea: Use first-order logic to compactly specify large non-i.i.d. models
- **MLN** = Set of formulas with weights
- Formula = Feature template (Vars→Objects)
- E.g., social network:

```
Smokes(x) ^ Friends(x,y) => Smokes(y)
```

$$P(x) = \frac{1}{Z} \exp\left( \sum_i \boxed{w_i} \boxed{n_i(x)} \right)$$

Weight of formula *i*

No. of true instances of formula *i* in *x*

# MLN Algorithms: The First Three Generations

| Problem | First generation | Second generation | Third generation |
|---|---|---|---|
| MAP inference | Weighted satisfiability | Lazy inference | Cutting planes |
| Marginal inference | Gibbs sampling | MC-SAT | Lifted inference |
| Weight learning | Pseudo-likelihood | Voted perceptron | Scaled conj. gradient |
| Structure learning | Inductive logic progr. | ILP + PL (etc.) | Clustering + pathfinding |

# Weighted Satisfiability

- **SAT:** Find truth assignment that makes all formulas (clauses) true
  - Huge amount of research on this problem
  - State of the art: Millions of vars/clauses in minutes
- **MaxSAT:** Make as many clauses true as possible
- **Weighted MaxSAT:** Clauses have weights; maximize satisfied weight
- MAP inference in MLNs is just weighted MaxSAT
- Best current solver: MaxWalkSAT

# Lazy Inference

- Exploits sparseness
- In most domains, most atoms (random variables) are false
  E.g: `Friends(x,y)`
- As a result, most clauses are trivially true
  E.g.: `Smokes(x) ^ Friends(x,y) => Smokes(y)`
- Materialize only atoms and clauses with non-default values
- Start with evidence and extend lazily
- Vastly reduces time and memory

# Cutting Plane Inference

- Basic idea:
  - Solve problem with small subset of constraints
  - Add violated constraints and repeat
  - Redundant constraints → Small fraction suffices
- In Markov logic, violated constraints are false clauses ( = true conjunctions)
- Use database queries to efficiently find them
- Applicable with any base solver, not just LP
- Much more scalable than Sontag & Jaakkola (2008)

# MC-SAT

- Deterministic dependences break MCMC
- In practice, even strong probabilistic ones do
- **Swendsen-Wang**:
  - Introduce aux. vars. ***u*** to represent constraints among ***x***
  - Alternately sample ***u*** | ***x*** and ***x*** | ***u***.
- But Swendsen-Wang only works for Ising models
- **MC-SAT**: Generalize S-W to arbitrary clauses
- Uses SAT solver to sample ***x*** | ***u***.
- Orders of magnitude faster than Gibbs sampling, etc.

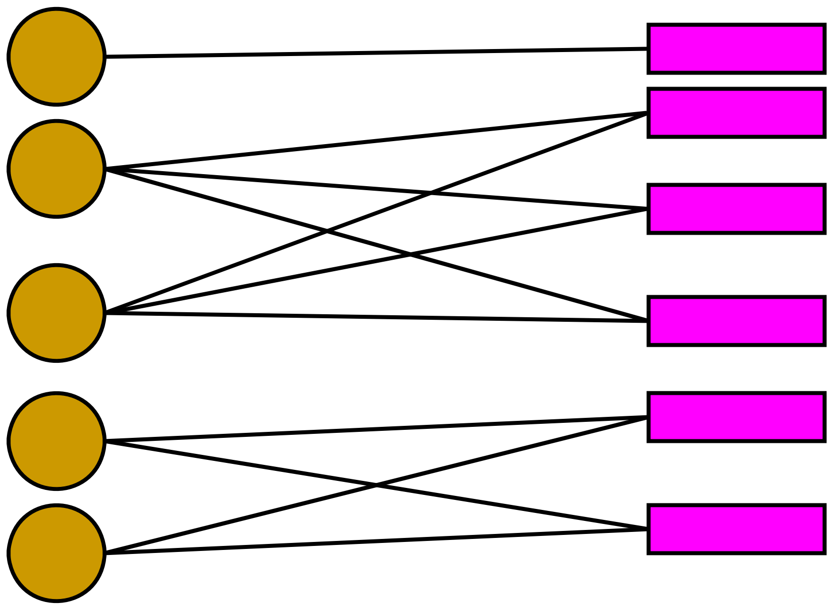
# Lifted Inference

- Consider belief propagation (BP)
- Often in large problems, many nodes are *interchangeable*:
  They send and receive the same messages throughout BP
- Basic idea: Group them into *supernodes*, forming *lifted* network
- Smaller network → Faster inference

# Belief Propagation

$$\mu_{x\to f}(x) = \prod_{h\in n(x)\backslash\{f\}} \mu_{h\to x}(x)$$

Nodes (x)



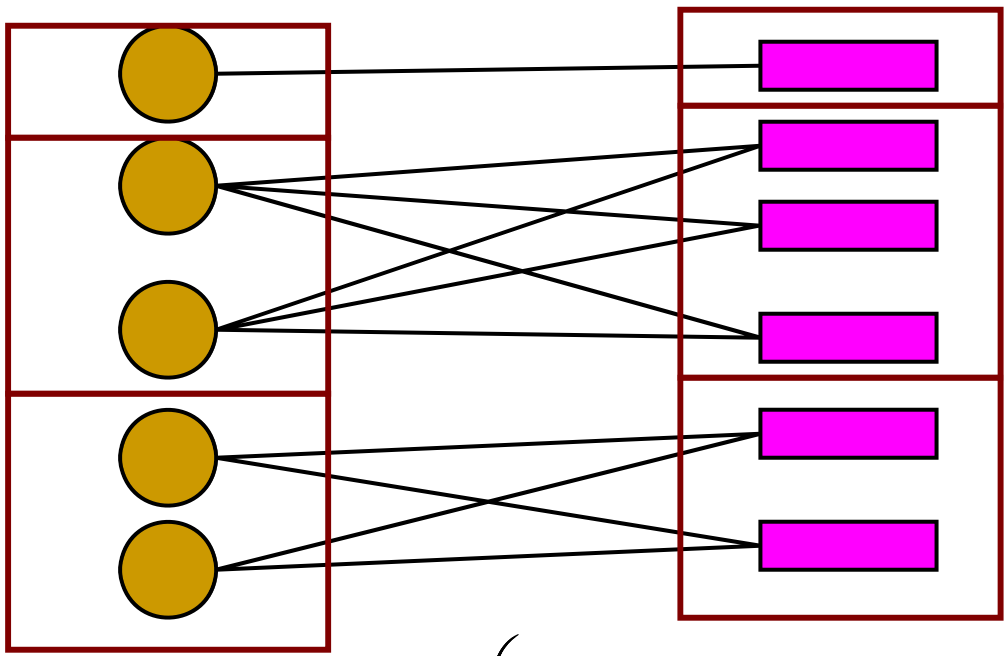
Features (f)

$$\mu_{f\to x}(x) = \sum_{\sim\{x\}} \left( e^{wf(x)} \prod_{y\in n(f)\backslash\{x\}} \mu_{y\to f}(y) \right)$$

# Lifted Belief Propagation

$$\mu_{x\to f}(x) = \prod_{h\in n(x)\setminus\{f\}} \mu_{h\to x}(x)$$

**Nodes (x)**



**Features (f)**

$$\mu_{f\to x}(x) = \sum_{\sim\{x\}} \left( e^{wf(x)} \prod_{y\in n(f)\setminus\{x\}} \mu_{y\to f}(y) \right)$$

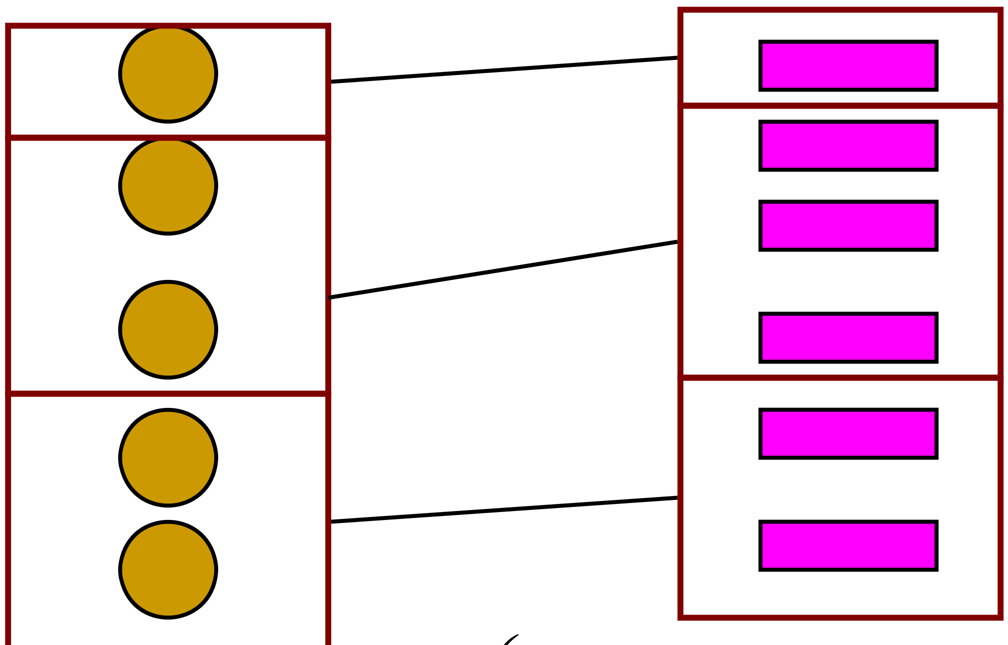
# Lifted Belief Propagation

α,β :
Functions of edge counts

α

$$\mu_{x\to f}(x) = \beta \prod_{h\in n(x)\setminus\{f\}} \mu_{h\to x}(x)$$

Nodes (x)



Features (f)

$$\mu_{f\to x}(x) = \sum_{\sim\{x\}} \left( e^{wf(x)} \prod_{y\in n(f)\setminus\{x\}} \mu_{y\to f}(y) \right)$$

# Forming the Lifted Network

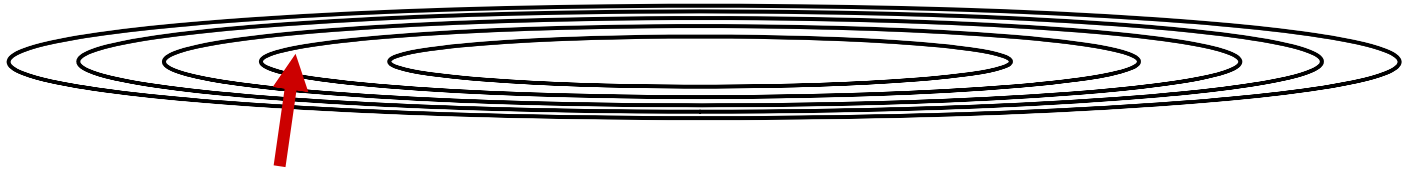
1. **Form initial supernodes**
   One per predicate and truth value
   (true, false, unknown)
2. **Form superfeatures by doing joins of their supernodes**
3. **Form supernodes by projecting superfeatures down to their predicates**
   Supernode = Groundings of a predicate with same number of projections from each superfeature
4. **Repeat until convergence**

# Weight Learning

- Pseudo-likelihood + L-BFGS is fast and robust but can give poor inference results
- Voted perceptron:
  Gradient descent + MAP inference
- Problem: Multiple modes
  - Not alleviated by contrastive divergence
  - Alleviated by MC-SAT
  - Start each MC-SAT run at previous end state

# Weight Learning (contd.)

- Problem: Extreme ill-conditioning



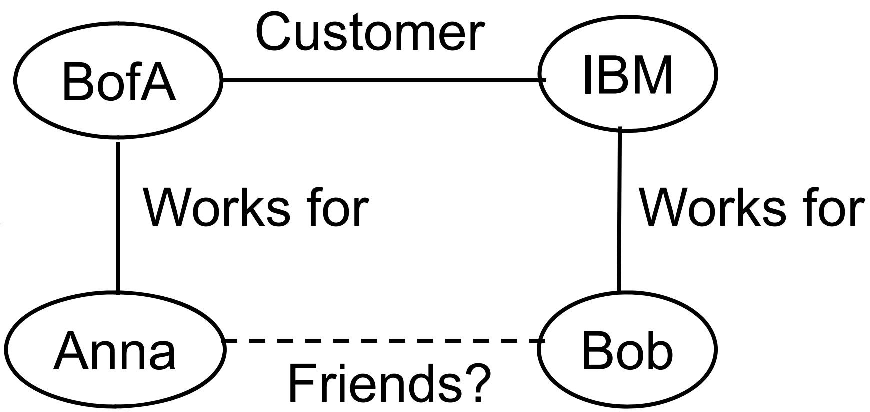
- Solvable by quasi-Newton, conjugate gradient, etc.
- But line searches require exact inference
- Stochastic gradient not applicable because data not i.i.d.
- Solution: Scaled conjugate gradient
- Use Hessian to choose step size
- Compute quadratic form inside MC-SAT
- Use inverse diagonal Hessian as preconditioner

# Structure Learning

- Standard inductive logic programming optimizes the wrong thing
- But can be used to overgenerate for L1 pruning
- Our approach:
  ILP + Pseudo-likelihood + Structure priors
- For each candidate structure change:
  Start from current weights & relax convergence
- Use subsampling to compute sufficient statistics
- Search methods: Beam, shortest-first, etc.

# Clustering + Pathfinding

- Standard search methods generate a huge number of useless candidates
- **Relational pathfinding:** Find paths of true atoms between objects & generalize them




- **Relational clustering:** Cluster objects with similar relations to similar objects
- **Clustering + Pathfinding:** Cluster objects, find paths between clusters & extract rules

# MLNs: The Next Generation

- Weighted model counting
- Compilation to arithmetic circuits/BDDs/etc.
- Approximate compilation
- Generalized, approximate, incremental lifting
- Coarse-to-fine inference and learning
- Bottom-up learning of MLN structure
- Learning tractable high-treewidth MLNs
- Learning from data streams
- Etc.

# Resources

- Open-source software/Web site: Alchemy
  - Learning and inference algorithms
  - Tutorials, manuals, etc.
  - MLNs, datasets, etc.
  - Publications

**alchemy.cs.washington.edu**

- Book: Domingos & Lowd, ***Markov Logic***, Morgan & Claypool, 2009.